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SURVEYING IMPROVED:

OR, THE

W H O L E A R T,

BOTH IN

THEORY and PRACTICE,

FULLY DEMONSTRATED.

IN FOUR PARTS.

I. ARITHMETIC, Vulgar and Decimal.

II. All DEFINITIONS, THEOREMS, and PROBLEMS; with Plain TRIGONOMETRY, and whatsoever is useful in the THEORY of SURVEYING.

III. The DESCRIPTION and USE of INSTRUMENTS, necessary in the Practice of SURVEYING; particularly *Gunter's Chain*, the *Surveying-Wheel*, the *Theodolite*, the *Sector*, the *Circumferentor*, the *Protractor*, the *Semicircle*, and the *Plain-Table*; which last, of all others, is esteemed universally useful in every Part of this Art.

IV. How to Measure, Cast up, Plot, and Divide Parcels of LAND, of what Denomination soever, *viz.* Meadows, Pastures, Fields, Forests, Woods, Commons, Lordships, Manors, &c. by any one or more of the abovesaid Instruments. Also, to take inaccessible Heights and Distances, with surveying Compasses, Roads, Rivers, &c. Likewise to reduce a Plan to a Prospect, and to correct any Survey by Astronomical Calculation: With DIRECTIONS for making transparent COLOURS for Maps, &c.

With an APPENDIX concerning LEVELLING and CONVEYING of WATER to any possible Place assigned.

The SIXTH EDITION, with ADDITIONS; illustrated with Cuts, &c. and also accurately revised, improved, and augmented.

By HENRY WILSON, Mathematician.

To which is now added,

GEODOESIA ACCURATA:

OR,

SURVEYING made EASY by the CHAIN only.

S H E W I N G,

How to Measure, Plot, Divide, and Delineate any Parcel of LAND, without the Trouble and Charge of any other Instrument but the CHAIN; With proper Directions for Reducing, Mapping, Colouring, and finding the Content, after an accurate and expeditious Manner.

A L S O,

A NEW ESSAY UPON SOLIDS.

Shewing how the Dimensions of Timber-Trees, &c. both standing and lying, may be exactly taken, and readily measured various Ways, *viz.* Vulgarly, Decimally, Duodecimally, Instrumentally, and Tabularly.

By WILLIAM HUME, Philomath.

*Judicium populi nunquam contempseris unius,
Ne nulli placeas, dum vis contemnere multos.*

CATO.

L O N D O N,

Printed for J. and F. RIVINGTON, L. HAWES and Co. W. JOHNSTON,
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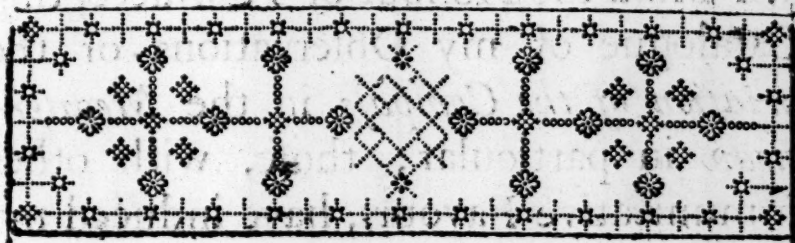
INVESTIGATING

W. H. O. L. A. R. T.

THEORY AND PRACTICE

OF THE ART





To the Celebrated

Dr. EDMUND HALLEY,

Regius Professor of Astronomy at

GREENWICH.

S I R,

ALTHOUGH your great Acquirements in the more sublime Parts of the Mathematics, have advanced you to a Station of much more Honour, and less Toil, than that of a *Practical Surveyor*; yet, that is so far from discouraging me, that Your good Intentions for promoting useful Improvements in general, and the Satisfaction you were pleased to express,

DEDICATION.

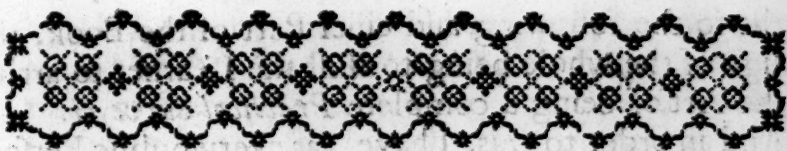
when I had the Honour of Your accepting a Catalogue of my Observations of the *Variation of the Compass* in the *Mediterranean* in particular; those, with other communicative Favours, have induced me to presume upon Your Acceptance of this Dedication of my SURVEYING IMPROVED from,

S I R,

Your most humble,

obliged Servant,

HEN. WILSON.



T H E

P R E F A C E.

IF we judge of the Necessity of writing new Books by the Number already written upon the same Subject, it would render it needless to publish a Book of *Surveying*, in a Year that has been so productive of new Authors and Books of that kind; but when we find so large a Part of one of our New Treatises filled up with a literal Transcript of Mr. *Cunn's* Appendix to *Leybourn's Surveying*, so that many, who thought they had got something new, found the same thing over again *verbatim* that they had before: Such Books seem to be calculated to serve as an Advertisement to promote the Maker's Instruments, rather than for the Information and Advantage of the Public, and rather prompted me to, than deterred me from, publishing the following Treatise.

I shall not enumerate the Particulars contained in this Book, the Title, the Contents, and the Book itself, being the most proper to be consulted upon that Head, but presume there is nothing wanting in it that could have been couched in so small a Volume, to render it a complete Manual of Surveying, both in Theory and Practice.

Indeed, I own, let a Treatise be never so familiarly adapted to the Learner's Capacity, yet Practice and Experience is the Life of Improvement; but such sound Notions and Demonstrations, as are previously necessary to the Theory of any Science, are the best Preparative to, and the most inseparable Companion of Practice.

For this Reason, I have introduced this Work with such Definitions, Theorems, Problems, &c. as give

A 3

Light

The P R E F A C E.

Light to the following and chief Part of the Book, the Design of the whole being to assist the Learner to arrive at the Art of being a complete *Practical Surveyor*.

And in order to this, I have, (as every public Person, that aims at Improvements, ought to do) consulted the best Authors; and, where I have found Deficiencies, supplied them; and, where I found Errors that require a Correction, or Caution, not there expressed, I have given a Demonstration of the Truth itself, and applied those Demonstrations to Practice in actual Surveying.

But though this Book be a complete Treatise of *Surveying*, both in Theory and Practice, yet the following Abstract of *Practical Surveying* cannot be amiss.

County Surveying, especially that of Roads and Rivers, has some Resemblance with Practical Navigation, or the conducting a Ship upon the vast Ocean, *one being a Traverse by Sea, and the other by Land*, yet with this Difference, that in a Traverse by Sea, we only require, by the Help of the various Courses described in twenty-four Hours, to determine the Point the Ship is in each Day at Noon, with respect to Longitude and Latitude, &c. except coming near the Land, or some Danger, cause a more frequent, or perhaps constant Look-out: But, in surveying Roads, Rivers, &c. and in making Maps or Draughts of such Surveys, we are obliged to describe every Winding, Curving, or Angle, or whatsoever else occurs, as it appears upon the Spot, and to a Scale, bearing every-where the same Proportion to the Space to be represented.

Besides, the carefully taking of Distances and Angles, or Bearings, where the Roads are straight, and the Ground plain between the Angles, which but rarely happens: Yet more frequently Irregularities occur, and are to be accounted for; as,

I. When a Road goes winding or curving between Station and Station, it is to be accounted for by the Directions in *Page 144*.

II. When Ascents and Descents fall in the Way, as is common to the Roads, it is to be reduced to a horizontal Plane by *Chap. VI. Page 145*.

III. Left

The P R E F A C E.

III. Lest any Error should be contracted in the frequent taking of Angles, where you are obliged to take Stations at short Distances, always remark the Course between every two Stations, by the Compass as well as the Angle that one stationary Line makes with the other.

IV. Lest any Error should yet be contracted by the Compass not being well touched, or some Iron or the like affecting it, &c. correct the whole, as often as Opportunity offers and Occasion requires, as you are taught to do it by an Astronomical Observation, *Chap. XV. Page 249.*

V. If some Church, or Gentleman's Seat, &c. appear at some Distance from the Road, take a convenient Opportunity to observe what Angle or Line drawn from your Station to the said Church, &c. makes with the Road; and when you have gone a competent Distance from the said Station, so as to have altered considerably the Position of the said Church, House, &c. observe again what Angle it makes with the Road; and setting down carefully the two Angles, and the Distance between your two Stations, you may thereby project two Lines, whose Intersection is the Place of the Object observed, and gives its true Bearing from either Station, and its Distance from the Road to any Scale that you project from, although you come not near the Object.

VI. When three, four, or more Objects, as Villages, Churches, Castles, &c. appear at some considerable Distance from one Station upon the Road, and there is another upon the Road to be seen at a Distance from the former, and from whence you can see also all the Objects before mentioned, be very careful in allowing for Ascents and Descents, and Curving of the Road between the two Stations, as before directed, and thereby gain the true horizontal Distance between them; and you may thereby determine the true horizontal Distance of each of the other from either Station, and from one another, by the Rules laid down, *Chap. III. Sect. 3. Page 125.*

All these Helps ought carefully to be made use of, as Opportunity offers; for although they do not save the Labour of going to the said Villages, Churches, &c. yet
where

THE PREFACE.

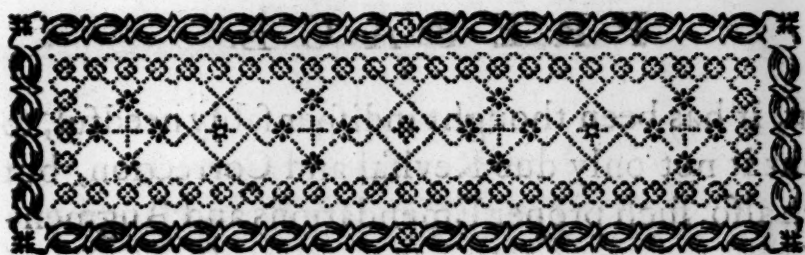
where Roads are crooked and uneven, or attended with other Difficulties, they are great Helps towards correcting the true horizontal Distances, according to which all Maps ought to be laid down, although the Country be very irregular.

But besides the Survey, and from thence a true Map of the Roads, Rivers, Towns, &c. in their true Form and Situation, it would add very much to the Value of the Performance, and the Honour of the Performer, to keep a curious Journal of all his Perambulations, and therein observe what Rarities, Antiquities, or other remarkable Things he meets with; also of the Product of the Country, or the several Parts of it, and how manufactured; together with their inland Trade; and (if a maritime Country) of their Exports and Imports; all which will be looked upon as very entertaining, as well, or, to some Gentlemen, more than the Map itself; but it is both together that makes a Survey complete.

This Compendium of the Whole I thought more helpful to the Memory of young Practitioners, than being obliged to read the whole Book, or to be at the Pains of tumbling it, to find particular Instructions, whereby to be completed in the Art of PRACTICAL SURVEYING, which with a Presumption that these my Labours will be instrumental to, are published to the World by,

A true Lover of Improvements,

HENRY WILSON.



THE
CORRECTOR'S
PREFACE.

Courteous Reader,

AS the Author of this Treatise, intituled, *Surveying Improved, &c.* has already met with the Approbation of the Public, as well as the Experience of the accomplished Surveyor; so all further Encomiums herein must be esteemed needless and impertinent; seeing the Author's Character can receive no Addition by any such Embellishment: Only I shall observe, that as this Essay, in former Impressions, has contracted several Press-errors, particularly in the Table of Logarithms, Sines, Tangents, &c. so, upon this Edition,

P R E F A C E.

tion, it has been thought indispensably needful, to grant it not only due Revival and Correction, but even also, such proper Emendations and Augmentations, as may render it every way perfect and complete, as well as conducive to the public Benefit and Satisfaction; which is the zealous Endeavour of,

Your Humble Servant,

London, June 30,
1740.

W. H.

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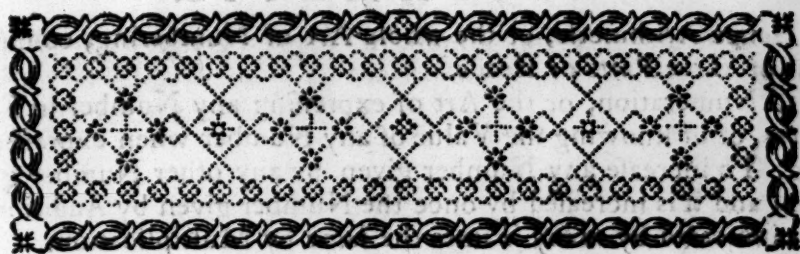
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SURVEYING

IMPROVED.

PART I.

Of ARITHMETIC.



ARITHMETIC is the Art of Numbering, and may be said to consist of two Parts, viz. Whole Numbers and Fractions; for by one or both of these, are all Numbers whatsoever expressed. Altho' there are many Rules (commonly so called) in Arithmetic, yet they are all performed and expressed by the first Five, to wit, Numeration, Addition, Subtraction, Multiplication, and Division; all the rest, tho' called different Rules, may more properly be said to be some or all the abovementioned Rules differently applied.

These five Rules may again be reduced to three Heads; for Multiplication is the same in effect as Addition, but more compendious; it increasing a Number at one Operation, which Addition performs at many.

Division also serves instead of many Subtractions, and takes away from a Number at one Operation, what Subtraction would take away at many.

B

Hence

Hence Arithmetic, or the whole Art of Numbering, consists but of three Parts or Heads.

1. Numeration, or the Art of expressing any Number in Figures, or of knowing the Value of any Number when expressed.

2. To increase any Number given by any other Number given; and it is increased by once the Number given by Addition, but increased by many Times the same by Multiplication.

3. To diminish or lessen a Number given; and this is done at once by Subtraction, but lessened by many Times the Number given by Division.

CHAP. I.

Of NUMERATION.

IN order more compendiously to express any Number, the Ancients invented ten Characters, nine of which are called Digits, or Figures, and the other is called a Cypher, as below.

One, two, three, four, five, six, seven, eight, nine, cypher.
 1 2 3 4 5 6 7 8 9 0

Any one of the Figures single, signifies no more than the Number set over it; as 3 signifies Three, 7 signifies Seven, &c. and 0 alone signifies nothing.

But by the various disposing of these Figures, or placing more or less of them in a Row, as the following Rules will direct, all Numbers, how great or little soever, may be expressed.

In any Number, of how many Figures soever it consists, the first (towards the Right-hand) signifies no more than its own simple Value; as 2 two, 5 five, and the like.

The figure next to that signifies ten times its own Value; so three on the Left-hand of another Figure signifies ten times 3, viz. thirty. As for Instance; let the two Figures stand thus, 37, the 3 on the Left-hand signifies ten times three, that is thirty, and the 7 being on the Right-Hand, signifies only seven, so that both together so placed stand for thirty-seven; but if they had stood thus (73) they had signified seventy-three (vulgarly, three score and thirteen.)

In all Rows of Figures, consisting of how many soever, (beginning at the Right-Hand, and reckoning backward toward the Left) the second Place is ten times the Value of the first; the third

third is ten times the Value of the second; the fourth of the third, and so on through all the Figures; and hence the Places are distinguished as follows:

Hund. of Mill.	Tens of Mill.	Millions.	Hund. of Thouf.	Tens of Thouf.	Thousands.	Hundreds.	Tens.	Units.
3	3	3	3	3	3	3	3	3

Begin at the Right, and call that Units; the second Place is Tens; the third Ten times Ten, or Hundreds; the fourth Place is Ten times Hundreds, that is, Thousands, &c. to Tens of Thousands, Hundreds of Thousands, Millions, Tens of Millions, Hundreds of Millions, which is the last or highest Place in this Row.

The first Figure towards the Right-hand signifies only Three, the second signifies Thirty; the third Three hundred; the fourth Three thousand; the fifth Thirty Thousand; the sixth Three Hundred Thousand; &c. So that the whole nine Figures together, 333 333 333, signify, Three hundred and Thirty-three Millions, Three Hundred and Thirty-three Thousand, Three Hundred and Thirty-three.

For the first to the Right-hand signifies Three; the first Two is Thirty-three; the first Three is Three Hundred, Thirty-three; the first Four is Three Thousand, Three Hundred, Thirty-three; the first five Figures are Thirty-three Thousand, Three Hundred and Thirty-three; and so on to the Ninth, as above.

But if any Cyphers happen, they are of no Use, but only to advance the Figures, on the Left-Hand of them, a Place or Places higher, according to the Number of Cyphers; and hence a Cypher on the Left-hand of a whole Number adds nothing to its Value, because there is no Figure on the Left-hand to be advanced by it.

If there be a Cypher on the Right-hand of a Figure, it puts that Figure a Place higher than it was without the Cypher, and makes its Value ten times as much; thus 4 signifies but four; but 40 signifies forty, because the Cypher on the Right-hand puts the four into the Place of Tens.

Thus 400 is Four Hundred, 4000 is Four Thousand, &c.

And when a Cypher falls in the Middle of a Number, it is no Number of itself, and only advances the Figures on the Left-

hand of it; thus 404 signifies Four Hundred and Four, as in the following Examples.

505 Five Hundred and Five.
 550 Five Hundred and Fifty.
 055 Fifty-five (the Cypher here being of no Use.)
 6006 Six Thousand and Six.
 6600 Six Thousand Six Hundred.
 6060 Six Thousand and Sixty.
 7000809 Seven Millions, Eight Hundred and Nine.
 607000089 Six Hundred and seven Millions, and Eighty-nine.

Although real Numbers in Business seldom amount to above nine or ten Figures, yet where they do go higher, as to Millions of Millions, and Millions of Millions of Millions, &c. it is best, instead of confusing the Memory with the Repetition of the Word *Millions* so often, to reduce it to one Word; as instead of Millions of Millions, say *Billions*; and where the Word should be three times repeated, as Millions of Millions of Millions, say, *Trillions*; and for Millions of Millions of Millions of Millions, viz. four Times repeated, say, *Quadrillions*; and so to *Quintillions*, &c. as in the following Table.

NUMERATION TABLE.

- 1 Hundreds of Thousands of *Trillions*.
- 2 Tens of Thousands of *Trillions*.
- 3 Thousands of *Trillions*.
- 4 Hundreds of *Trillions*.
- 5 Tens of *Trillions*.
- 4 *Trillions*.
- 3 Hundreds of Thousands of *Billions*.
- 2 Tens of Thousands of *Billions*.
- 3 Thousands of *Billions*.
- 4 Hundreds of *Billions*.
- 5 Tens of *Billions*.
- 6 *Billions*.
- 7 Hundreds of Thousands of *Millions*.
- 8 Tens of Thousands of *Millions*.
- 9 Thousands of *Millions*.
- 8 Hundreds of *Millions*.
- 7 Tens of *Millions*.
- 6 *Millions*.
- 5 Hundreds of Thousands.

Ten

- 4 Tens of Thousands.
- 3 Thousands.
- 2 Hundreds.
- 1 Tens.
- 0 Units.

These Figures, placed as above, may be read downwards, thus:

One Hundred and Twenty-three Thousand, Four Hundred and Fifty-four *Trillions*, Three Hundred and Twenty-three Thousand, Four Hundred and Fifty-six *Billions*, Seven Hundred and Eighty-nine Thousand, Eight Hundred and Seventy-six Millions, Five Hundred and Forty-three Thousand, Two Hundred and Ten.

But such large Numbers being more for Curiosity than Business, I shall not enlarge upon them, but confine myself to what is most useful for introducing the Learner to the Practice of Surveying.

CHAP. II.

TABLES of REDUCTION.

ALTHOUGH it is not common with Authors to place the Reduction-Table before Addition, yet it is most convenient, because it is impossible to perform Addition of several Denominations, without knowing how many of one Denomination makes, or is contained in another; as how many Inches make one Foot, or how many Rood make an Acre, &c.

Of Sterling or English Coin.

4 Farthings	}	make one	{	Penny
12 Pence				Shilling
20 Shillings				Pound.

Of Long Measure.

3 Barley Corns	}	make one	{	Inch
12 Inches				Foot
3 Feet				Yard
2 Yards				Fathom
5½ Yards or 16½ Feet				Perch
40 Perches				Furlong
8 Furlongs				Mile
3 Miles				League.

Of

Of ARITHMETIC.

Of Cloth Measure.

4 Nails	}	make one	{	Quarter
4 Quarters				Yard
5 Quarters				Ell <i>English</i>
3 Quarters				Ell <i>Flemish</i> .

Of Troy Weight.

24 Grains	}	make one	{	Penny-weight
20 Penny-weight				Ounce
12 Ounces				Pound.

Of Averdupoise Weight.

16 Drams	}	make one	{	Ounce
16 Ounces				Pound
14 Pound				Stone
2 Stone				Quarter of a Hundred
4 Quarters				Hundred Weight
20 Hundred				Tun.

Note, It is not common to regard Stones in this Weight, but Hundreds, Quarters, and Pounds, and then 28 Pounds make one Quarter of a Hundred, &c.

Of Apothecaries Weight.

24 Grains	}	make one	{	Scruple
3 Scruples				Dram
8 Drams				Ounce
12 Ounces				Pound.

Of Land Measure.

40 Perches	}	make one	{	Rood
4 Rood				Acre
160 Perches				Acre.

100000 square Links of *Gunter's Chain*, make one Acre.

Of Dry Measure.

2 Pints	}	make one	{	Quart
2 Quarts				Pottle
2 Pottles				Gallon
2 Gallons				Peck
4 Pecks				Bushel Land Measure
5 Pecks				Bushel Water Measure
4 Bushels				Coomb
8 Bushels, or 2 Coombs				Quarter
4 Quarters				Chalder
5 Quarters				Tun, or Wey.

Of

Of ARITHMETIC.

Of Liquid Measure.

2 Pints	}	make one	Quart
2 Quarts			Pottle
2 Pottles			Gallon
8 Gallons			Firkin of Ale, Soap, or Herrings
9 Gallons			Firkin of Beer
2 Firkins			Kilderkin
2 Kilderkins			Barrel
42 Gallons			Tierce
63 Gallons			Hogshead
2 Hogsheads			Pipe
2 Pipes			Butt.

Of Time.

60 Seconds	}	make one	Minute
60 Minutes			Hour
24 Hours			Day natural
7 Days			Week
4 Weeks			Month
13 Months and a Day			Year.

CHAP. III.

Of ADDITION.

ADDITION teacheth how to bring two or more Numbers or Quantities into one, and to know the Value or Sum of the whole.

If you are to add together several Sums or Numbers of one Denomination, as Men, Bricks, or the like; or of the highest of several Denominations, as Pounds (in Money) Acres of Land, &c. set down the Numbers one under another, and if some of the Numbers consist of more Places than others, keep the Figures next the Right-Hand one exactly under another, and what Number of Figures is in one Sum more than the other, let that possess so many Places towards the Left-hand more than the other.

Then begin at the Right-Hand at the Bottom, and go upwards, adding all the Figures together that are one over another, and for every Ten carry one to the next Place, and set down the

Remainder, or what is above 10, 20, 30, &c. under the Sum, and proceed so through the whole, and you will have the Sum total required.

Example 1. A Gentleman has three Bags of Money, as follows :

In the first Bag 27 Pounds, in the second 65 Pounds, and in the third 94 Pounds; how much is in all the three Bags?

Set them down one under another, and draw a Line, as you see.

Then begin at the bottom Figure on the Right-Hand, and go upwards, saying, 4 and 5 is 9, and 7 is 16, set 6 under the Line under 4, and for the 10 carry one, saying, one that I carry and 9 is 10, and 6 is 16, and 2 is 18, which being the last Figures you have to add, set down 18 under the Line before the 6, then you have under the Line 186, which is the Number of Pounds contained in the three Bags.

27
65
94
—
186

Example 2. A Farmer hath four Fields of Meadow-Ground, one of them contains 7 Acres, another 23 Acres, another 47 Acres, and the fourth 132 Acres; how many Acres do they all contain?

Set them down as before directed, keeping the Right-Hand Figures over one another, and they will stand thus :

Then say 2 and 7 is 9, and 3 is 12, and 7 is 19, set 9 under the Line (under that Row of Figures next the Right-Hand where it was taken from) and carry one, saying, 1 and 3 is 4, and 4 is 8, and 2 is 10, set down 0 under the Line, and carry 1, then one that I carry and one is 2, set down 2 under the 1, then have you under the Line 209, the Number of Acres in all the four Fields.

7
23
47
132
—
209

Example 3. A Gentleman hath five Parcels of Bricks: In one Parcel there is 9246, and in another Parcel is 5724, in a third Parcel is 947, in another Parcel is 762, and in the last Parcel is only 96; how many is there in all the Parcels?

It is the most convenient for the Eye, (tho' of no other Consequence) to set down the least Numbers uppermost, which when done, proceed as in the two foregoing Examples, and you will find the five Parcels amounted to 16775 Bricks.

But if you have so many Numbers to add, that what you have to carry is like to overcharge the Memory,

96
762
947
5724
9246
—
16775
you

you may put down a Prick or Dot at every Figure where you have 10 or more, and carry the odds up to the next Figure, 'till you get through the whole Row, and then counting your Dots carry so many to the next Row as you have Dots, and so proceed through all the Rows.

EXAMPLES.

I would add the Figures in the Margin into one Sum.

Begin as before and say, 4 and 8 is 12, make a Prick or Dot at 8 and bear two in Mind, saying 2 and 6 is 8 and 8 is 16, make a Dot at 8 and carry the 6, saying, 6
and 7 is 13, Dot at 7 and carry three, then say 3 and
2 is 5 and 6 is 11, make a Dot at 6 and carry 1, say-
ing, 1 and 8 is 9, which being less than 10 set down 9
at the Bottom under the Line.

Then count the Dots in that Row which you have
finished, and you find four of them, which carry to the
next Row, saying, 4 and 7 is 11, Dot and carry 1,
then 1 and 5 is 6 and 6 is 12, Dot and carry 2, then
2 and 9 is 11, Dot and carry 1, again 1 and 5 is 6
and 8 is 14, Dot at 8 and carry 4, saying 4 and 4 is
8 and 7 is 15, make a Dot at 7 and set 5 under the Line.

Then count the Dots in that Line, which you find to be five;
therefore set 5 also under the Line, and the Number 559 is the
Sum of all the other Numbers.

Addition of several Denominations.

Set down all the same Denominations under each other: In Pounds, Shillings, and Pence, set Pounds under Pounds, Shillings under Shillings, and Pence under Pence, and begin at the least Denomination (Pence) and add them together, as before directed, only instead of carrying 1 at every 10, you must carry 1 at every such Number as that lesser Denomination is contained in the next greater; as in Pounds, Shillings, and Pence, you must at every 12 in the Pence carry 1 to the Place of Shillings, because 12 Pence is one Shilling, also at every 20 in the Place of Shillings you must carry one to the Place of Pounds, because 20 Shillings is one Pound, &c.

Example. Three Men owe me the following Sums, I desire to know how much it is all.

Begin at the Bottom at Pence, and say, 10 and 3 is 13, and 9 is 22, which being 1 Shilling and 10 Pence, set 10 under Pence and carry one to the Shillings, saying, 1 and 18 is 19 and 9 is

9 is 28 and 17 is 45, which being 2 Pound 5 Shillings, set 5 under Shillings, and carry 2 to the Place of Pounds, saying 2 and 4 is 6 and 2 is 8 and 5 is 13, set down 3 and carry 1, then 1 and 8 is 9 and 6 is 15 and 3 is 18, which being the last Figure, set down 18 before the 3, and then 183 Pounds, 5 Shillings, and 10 Pence is the Sum of the three Debts required to be known.

l.	s.	d.
35	17	9
62	9	3
84	18	10
183	5	10

But in this Case, as before, if the Sum consists of so many Articles that it is over-burdenfome to the Memory to remember what you have to carry to the next Denomination, you may make Dots as you go along, as before directed.

Example. A Farmer had 7 Fields, containing the several Numbers of Acres, Roods, and Perches as follows, 'tis desired to know how much they all contain.

Note, You may see in the Table of Reduction beforegoing, that 40 Perches make one Rood, and 4 Rood make 1 Acre.

Begin at the Bottom of the least Denomination, saying, 10 and 15 is 25 and 37 is 62, which being 1 Rood and 22 Perches, make a Dot at 37 and proceed, saying, 22 and 29 is 51, viz. 1 Rood and 11 Perches, make a Dot at 29 and carry 11, saying, 11 and 6 is 17 and 20 is 37 and 27 is 64, viz. 1 Rood and 24 Perches, make a Dot at 27 and set down 24 under the Line.

Then counting your Dots, you find three, viz. 3 Rood, which carry to the Place of Roods, saying, 3 and 3 is 6, viz. 1 Acre and 2 Rood; make a Dot at the lowest 3 and carry 2, saying, 2 and 3 is 5, that is, one Acre and one Rood, make a Dot at 3 and carry 1, saying, 1 and 1 is 2 and 2 is 4, to wit, 4 Rood or one Acre; make a Dot at 2, and having nothing to carry, begin at the next 2, saying, 2 and 3 is 5, make a Dot at 3 and set 1 under the Line.

Then counting the Dots under Roods you find 4, therefore say 4 and 8 is 12 and 2 is 14 and 4 is 18 and 9 is 27 and 8 is 35 and 2 is 37 and 5 is 42, set down 42 under the Column of Acres, and then you have the Content or Quantity of all the 7 Fields, viz. 42 Acres 1 Rood and 24 Perches,

A.	R.	P.
5	3	27
2	2	20
8	2	6
9	1	29
4	3	37
2	0	15
8	3	10
42	1	24

It will be needless to enumerate Questions in Addition, the Method of Working being the same in all, if you do but take Care to consider how many of one Denomination make one of another

another Denomination, and in this the Reduction Tables prefixed will fully instruct you.

C H A P. IV.

Of SUBTRACTION.

SUBTRACTION teacheth how to find the Difference between any two Numbers, or to take the lesser Number from the greater and to tell what remains.

This may also be distinguished into two sorts ; of one Denomination, or of many.

To subtract a lesser Number from a greater of one Denomination, place the lesser under the greater so as (whether they consist of an equal Number of Figures or no it matters not, but) that the first figure towards the Right-Hand in one Number may be just under the first Figure to the Right-Hand in the other, and then subtract the Figures one by one, the undermost from the uppermost, and set down the Remainder, which expressed according to the Rules of Numeration, gives the Number that remains.

Example. A General had an Army of 45632 Soldiers, and having drawn off 2131, I desire to know how many remained.

Set the Numbers one under the other, as above directed, and draw a Line under them as in the Margin.

Then beginning at the Right-Hand say 1 from 2 and there remains 1, set that 1 that remains under the Line, and proceed, saying, 3 from 3 and there remains nothing, set down 0 under 3, then 1 from 6 and 5 remains, set 5 under 1, then 2 from 5 and there remains 3, set 3 under 2, and lastly, as there is no Figure under the 4, take nothing from 4 and there remains 4, so that there were 43501 Men left.

45632
2131

43501

But if (as it frequently happens) some or all of the lowest Figures be greater than its correspondent Figure from whence it should be taken, except the first Figure on the Left-Hand, which must be less than the upper Line ; in this Case you must borrow 10 of the next Figure towards the Left-Hand, and pay 1 again for it when you come to that Figure, &c.

Example.

Example. Suppose out of the aforesaid Army of 45632 Men there were 9999 to be drawn, how many would be left?

Place them as before directed, and begin at the Right-Hand, saying, 9 from 2 I cannot, but (borrowing 10) 9 from 12 and there remains 3, which 3 set under the first 9 towards the Right-Hand; then proceed, saying, 1 that I borrowed and 9 is 10, but 10 from 3 I cannot, therefore (again borrowing 10) I say 10 from 13 there remains 3, which set down in the second Place from the Right-Hand; then 1 that I borrowed and 9 is 10 from 6 I cannot, but (borrowing 10) 10 from 16 there remains 6; again, 1 that I borrowed and 9 is 10 from 5 I cannot, but 10 from 15 and there remains 5; and lastly, there being no figure under 4, I say, 1 that I borrowed from 4 and there remains 3, so that the Remainder of the Men is 35633.

$$\begin{array}{r} 45632 \\ 9999 \\ \hline 35633 \end{array}$$

The Reason of paying but 1 when we borrow 10, is because it is paid in a Place higher, where every figure has 10 times the Value of what they had in the Place where it was borrowed; thus, if I borrow 10 in the Place of Hundreds, paying 1 in the Place of Thousands is equal to it, for 1 time 1000 is equal to 10 times 100.

The Proof of Subtraction is by Addition; for if you add the lesser Number and the Remainder together, if their Sum be equal to the greater Number, the Operation is right, otherwise not.

In the last Question add the lesser Number 9999 to the Remainder 35633, by the common Rules given in Addition; and because the Sum is 45632, equal to the uppermost and greater Number, we are sure the Work is right.

$$\begin{array}{r} 45632 \\ 9999 \\ \hline 35633 \\ \hline 45632 \end{array}$$

Subtraction of several Denominations.

Here, whether your Question be in Pounds, Shillings, and Pence, or in Acres, Roods, and Perches, or whatever else, place every Denomination in the lesser Number under the same Denomination in the greater, as Pence under Pence, Pounds under Pounds, Perches under Perches, &c. and then proceed to take the lesser Number out of the greater of the same Denomination, and set the Remainder under the Line drawn below for that Purpose.

Example.

Example.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Received - - - - -	96	15	8
Paid out of it - - - - -	34	11	3
Left behind - - - - -	62	4	5
Proof - - - - -	96	15	8

Place the two Sums as you see in the uppermost Lines, and beginning at Pence, say, 3 from 8 and there remains 5, which set down under the Line under Pence, and proceed to Shillings, saying, 11 from 15 there rests 4, which set under Shillings, and then come to Pounds, saying, 4 from 6 and there remains 2, which set under 4; then 3 from 9, there remains 6, which having set down as you see, there is left 62 Pounds, 4 Shillings, and 5 Pence.

For Proof add the Remainder 62 Pounds 4 Shillings and 5 Pence, to the lesser Number 34 Pounds 11 Shillings 3 Pence, and the Sum being 96 Pounds 15 Shillings and 8 Pence, agreeing with the uppermost Number, proves the Work to be right.

But as in Subtraction of one Denomination, so here, when the lowest Figures are bigger than their correspondent uppermost Figures, you must borrow 1 from the next denomination; as, if you are in Pence, you must borrow 12, because 12 Pence is a Shilling, and pay 1 for it in the Place of Shillings; or in Shillings, you must borrow 20, and pay 1 in the Place of Pounds, &c.

Example. If I borrow 100 Pounds, and pay again 76 : 13 : 5, how much is to pay?

Place them as in the Margin.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Begin at Pence, and say, 5 from 0 I cannot, but I borrow 12 Pence, (which makes 1 Shilling) then 5 from 12 there remains 7; and so I proceed to Shillings, saying, 1 that I borrowed and 13 is 14, but 14 from 0 I cannot, but I borrow 20 Shillings (one Pound) saying, 14 from 20 there remains 6, which put under the Line in the Place of Shillings, and proceed to Pounds, saying, 1 that I borrowed and 6 is 7 from 0 I cannot, but 7 from 10 there rests 3, which place under 6, and then say, 1 that I borrowed and 7 is 8 from 10 (the two last Figures of the uppermost Sum) there remains 2, which	100	0	0
	76	13	5
	23	6	7

being

being set before the 3, makes the Remainder in all 23 Pounds, 6 Shillings, 7 Pence.

The Proof is the same as the former.

Example 2. There is a certain Manor containing just 800 Acres, and it is agreed to set off from it 327 Acres, 2 Roods, and 33 Perches; how much will the other Part contain after that is set off?

To solve this, you must mind in Perches, to borrow 40, because 40 Perches make a Rood, and in Roods to borrow 4, because 4 Roods make an Acre, and then the Work will stand thus with its Proof.

<i>Acres.</i>	<i>Roods.</i>	<i>Perches.</i>	Begin at Perches, 33 from 00 I cannot, (but borrowing 40) 33 from 40 there remains 7, then 1 that I borrowed and 2 is 3 from 0 I cannot, but borrowing 4, 3 from 4 there remains 1; again, 1 that I borrow and 7 is 8 from 10 there rests 2, and 1 that I borrowed and 2 is 3 from 10 there rests 7; and lastly, 1 that I borrowed and 3 is 4 from 8 there remains 4: So that there remains 472 Acres, 1 Rood, 7 Perches, out of 800 Acres, after 327 Acres, 2 Roods and 33 Perches has been taken off.
800	: 00	: 00	
327	: 2	: 33	
472	: 1	: 7	
800	: 0	: 0	

CHAP. V.

Of MULTIPLICATION.

MULTIPLICATION, as I hinted before, serves instead of many *Additions*, and shews at one Operation, how much any Number amounts to when it is any Number of Times repeated.

The Operation is composed of three Numbers, *viz.* the Multiplicand, or Number to be multiplied; the Multiplier, or Number to multiply by; and the Product or Number produced by the Multiplication; as if I say 3 times 4 is 12, here 4 is the Multiplicand, 3 the Multiplier, and 12 the Product.

The Multiplicand and Multiplier are always given, and the Product is thereby required to be found.

It is most proper to make the greater of the two given Numbers the Multiplicand ; nevertheless, whichsoever be made Multiplicand, it amounts to the same in effect, and the Product will be the same.

In order to the more expeditiously performing Multiplication, it is necessary that the Substance of the following Table be got by heart.

MULTIPLICATION TABLE.

2 times 2 is 4	5 times 5 is 25
2 times 3 is 6	5 times 6 is 30
2 times 4 is 8	5 times 7 is 35
2 times 5 is 10	5 times 8 is 40
2 times 6 is 12	5 times 9 is 45
2 times 7 is 14	
2 times 8 is 16	6 times 6 is 36
2 times 9 is 18	6 times 7 is 42
	6 times 8 is 48
3 times 3 is 9	6 times 9 is 54
3 times 4 is 12	
3 times 5 is 15	7 times 7 is 49
3 times 6 is 18	7 times 8 is 56
3 times 7 is 21	7 times 9 is 63
3 times 8 is 24	
3 times 9 is 27	8 times 8 is 64
	8 times 9 is 72
4 times 4 is 16	
4 times 5 is 20	9 times 9 is 81
4 times 6 is 24	
4 times 7 is 28	
4 times 8 is 32	
4 times 9 is 36	

The Multiplication Table is composd in such a plain Method, that it admits of no Explanation, it being expressd in Words that 3 times 4 is 12, and 6 times 8 is 48, &c.

Let

Let it be required to multiply 368 by 7, set them down as in the Margin (the least Number under the greatest) and say (as you learn in the Multiplication Table) 7 times 8 is 56, set down 6 and carry 5; then 7 times 6 is 42, and 5 that I carry is 47, set down 7 and carry 4; then 7 times 3 is 21, and 4 I carry is 25, set down the 25, and the Figures as they then stand make 2576, the Product of 368 multiplied by 7.

But if you multiply by more Figures than one, you generally multiply by one Figure at a time, beginning at the Figure next the Right-Hand, and set the Product down as in the Example above; and then proceed to the second Figure of the Multiplier, and set the Product that comes of it, under the Product that comes of the first, only one Place more towards the Left-Hand; and continually follow this Method, how many Figures soever you multiply by; and then adding the several Products together as they stand, the Sum is the Product of the whole Multiplication.

Note, The Reason of setting every Product a Place higher toward the Left-Hand is because every Figure is oftentimes the Value that the same Figure would have been, if standing one Place lower, or towards the Right-Hand, as the Product of any Number multiplied by 30, is ten times as much as the Product of the same Number multiplied by 3.

Multiply 3581 by 27.

Place them as in the Margin, and first multiply 3581 by 7, as in the foregoing Example is directed, and place the Product 25067 under the Line. Then say, twice 1 is 2, which set (not under 7 but) under 6, and twice 8 is 16, set down 6 and carry 1, and so proceed; and having found the Product of the latter Figure (7162) add them together, saying, 7 is 7, 6 and 2 is 8, and so proceed as in common Addition, you have 96687, which is the Product of 3581 multiplied by 27.

If you multiply by any greater Number of Figures, the Rule is the same, always observing to place the first Figure of each Product a Place higher towards the Left-Hand; and all the rest successively, as in the adjoining Example, which I shall only put down the Answer to, because the Method of Working is the same as above.

But in some Cases of multiplying by two Figures, some Labour may be saved by help of the following Table being got by Heart, and is of great Use in reducing Feet into Inches, or Shillings into Pence; as I shall shew when I come to *Reduction*.

I	10	11	12
2	20	22	24
3	30	33	36
4	40	44	48
5	50	55	60
6	60	66	72
7	70	77	84
8	80	88	96
9	90	99	108
10	100	110	120
11	110	121	132
12	120	132	144

Any Number multiplied by 11, may be done yet a nearer Way: as suppose I would multiply 2785 by 11, first by the common Way the Work would stand as below.

$$\begin{array}{r}
 2785 \\
 \times 11 \\
 \hline
 2785 \\
 2785 \\
 \hline
 30635 \\
 \text{C}
 \end{array}$$

But

$$\begin{array}{r}
 135798642 \\
 2589 \\
 \hline
 1222187778 \\
 1086389136 \\
 678993210 \\
 271597284 \\
 \hline
 351582684138
 \end{array}$$

As the former Table of Multiplication reaches to 10 times 10, so this runs as high as 12 times 12; as for Instance,

If I desire to know how much is 7 times 11, find 11 at the Top of the Table, and under it against 7, you have 77, and so of the rest.

Multiply 2785 by 12.

Set them down as before taught, only instead of saying, 2 times 5 is 10, &c. say, 12 times 5 is 60, set down 0 and carry 6; then 12 times 8 is 96, and 6 I carried is 102, set down 2 and carry 10; then 12 times 7 is 84, and 10 I carry, is 94, set down 4 and carry 9; then 12 times 2 is 24, and 9 I carry is 33, which set down, and then you have 33420 for the Product of 2785, multiplied by 12.

$$\begin{array}{r}
 2785 \\
 \times 12 \\
 \hline
 33420
 \end{array}$$

But because the Figures in 11 are both Figures of 1, which neither multiplies nor divides, and therefore does but produce the Multiplicand twice over, as in the Question above, you may as well set only the Multiplier down twice (one in a Place higher than the other) and add them together as in common Multiplication, and the abovementioned Question will stand as in the Margin.

2785

2785

30635

Contractions in Multiplication.

1. If you have a Number given to multiply by 10, 100, 1000, or any Number, consisting of 1 and Cyphers, you need but add so many Cyphers to the Multiplicand, as in the Multiplier, and that shall be the Product.

Example. 5324 multiplied by 1000 is 5324000.

2. If any Number be to be multiplied by a Number that hath Cyphers at the latter end of it, multiply by the Figures in the Multiplier as in common Multiplication, as if there were no Cyphers, and then add as many Cyphers to the Product as are in the Multiplier, and that shall be the true Product.

Example. Multiply 2785 by 1200, first multiply 2785 by 12, as before taught, the Product is 33420, to which annex the two Cyphers, it makes 3342000 for the Product required.

3. If there be Cyphers at the latter-end of both Multiplicand and Multiplier, reject all the Cyphers in both, and work with the Figures; only to the latter-end of the Product so found, annex a Number of Cyphers equal to all the Cyphers in both Multiplicand and Multiplier, and that shall make the true Product required.

Example. To multiply 358100 by 27000, the Product will be 9668700000.

4. If there be a Cypher in any middle Place of the Multiplier, you need not set a Row of Cyphers down for it, but place the Product of the next Figure two Places more towards the Left-Hand; or if there be two Cyphers, three Places, &c. And thus 3581 multiplied by 207, will stand thus:

$$\begin{array}{r}
 3581 \\
 207 \\
 \hline
 25067 \\
 7162 \\
 \hline
 741267
 \end{array}$$

But:

But to multiply 3581 by 2007, it will stand as follows :

$$\begin{array}{r} 3581 \\ 2007 \\ \hline 25067 \\ 7162 \\ \hline 7187067 \end{array}$$

And so for a greater Number of Cyphers, whether they fall all together or not.

CHAP VI.

Of DIVISION.

DIVISION teacheth to find how often a lesser Number may be taken out of, or subtracted from a greater, and what remains, and consequently, performs at one Operation what Subtraction does at many.

An Operation in Division consists of four Numbers, the Dividend, the Divisor, the Quotient, and the Remainder.

The Dividend is always the biggest Number of the three, and is the Number to be divided.

The Divisor is the Number by which the Dividend is to be divided.

The Quotient is the Number of times in which the Divisor is contained in the Dividend; and

The Remainder is the odd Number left after the Divisor hath been taken out of the Dividend as often as possible; as for Instance, If it is asked how often I can have 3 in 14, the Answer will be 4 times and 2 remaining; here 14 is the Dividend, 3 the Divisor, 4 is the Quotient, and 2 is the Remainder required.

Multiplication and Division mutually unravel each other; for when two Numbers have been multiplied together, and a Product found, divide the Product by the Multiplicand, and it shall bring for a Quotient the Multiplier; or divide the Product by the Multiplier, and the Quotient shall be the Multiplicand.

Again, when a greater Number hath been divided by a less, and a Quotient and Remainder found, if you again multiply the Quotient by the Divisor (either taking in carefully the Fraction

of the Quotient, or adding the Remainder, if any be, to the Product) it shall again unravel the Division and produce the first Dividend.

Hence the best way of proving either Division or Multiplication, is to make them reciprocally (by unraveling) prove each other, of which I shall give some instances at the Conclusion of this Chapter.

There are several Methods of working Division; I shall instance in one of the most useful and plain of them, and that which may be done with the fewest Figures, or with the least Trouble, and may be proved by common Addition; and first to divide by one Figure.

It is required to know how often I can have 5 in 793, and how much (if any thing) remains.

$$\begin{array}{r} 158 \\ 5 \overline{) 793} \end{array}$$

Set down the Dividend, and on the Left-Hand of it the Divisor, having room also on the Right-hand for the Quotient, as you see.

Then, beginning at the Left-Hand, say, how oft 5 (the Divisor) in (the first Figure of the Dividend) 7, which can be but once, set 1 in the

Quotient, and say, once 5 is 5, which set under 7, and say, 5 from 7 there remains 2, which set over 7 and cancel the 7.

Then say, how oft 5 (the Divisor) in 29 (the 2 that stands over the 7, and 9, the second Figure in the Dividend) which you may have 5 times and no more; set 5 in the Quotient, and by that 5 multiply the Divisor (5) saying, 5 times 5 is 25, which set down so, as that the 25 may stand under the 29, Units under Units, and Tens under Tens; then say 25 from 29 there remains 4, which set over the 9, and cancel the 2 and the 9.

Then the last Figure in the Dividend 3, in the Units Place, and 4 (over the 9) in the Tens Place make 43, say, how oft 5 in 43, answer 8 times, put 8 in the Quotient, and say, 8 times 5 is 40, set that under the Dividend, and under the 43, then as before directed, and say, 40 from 43 there remains 3, which set over the last Figure in the Dividend, and cancel the 43, then is 158 the Quotient, and 3 the Remainder sought.

For the Proof of this Division, add together all the Figures that are below the Dividend, (taking in none above but the Remainder 3) in the Order as they stand, saying, 0 and 3 is 3, and 5 and 4 is 9, also 2 and 5 is 7; set them below the Line, as in common Addition, and if the Work is right, the sum will be the same as the Dividend, (as is here) or if it is not the same as the Dividend, the Work is false.

For the Figures below the Dividend are the several Products of the Divisor and Quotient multiplied together, and I have observed

served before, that the Quotient multiplied by the Divisor (taking in the Remainder) will produce the Dividend, therefore, &c.

But they that do not regard this Way of proving Division, may do the same Question with few Figures, yet much in the same Form, thus:

Place the Dividend, Divisor, and Place for the Quotient as before, and say, how oft 5 in 7, which being once, set 1 in the Quotient, and say, once 5 is 5, from 7 there remains 2, set 2 over 7, and cancel the 7; then 9, the second Figure in the Dividend, in the Units Place, and 2 (over the 7, in the Tens Place) make 29, say, how oft 5 in 29, which is 5 times, and 4 remains, set 5 in the Quotient, and 4 over 9, and cancel the 2 and 9; then 4 (over the 9) in the Tens Place, and 3 (the last Figure in the Dividend) in the Units Place, make 43, say, how oft 5 in 43, which is 8 times, set 8 in the Quotient, and say, 8 times 5 is 40, from 43 and there remains 3, which set over the last Figure in the Dividend, then is the Quotient 158, and the Remainder 3, as before.

$$\begin{array}{r} 158 \\ 5 \overline{) 793} \end{array}$$

I desire to know how oft I can have 72 in 57289.

Place the Dividend, &c. as before, and say, how oft 7 (the first Figure of the Divisor) in 5 (the first figure of the Quotient) but because it cannot be, take in one Figure more, and say, how oft 7 in 57, which may be 8 times, because 8 times 7 is but 56, which taken from 57 there would remain 1, but then 8 times 2 is 16, but 1 that remained before, and 2 (the third Figure in the Dividend) taken as they stand, make but 12, because we cannot take 16 from 12, we must put 1 less in the Quotient, and instead of 8 put 7 in the Quotient.

Note, When we divide by more Figures than one, we must previously consult whether the rest of the Figures of the Divisor (as well as the first) will bear the Figures we propose to put in the Quotient, before we set it down.

Because I could not have the two Figures of the Divisor 72 in the first two Figures of the Dividend 57, I must make use of the first three Figures of the Dividend 572, in the first Operation, and then beginning the Work at the Right-Hand, and multiply 2 (the last Figure of the Divisor) by 7, (the Quotient) saying, 7 times 2 is 14 from 2 (the third Figure in the Quotient) I cannot, I borrow 20 and say, 14 from 22 there remains 8, which set over 2, and say 7 times 7 is 49, and 2 I borrowed is 51, from 57, there re-

$$\begin{array}{r} 795 \\ 72 \overline{) 57289} \end{array}$$

mains 6, which set over 7, and cancelling the 572, the first Operation is done,

Then you must take in another Figure of the Dividend, which is 8, which with 68 above, and taken in order as they stand, make 688, then say, how oft 7 in 68, which suppose 9 times, but before you set it down in the Quotient, consider 9 times 7 is 63, from 68 there remains 5, and 9 times 2 is 18, which, because I can have from 50, I place 9 in the Quotient, and say, 9 times 2 is 18 from 8 I cannot, but (borrowing 10) 18 from 18 there remains nothing, set 0 over 8, and cancel the 8, then 9 times 7 is 63, and one that I borrowed is 64, from 68, and there remains 4, which set over the 8, then the Figures you have to work with are 409, say, how oft 7 in 40, which is 5 times, say, 5 times 2 is 10, from 9 I cannot, but 10 from 19 there remains 9, which set over 9 (the last Figure in the Quotient) and then 5 times 7 is 35, and one that I borrowed is 36, from 40 there remains 4, which set over 0 (the last Figure of 40) and the Question is answered, the Quotient is 795, and the Remainder 49.

Note, The Reason of cancelling the Figures, is to prevent making use of one twice over, or puzzling the Memory to know which have been made use of.

There is another Method of Division much easier to the Memory, but requires more Room and Figures for the Operation; I thought fit to insert it, because some People do not fancy scratched Division, as they call it.

We will instance in the foregoing Question of dividing 57289 by 72.

Place the Dividend, &c. as in the foregoing Example; and as before, because you cannot have 7 in 5, say, how oft 7 in 57, which with the Considerations before mentioned, you will find to be 7 times, put 7 in the Quotient, and say, 7 times 2 is 14, therefore (because in the first Operation you use the first three Figures of the Dividend) set 4 under (the third Figure of the Dividend) 2, and carry 1, saying, 7 times 7 is 49, and 1 I carry is 50, which set under 57, and then by the common Rules of Subtraction, take 504 from 572, there remains 68, to which bring down the fourth Figure in the Dividend, which is 8, and it stands as you see (688).

Then say, how oft 7 in 68, which you have 9 times, place 9 in the Quotient, and say, 9 times 2 is 18, set down 8 under 8,

and 9 times 7 is 63, and 1 is 64, which set down as you see, and subtracting 648 from 688, there remains 40, which set below, and bring down 9 (the last Figure in the Dividend) it makes 409, in which proceeding as in the other Operations you find in the Quotient 795 and 49 remaining.

The best Proof of Division is by Multiplication, for if you multiply the Divisor and the Quotient together, and to the Product add the Remainder (if any be) that Sum shall be the same with the Dividend, if the Work be right.

Example in the foregoing Question.

The Quotient is	795
To be multiplied by the Divisor	72
	1590
	5565
	57240
The Product	57240
To which add the Remainder	49
	57289
The Sum is	57289

Which being equal to the Dividend, proves the Division to be right.

Contractions in Division.

1. If a Number is to be divided by a Number consisting of 1 and any Number of Cyphers annexed, cut off as many Places from the Dividend, towards the Right-Hand, as there are Cyphers annexed to the Divisor, and the Figures left in the Dividend is the Quotient, and those cut off are the Remainder.

Example. Divide 5738 by 100, the Quotient is 57, and the Remainder 38; or if it was to be divided by 10, the Quotient is 573, and 8 remains, &c.

2. If any Number is to be divided by any Number of one or more Figures with Cyphers annexed, reject the Cyphers, and cut off from the Right-Hand of the Dividend as many Places (whether Figures or Cyphers it matters not) and work, as in common Division, the Quotient so found is the true Quotient, and the Remainder, with the Figures cut off, is the true Remainder.

Divide 3748 by 240, the Quotient is 15, and the Remainder, 14 (with the 8 that was cut off annexed to it on the Right-Hand) is 148.

$$\begin{array}{r}
 24 \overline{) 0374} 8 (15 \\
 \underline{134} \\
 148
 \end{array}$$

CHAP. VII.

Of REDUCTION.

REDUCTION is bringing the same thing into a Different Denomination, as Pounds into Shillings or Pence, Acres into Roods or Perches; or the contrary, Perches into Acres, &c. and it is of two Kinds, *viz.* to bring a greater Denomination into a lesser, as Shillings into Pence, and is performed by Multiplication.

The other to bring a lesser Denomination into a greater, as Pence into Shillings, &c. and is done by Division.

Reduction Descending.

Quest. 1. In 1728 Shillings, how many Pence?

Because 12 Pence make one Shilling, multiply 1728 by 12, and if you work by the Direction given in *Chap. V.* towards the Conclusion (where you are taught to multiply by both the Figures at once) the Work will stand as in the Margin, and the Answer then is 20736 Pence in 1728 Shillings.

1728
12
——
20736

Quest. 2. In 1728 Pounds, how many Shillings?

We are to consider that in one Pound there is 20 Shillings, therefore multiply 1728 by 20, and the Product 34560 is the Number of Shillings in 1728 Pounds required.

1728
20
——
34560

Quest. 3. In 49632 Acres, how many Perches?

You see in the Reduction Tables, at the Beginning of the Book, that 160 Perches is one Acre, therefore multiply by 160, and you have the Answer, 7941120, the Answer to the Question.

49632
160
——
2977920
49632
——
7941120

Quest.

Quest. 4. In 3694 Rood how many Perches?

$$\begin{array}{r} 3694 \\ \text{Perches in a Rood} \quad 40 \\ \hline 147760 \end{array}$$

Answer. 147760 Perches in 3694 Rood.

Quest. 5. In 426 Acres how many Square Links?

You see by the Reduction Table, that 100000 Links make an Acre, therefore by the first Contraction in Multiplication (by adding five Cyphers to the Number of Acres) you have 42600000 the Number of Square Links in 426 Acres.

These are sufficient for Examples of this Kind.

Reduction Ascending.

I shall next come to that Kind of Reduction which changeth from a lesser Denomination to a greater, and is performed by Division, and shall make use of the last five Questions to shew how one proves the other.

Quest. 1. In 20736 Pence, how many Shillings?

This Part of Reduction, requiring to bring a less Denomination into a greater, is consequently to be performed by Division:

$$\begin{array}{r} \text{Consider the Number of Pence in} \\ \text{one Shilling, which is 12; therefore} \\ \text{divide 20736 by 12, the Quotient is} \\ \text{1728, the Number of Shillings in} \\ \text{20736 Pence.} \end{array} \quad \begin{array}{r} 12)20736(1728 \\ 87.. \\ 33 \\ 96 \\ \hline 00 \end{array}$$

Quest. 2. In 34560 Shillings, how many Pounds?

Divide 34560 by 20 (the Shillings in one Pound) and the Quotient is 1728, the Number of Pounds required.

$$\begin{array}{r} 20)34560(1728 \\ 14.. \\ 05 \\ 16 \\ \hline 00 \end{array}$$

Note, This Question having a Cypher in the last Place of the Divisor, the Work is shortened by the Directions given for Contraction in Division, Page 23.

Quest.

Quest. 3. In 7941120 Perches, how many Acres?

The Number of Perches in an Acre is 160, as you see by the Reduction Tables at the Beginning of this Book. Therefore divide the Number of Perches proposed, viz. 7941120 by 160 (the Perches in a square Acre) the Quotient 49632 is the Number of Acres required.

$$\begin{array}{r} 16\overline{)0}794112\overline{)0}(49632 \\ 154\ldots \\ 101 \\ 051 \\ 32 \\ \hline 00 \end{array}$$

But if you have any Number of Perches given, and you would know how many Acres, and also how many odd Roods and Perches it contains, it is best to divide it by 40, and then the Quotient is Roods, and the Remainder is odd Perches; and the Roods so found divided by 4 (because 4 Roods is an Acre) the Quotient is the Acres, and the Remainder is odd Roods: And thus you have the Acres, Roods, and Perches, (if any odds remain) of any Number of Perches proposed.

Example. In 2535 Perches, how many Acres, and what odd Roods and Perches?

Divide 2535 by 40 (the Perches in a Rood) and the Quotient 63 is the Number of Roods, and the Remainder 15 is the odd Perches. Again,

Divide 63 by 4 (the Roods in an Acre) and the Quotient 15 is the Number of Acres, and the Remainder 3 is odd Roods; so that the whole is 15 Acres, 3 Roods, 15 Perches, as above, the Perches being to be brought into Roods by dividing by 40, as in the following Example.

$$\begin{array}{r} 4\overline{)0}253\overline{)5}(63 \\ 13 \\ \hline 15 \end{array}$$

$$\begin{array}{r} 4\overline{)63}(15 \\ 3 \end{array}$$

Acres. Roods. Perches.
Ans. 15 : 3 : 15

Quest. 4. In 147760 Perches, how many Roods?

Divide 147760 by 40, and the Quotient 3694 is the Number of Roods, and nothing remains.

$$\begin{array}{r} 4\overline{)0}14776\overline{)0}(3694 \\ 27\ldots \\ 37 \\ 16 \\ \hline 00 \end{array}$$

Quest.

Quest. 6. In 42600000 square Links, how many Acres?

An Acre contains 100000 square Links; thereby divide 42600000, the Quotient is 426; and this is performed without any Trouble but cutting off the Cyphers, as is shewed in the Contractions in Division, taught *Page 23* of this Book.

Quest. 6. In long Measure

In 4105728000 Barley-Corns, how many Inches, Feet, Yards, Furlongs, and Miles?

Barley-Corns.	Inches.
3)4105728000	(1368576000
11	
20	
25	
17	
22	
18	
0000	

	Feet.
12)1368576000	(114048000
16	
48	
05	
57	
96	
0000	

	Yards.
3)114048000	(38016000
24	
004	
18	
0000	

	8) Furl. Miles.
22)0 3801600 0	(172800(21600
160	12
61	48
176	000
0000	

For the Poof of *Reduction*.

Reduction Ascending and Reduction Descending alternately prove one another, by making that a given Term in one, which (till found) was required in the other; and this Method I have taken in the five Questions of each Kind of Reduction; as, for Instance, *Question* the first of Reduction Descending is—

In 1728 Shillings, how many Pence? And the Answer is 20736 Pence.

Therefore the first Question I have proposed in Reduction Ascending is—

In 20736 Pence, how many Shillings? And the Answer is 1728.

This brings the Question into its first Denomination, and proves the Work to be right.

CHAP. VIII.

The Golden Rule, or Rule of Three.

THE Use of the *Golden Rule Direct* is, when three Numbers are given, to find a Fourth that bears the same Proportion to the Third, that the second does to the First; and the Rule is—

Observe, in stating your Question, that the first and third Terms be of one Denomination; and the Fourth, which is always the required Term, will be of the same Denomination with the second—

Then multiply the second and third Numbers together, and divide that Product by the first Number or Term; the Quotient arising from thence is the fourth Term, or proportional Number sought.

The Reason of this Rule is, because in any four Numbers that are proportional, the Rectangle under the two Means is equal to the Rectangle under the two Extremes; that is, the two middle Terms or Numbers multiplied together, their Product is equal to the Product of the two Extremes, viz. the first and last Terms. *Euc. Lib. VII. Prop. 19.*

Example. If we imagine any Rank of Numbers in a Geometrical Proportion, as 1, 2, 4, 8, 16, 32, 64, &c. Or, 1, 3, 9, 27, 81, &c. Take any four of these Numbers together; as for
6 Instance,

Instance, 4, 8, 16, 32, it is, as 4 to 8, so 16 to 32: And if so, then 32, multiplied by 4, is equal to 16 multiplied by 8.

Also, as 3 to 9, so 27 to 81; therefore the product of 81 multiplied by 3, is equal to the Product of 27 multiplied by 9, and so in any other Case; for if four Numbers are proportional, the Second must contain the First as often as the Fourth contains the Third, as in the last Instance, it is 3 Times, for the second Term 9 contains the first Term 3 three Times; and therefore (the Numbers being proportional) the fourth Term 81 contains the third Term 27 also 3 Times. Now, if 27 be multiplied by 9, and 81, which is 3 Times 27, be multiplied by 3, which is but one Third of 9, it is certain the Products must be equal.

And for the same Reason, in any three proportional Numbers, the Square of the Mean is equal to the Rectangle, or Product of the Extremes.

$$\begin{array}{r}
 32 \quad 16 \\
 \underline{4 \quad 8} \\
 128 \quad 128 \\
 \hline
 81 \quad 27 \\
 \underline{3 \quad 9} \\
 243 \quad 243
 \end{array}$$

$$\begin{array}{r}
 35 \quad 175 \\
 \underline{35 \quad 7} \\
 175 \quad 1225 \\
 \hline
 105
 \end{array}$$

As 7 to 35, so is 35 to 175:

1225

For 35, multiplied by itself, is equal to a Number 5 Times as great multiplied by another Number, that is but one Fifth as great, &c.

Now it is evident from what hath been said of the Nature of *Multiplication* and *Divison*, that if the Product of the First and Last be equal to the Product of the Second and Third, then if the Product of the Second and Third be divided by the First, the Quotient shall be the fourth Number required; this is a necessary Consequence of *Euc. Lib. VII. Prop. 19.* before cited.

This Demonstration of the Rule of Three, I recited in my London Accomptant, Pag. 108, 109, and 110.

Example. Question 1.

If 9 Pound of Tobacco cost 18 Shillings, what will 85 Pound cost?

lb. s. lb.

State it thus, If 9 : 18 :: 85

$$\begin{array}{r}
 18 \\
 \underline{85} \\
 680 \\
 85 \\
 \hline
 9)1530(170 \\
 63 \\
 \hline
 00
 \end{array}$$

$$\begin{array}{r}
 2)0170(8 \\
 \underline{110}
 \end{array}$$

Accord

According to the Rule, multiply the two last Terms together, viz. 85 by 18, the Product 1530 divide by the first Term 9, and the Quotient 170 is the Shillings that 85 Pound of Tobacco will cost at that Price, which is 8 Pound 10 Shillings.

Sometimes there is a Remainder after *Division*, whose Value is required to be known in Quarters and Pounds, or Shillings and Pence, &c. according to the Quality of the fourth Term, whether Money, Weight, or Measure; and this is to be done by multiplying the first Remainder by the Number of Integers of the next lower Denomination, contained in the first; as, if the second Term be Pounds, the Quotient or Answer will also be Pounds: and if there be a Remainder, multiply it by 20, the Shillings in a Pound; and divide by the first Divisor, the Quotient is Shillings: And if any thing yet remains, multiply by 12, the Pence in a Shilling, and divide still by the first Divisor, the Quotient is Pence, &c.

Example. Question 2.

If 19 Hundred Weight of any Sort of Goods cost 29 Pounds Sterling, what will 359 Hundred Weight cost?

	C.	l.	C.
If	19	: 29	:: 359
			<u>29</u>
			3231
			<u>718</u>
	19	10411	(544 Pounds.
		91	
		<u>151</u>	
Remains	-	-	18
Shillings in a Pound	-	-	<u>20</u>
			19)360(18 Shillings.
			<u>170</u>
Remains	-	-	18
Pence in a Shilling	-	-	<u>12</u>
			19)216(11 Pence.
			<u>26</u>
			7
			<u>4</u>
			19)28(1 Farthing.
			<u>9</u>

Ans.

Ans. 359 Hundred Weight will cost $547 \frac{1}{2}$ Pounds; but to know what Shillings and Pence this Fraction amounts to, take this

Example. In the foregoing Question, the Remainder after the first Division is 18, which multiplied by 20 (the Shillings in a Pound) is 360, that divided by 19 (the first Divisor) the Quotient 18 is Shillings, and there again remains 18, which multiplied by 12, the Pence in a Shilling, produceth 216, which divided by still the same Divisor 19, the Quotient 11 is Pence, and the Remainder is 7, which multiplied by 4, the Farthings in one Penny, the Product is 28, which divided by the first Divisor 19, the Quotient is 1, and Remainder 9; so that if 19 C. cost 29 $l.$ 359 C. will cost $547 \text{ } l. \text{ } 18 \text{ } s. \text{ } 11 \text{ } d. \text{ } 1 \text{ } q.$ ² Observe the same Method in knowing the Value of a Fraction in any other Denomination.

Example. Question 3.

If 36 Pound Sterling will pay for 21 Quarters of Wheat; how much will 137 Pound Sterling pay for at that Price?

$l. \quad 2. \quad l.$
If $36 : 21 :: 137$

$$\begin{array}{r}
 21 \\
 \hline
 137 \\
 274 \\
 \hline
 36)2877(79 \text{ Quarters.} \\
 \underline{357} \\
 338 \\
 \hline
 36)264(7 \text{ Bushels.} \\
 \underline{12} \\
 4 \\
 \hline
 36)48(1 \frac{1}{3} \text{ Pecks.} \\
 \underline{12}
 \end{array}$$

The Remainder after the first Division is 33, which multiply by 8 (the Bushels in one Quarter) and the Product (264) divided by the first Divisor 36, gives 7 Bushels; proceed in the same Manner by 4, for the odd Pecks, and the Answer is 79 $Qrs.$ 7 B. $1 \frac{1}{3}$ P. viz. 79 Quarters, 7 Bushels, $1 \frac{1}{3}$ Peck, &c. —

But in Case one of the Terms be in several Denominations, that, together with its correspondent Term of the same Name, must also come under the lowest Denomination mentioned, and then proceed as before: But this I think a sufficient Specimen of the

According to this Table, 1 signifies One, and 10 signifies Ten, &c. as in whole Numbers; and if a Cypher be added on the Left-hand, it neither increases nor diminishes it, as 01, or 001, in whole Numbers, signify 1; but in Decimals, being distinguished by a Point on the Left-hand, .1 signifies $\frac{1}{10}$, and .01 signifies $\frac{1}{100}$, and .001 signifies $\frac{1}{1000}$, &c. Or, if the Places are supplied with Figures, it is the same, as

$$\left. \begin{array}{l} 35786 \\ 3578.6 \\ 357.86 \\ 35.786 \\ 3.5786 \\ .35786 \end{array} \right\} \text{ signifies } \left\{ \begin{array}{l} 35786 \\ 3578 \frac{6}{10} \\ 357 \frac{86}{100} \\ 35 \frac{786}{1000} \\ 3 \frac{5786}{10000} \\ \frac{35786}{100000} \end{array} \right.$$

Before I proceed to *Addition*, &c. I shall speak of *Reduction* under these two Heads, *viz.* To reduce a Vulgar Fraction to a Decimal, and to Reduce a Decimal Fraction to a Part of Coin, Weight, Measure, &c.

PROP. I.

To reduce a *Vulgar Fraction* to a *Decimal*.

RULE.

Add so many Cyphers to the Numerator, that dividing by the Denominator, nothing may remain, (if possible) the Quotient is the Decimal required.

Example.

Quotient 875.

I desire to express $\frac{7}{8}$ in Decimals.

$$\begin{array}{r} 8 \overline{)70} \\ 60 \\ \hline 40 \\ \hline 0 \end{array}$$

Ans. It is .875, or $\frac{875}{1000}$

But you may observe sometimes the Numbers will circulate, that is, in the *Division*, you will after some Time have the same Figure remaining that you have had before; and, consequently, that with a Cypher will bring the same in the Quotient as it did before; and when you perceive this, you may be sure there will always be a Remainder, for the same Figures will follow perpetually.

D

Ex-

OF ARITHMETIC

Example. Reduce $\frac{5}{14}$ to a Decimal.

Quotient 35714285714285, &c.

$$\begin{array}{r}
 14 \overline{) 50} \\
 \underline{80} \\
 100 \\
 \underline{20} \\
 60 \\
 \underline{40} \\
 120 \\
 \underline{80} \\
 100 \\
 \underline{20} \\
 60 \\
 \underline{40} \\
 120 \\
 \underline{80}
 \end{array}$$

In the first Operation the Remainder is 8, which with a Cypher makes 80, and brings in the Quotient 5 and 10 remaining, &c. to the seventh Step, which brings in the Quotient 8, and after that, the same Figures repeated as was after 5 before, as you see in the Operation.

P R O P. II.

To reduce a Decimal Fraction to any Part of Coin, Weight, Measure, or the like.

Example. I desire to know how many Shillings and Pence is in .75 *l.* or 0.75, or, which is the same, $\frac{75}{100}$ of a Pound.

Because 20 Shillings make one Pound, multiply .75 by 20, and cut off two Figures, because the Decimal Fraction [.75] consists of two Figures, the rest is the Shillings required.

The Answer is	$\begin{array}{cc} s. & d. \\ 15 & : 0 \end{array}$	$\begin{array}{r} .75 \\ 20 \\ \hline 15 \overline{) 100} \end{array}$
---------------	---	--

But if any thing had remained beyond the Line, it must have been multiplied by 12 (the Pence in one Shilling) cutting off two Places towards the Right-hand, which (if not Cyphers) are again to be multiplied by 4, the Farthings in one Penny; and again cutting off two Places towards the Right-hand, the rest are Farthings,

things, and those cut off are hundredth Parts of a Farthing, &c. And, if the Fraction had been to three Places, the Remainder had been thousandth Parts, &c.

Example. In 25.786 *l.* how many Shillings, Pence, and Farthings?

Ans. In 25.786 *l.* is —
515.720, viz. 515 $\frac{720}{1000}$ *s.*
or 6188.640 Pence, or
24754.560 Farthings.

Pounds	- -	25.786
		20
Shillings	-	515 720
		12
Pence	- -	6188 640
		4
Farthings		24754 560

The Reason of this Rule may be thus explained, as in the first Example .75 is $\frac{75}{100}$; therefore the Proportion is, as 100 to 75; so 20 (the Shillings in one Pound) to the Shillings in .75, or $\frac{75}{100}$ of a Pound; and if by the *Golden Rule* you would find it, you must multiply the two last Terms together, viz. 75 by 20, and divide by the first Term 100, it brings the fourth Term required. But, in Decimals, the Denomination or first Term being always 1 with Cyphers, viz. 10, 100, 1000, &c. being only to cut off as many Figures to the Right-Hand as there are Cyphers in the Divisor: and the Cyphers in the Divisor or Denominator being always equal to the Places in the Numerator, it will follow, that cutting off as many Figures as there are Places in the Decimal, is the same in Effect that the whole Operation would bring forth.

And the same Reason holds in mixed Numbers, viz. a whole Number with a Decimal annexed, as 25.786; for as the Operation for the Fraction .786 is demonstrated, as above, so it is plain, that the whole Number of Pounds, viz. 25, multiplied by 20 gives the Shillings, &c. by the common Rules of *Reduction* before taught.

P R O P. III.

To reduce any known Part of Coin, Weight, Measure, &c. to a Decimal Fraction, or (which is the same) to express any known Part of Coin, Weight, Measure, &c. in Decimals.

This is but the Reverse of the former, and performed by *Division*, as the other is by *Multiplication*.

Example. I desire to express one Farthing in Decimals, one Pound being the Integer.

Divide 1 (the Number of Farthings given) with Cyphers annexed to it, by 960 (the Number of Farthings in one Pound) the Quotient is the Decimal required, as in the following Operation.

$$\begin{array}{r}
 96 \overline{) 1000000} 0010416 \\
 \underline{400} \\
 160 \\
 \underline{640} \\
 64
 \end{array}$$

By the same Rule, if you would express 3 Farthings in Decimals, divide 3 with Cyphers annexed by 960, and the Quotient .003125 is the Decimals required; and by the same Rule 4 Pence 3 Farthings, or 19 Farthings, thus expressed, 0197916.

ADDITION in DECIMALS.

R U L E.

PLACE always the Point at the Beginning of the Fraction, over each other, and add as if they were whole Numbers.

Example. Add 56.375 to 834.22.

The Reason why, in Decimals the Fraction 22 stands not under the last Figures of the Fraction 375, is, because 375 possessing three Places in Thousandths, but 22 possessing but two Places in Hundreths for Reasons given at the Beginning hereof; but 22 Hundreths is equal to 220 Thousandths; and to set down 22 under the two first Figures of 375 is the same, as to set down 220 under 375. Hence observe,

In Addition or Subtraction of any Decimals of whatsoever Number of Figures they consist, let them always be even at the Left-hand, and let the odds be at the Right-hand, contrary to whole Numbers; as in the following Examples.

Example.

Example 1.

$$\begin{array}{r} 1.24 \\ 65.96 \\ 329.658 \\ 6638.1 \\ \hline 7034.958 \end{array}$$

Example 2.

$$\begin{array}{r} 68.7 \\ 93.98 \\ 48.07 \\ 82.099 \\ \hline 292.849 \end{array}$$

The Reason is this (*Example 2.*) the 7 is 7 Tenths, which is equal to 700 Thousandths. The next is 98 Hundredths, or 980 Thousandths; the third is 7 Hundredths, or 70 Thousandths, the last is 99 Thousandths, which set down, and added up as if they were whole Numbers, will produce 1849 Thousandths, viz. 1 Integer to be carried over the Point, and 849 Thousandths as a Fraction.

$$\begin{array}{r} 700 \\ 980 \\ 70 \\ 99 \\ \hline 1849 \end{array}$$

SUBTRACTION in DECIMALS.

IN this, as in *Addition*, make the whole Numbers even at the Right-hand, and the Fraction even at the Left-hand, placing the Points right over each other, and proceed as if they were whole Numbers.

EXAMPLES.

$$\begin{array}{r} 1 \\ 198.73 \\ 136.21 \\ \hline 62.52 \end{array}$$

$$\begin{array}{r} 2 \\ 265.12 \\ 139.56 \\ \hline 125.56 \end{array}$$

$$\begin{array}{r} 3 \\ 300.7 \\ 96.95 \\ \hline 203.75 \end{array}$$

MULTIPLICATION in DECIMALS.

RULE.

SET down your Sums, and proceed as if they were whole Numbers, only from the Product cut off as many Figures as there were Places of Decimals in both Multiplicand and Multiplier, the Figures remaining towards the Left-hand is a whole Number, and those cut off are Decimals.

D 3

EX.

EXAMPLES.

1.	2.	3.
$\begin{array}{r} .375 \\ .289 \\ \hline 3375 \\ 3000 \\ 750 \\ \hline .108375 \end{array}$	$\begin{array}{r} 27.6 \\ 2.76 \\ \hline 1656 \\ 1932 \\ 552 \\ \hline 76.176 \end{array}$	$\begin{array}{r} 126. \\ .32 \\ \hline 252 \\ 378 \\ \hline 40.32 \end{array}$

Multiply 1728 by .327.

Here, 1728 is all a whole Number, and .327 is all a Fraction; therefore the Product is 565.056; for as the Multiplier is only a Fraction, the Product will be less than the Multiplicand.

$$\begin{array}{r} 1728 \\ .327 \\ \hline 12096 \\ 3456 \\ 5184 \\ \hline 565.056 \end{array}$$

They that understand Multiplication in whole Numbers cannot err in Decimals. Mind but to cut off as many Figures from the Product, as there was from both Multiplicand and Multiplier; and if there were more cut off from one than from the other, or if all from one, and none from the other, it matters not, the Rule is the

same as in the Example above.

DIVISION in DECIMALS.

IN *Division*, where the Dividend or Divisor, or both are Decimals, observe;

Place your Numbers as if they were whole Numbers, and if the Dividend have just as many Places of Decimals (or Figures after the Dot) as the Divisor, proceed as in the common Division, and the Quotient is a whole Number without any Figures to be cut off.

Example. Divide 565.056 by .327, the Quotient is 1728.

$$\begin{array}{r} .327 \overline{) 565.056} (1728 \\ 2380 \\ 0915 \\ 2616 \\ \hline 000 \end{array}$$

Now, because there is just as many Decimals in the Divisor as in the Dividend, the Quotient is a whole Number 1728,

But

But if there be more Places of Decimals in the Dividend, than in the Divisor, cut off as many Figures from the Quotient as there are Decimals in the Dividend more than in the Divisor, and the rest is the whole Number, and those so cut off the Fraction of the Quotient.

Example. Divide 4119.3016 by 42.31.

Here are two Decimals more in the Dividend than in the Divisor; therefore there will be two Decimals in the Quotient, which will then be 97.36.

$$\begin{array}{r} 42.31 \overline{) 4119.3016} \\ 311 40. \\ 15 231 \\ 2 5386 \\ \hline 0000 \end{array}$$

But if there be more Places of Decimals in the Divisor than in the Dividend, add Cyphers to the Dividend, to make as many Places of Decimals as in the Divisor, and proceed as before, and the Quotient will be a whole Number.

Example. Divide 92 by 5.75.

Set them down as in common Division; and because there are two Places of Decimals in the Divisor, and none in the Dividend, annex two Cyphers to 92, and then proceed in the Division, the Quotient is 16 a whole Number.

$$\begin{array}{r} 5.75 \overline{) 9200} \\ 3450 \\ \hline 000 \end{array}$$

But if, as it commonly happens, something remains after the Division, add a Cypher, and make another Operation, and if any thing yet remains, add another, &c. remembering, that that Part of the Quotient that comes out after the Addition of Cyphers, is to be cut off from the Quotient by a Dot, and accounted a Decimal.

Example. Divide 968 by 8.75

First, because there are two Places of Decimals in the Divisor, more than in the Dividend, add two Cyphers to the Dividend, and perform the Division.

But when I have gone thro' all this Dividend, I find 550 remaining, put a Dot at the Quotient, and annex a Cypher to the Remainder, and proceed; and the next Step you have 250 remaining, to which add a Cypher, it produces 2 in the Quotient.

$$\begin{array}{r} 8.75 \overline{) 968.00} \\ 930 \\ 5500 \\ 2500 \\ 7500 \\ 5000 \\ 6250 \\ 1250 \\ 3750 \\ 2500 \\ 750 \end{array}$$

You may continue to add Cyphers, as you see in the Margin, till it comes to 2 in the Quotient, and then you need not proceed any further, for the Numbers in the Quotient will circulate perpetually 285714, and the same over again.

But this brings a Quotient nearer to exactness; for when we have gone through the first Dividend, the Quotient is 110, and 550 remains; but the second Operation makes it 110.6; the third 110.62, &c. every one nearer the Truth than other, &c.

The Golden Rule in Decimals, being performed by Multiplication and Division, in the same Manner as in whole Numbers, I shall not need to instance in it, supposing, that what is done is sufficient for an Introduction to the *Art of Surveying*.

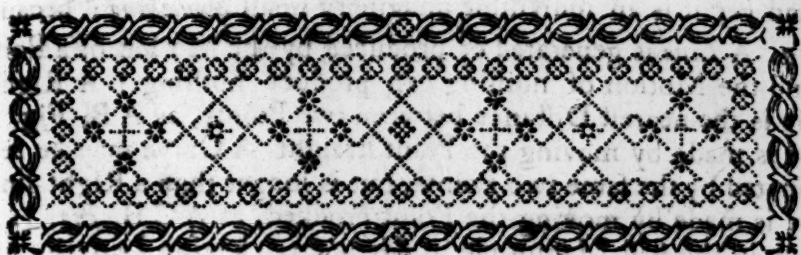


But if, as it commonly happens, something remains after the Division, add a Cypher to the Dividend, and perform the Division again; and if there is still a Remainder, add another Cypher, and so on, till you have brought it to the Degree of Exactness you desire. This is the Rule for dividing by a Denominator, and the same may be applied to the Division of Decimals.

Example. Divide 1000 by 2.75

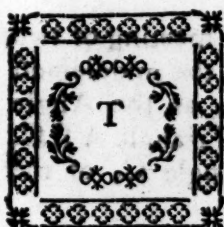
First, because there are two Places of Decimals in the Divisor, add two Cyphers to the Dividend, and perform the Division.

Now when I have done this, in this Division, I find 360 remaining, and a Denominator remaining, and annex a Cypher to the Remainder, and proceed, and the next step you have 250 remaining, to which add a Cypher, it produces 2500, and so on.



P A R T II.

Of the Theoric Part of Surveying, containing such Definitions, Theorems, and Problems, as are necessary to the understanding the Practice of Surveying in general, with Plain Trigonometry, Right-angled and Oblique.



THE Art of Surveying teacheth how to find the Area or superficial Content or Quantity of any Field, Close, Wood, Common, or any Plot or Parcel of Ground, however situate, or in whatsoever Form appearing, whether regular or irregular.

In order to the better understanding the Art of Surveying, both in Theory and Practice, it will be necessary to introduce the Directions therein, with the following Definitions, Theorems, and Problems: We shall begin with

C H A P. I.

Of DEFINITIONS.

- I. **A** Point is variously defined by different Authors, but all tending to the same Purpose: Some say it has no Part, because a Part implies Magnitude, and Magnitude may be (or imagined to be) divided; But a Point cannot, it being the least Thin

Thing that can be imagined, and so small, that it cannot be divided. And yet it is an indivisible imaginary small Something; because

2. A *Line* is generated or produced by the Motion of a Point (and the Motion of nothing can produce nothing.) A *Right-line* is the nearest Distance between two Points, as *A B*, Fig. 1. and is made by moving the Point streight. A *Curve* may be terminated by the same two Points, but is longer than a *Right-line*, and is made by moving the Point crooked, as *A C B*, Fig. 1. A *Line* is the first Magnitude in Geometry, having Length, but no Breadth or Thickness.

3. A *Superficies* is the second Kind of Magnitude in Geometry, having Length and Breadth, but no Thickness, and is produced by the Motion of a Line, as a Line is by the Fig. 2. Motion of a Point. The Extremes or Limits of a *Superficies* are Lines, and under the Denomination of a *Superficies* of one Kind or other are comprehended all Figures whatsoever, that can be the Subject of this Kind of *Surveying*, commonly called *Surveying of Land*. The Plain *E F G H* is a *Superficies*.

4. The third Geometrical Magnitude is a *Solid*, it has the three Dimensions of Length, Breadth, and Thickness, and is the Subject of one Kind of Mensuration; but not of this of *Surveying*.

5. An *Angle* is made by the Inclination, and meeting of two Lines and is commonly exprest by three Letters, the middlemost of which signifies the Angle meant, as the Angle Fig. 3. *A B C*, signifies the Angle *B*, of the Triangle *A B C*, and the Angle *A C B*, signifies the Angle *C*, of the Triangle *A B C*, &c.

There be three Kinds of Angles, viz. Right, Acute, and Obtuse.

6. A *Right-angle*, is that which is just 90 Degrees, or more vulgarly a true Square, as the Angle *A B C*.

7. An *Obtuse-angle* is greater than the *Right-angle*, or is without or more than a true Square, as *A B D*, which Fig. 3. is greater than the *Right-angle* *A B C*, because it contains both the Angle *A B C*, and the Angle *C B D*.

8. An *Acute* or *Sharp-angle* is less than 90 Degrees, or a *Right-angle* or true Square, as the Angle *A* or *D*.

9. A *Perpendicular* is a *Right-line*, which stands upon another *Right-line*, so as to make *Right-angles* with it, as the Fig. 4. Line *H F* is perpendicular to the Line *E G*, because the Angles *E H F* and *G H F* are *Right-angles*,

Of *Superficies*, some are regular, some irregular.

Regular Superficies, are either three-sided, four-sided, many-sided, or circular.

Three-sided Figures are called *Triangles*, of which there be two Sorts, Right-angled and Oblique.

10. *Right-angled Triangles*, are those that have one Right-angle, and two Acute-angles, as the Triangles GHI right-angled at H: In these the longest Side is called *Fig. 5.* the *Hypothenuse*, and the other two are called *Legs*.

11. *Oblique Triangles*, are those that have no Right-angle; but all Oblique-angles, as DEF, *Fig. 6.* and of these are the following kinds.

12. An *Equilateral Triangle*, which hath all its Sides equal, and consequently all its Angles, as GHI. *Fig. 7.*

13. An *Isoceles Triangle*, which hath two Sides (and consequently, the two Angles opposite to them) equal, as the Triangle KLM, whose two Sides KM and LM *Fig. 8.* are equal, and consequently their opposite Angles K and L are also equal.

14. A *Scalenum* is a Triangle with three unequal Sides, as DEF, *Fig. 6.*

These and all other Oblique Triangles (besides those particular Distinctions) come under the Denomination of *Acute-angled* or *Obtuse-angled Triangles*.

15. *Obtuse-angled Triangles*, are those that have one Angle Obtuse, or more than 90 Degrees, as the Triangle DEF, Obtuse-angled at E, *Fig. 6.*

16. *Acute-angled Triangles*, are those that have no Right-angle nor Obtuse-angle, as GHI, *Fig. 7.* Hence all Equilateral Triangles, are Acute-angled; but of Isoceles Triangles, some are Obtuse-angled, as NOP, *Fig. 9.* some Acute, as the Triangle KML, *Fig. 8.*

Of four-sided Figures are the following Kinds.

17. A *Geometrical Square*, Equal-sided, and Right-angled, as QRST, *Fig. 10.*

19. A *Parallelogram*, or Oblong, is a long Square, having four Right-angles, but not four equal Sides, only the opposite Sides equal, as EFGH, *Fig. 2.*

19. A *Rhombus* hath four equal Sides, but no Right-angles; The two opposite Angles being Acute, and the other two opposite Angles Obtuse, as the Figure VWXY, *Fig. 11.* the Angles V and X being Obtuse, and W and Y Acute.

20. A

20. A *Rhomboides* hath the opposite Sides equal, but no Right-angle, as *ABCD*, *Fig. 12*.

21. All four-sided Figures, that come not under some of the foregoing Denominations, are called *Trapezia* or *Trapezium*, as *EFGH*, *Fig. 13*.

22. The *Diagonal* of a four-sided Figure, is a Line drawn from one Corner to the opposite Corner of the said Figure, as in *Fig. 23*. the Right-line *AC*, is the *Diagonal* of the Figure *ABCD*.

All Figures of more than four Sides, are called *Polygons*, of which some are regular, some irregular.

23. A regular *Polygon* hath all its Sides equal, and all its Angles equal, and is commonly distinguished by a Name referring to the Number of its Sides: Thus a five-sided regular Polygon is called a *Pentagon*, *Fig. 14*. And an eight-sided regular Polygon is called an *Octagon*, &c. as follows.

A regular Polygon of	{	5	} Sides is called	{	a Pentagon,
		6			a Hexagon,
		7			a Heptagon,
		8			an Octagon,
		9			a Nonagon,
		10			a Decagon, &c.

24. All Figures of above four Sides, and whose Sides are not equal, are called irregular Polygons, and distinguished by the Number of their Sides, as *Fig. 16*. is a five-sided irregular Polygon, &c. and may be ten, twenty, or any possible Number of Sides.

25. A *Circle* is a Line continued round a point called the Center, and on every side equally distant from it; and when the Ends are brought to meet, it is called a perfect Circle. Every Circle contains 4 Right-angles, or 360 Degrees.

A Circle is the most comprehensive of all superficial Figures, a Line of the same length inclosing a greater Area or Square in it, than in any other Form or Figure whatsoever. *Fig. 17*. is a Circle.

26. The *Out-line* or *Bound* of a Circle is called the *Circumference* or *Periphery*, as the Out-Line *BDEFCG*. The Center is *A*, from which point every Part of the Circumference is equally distant.

27. The *Diameter* of a Circle, is a Right-line drawn quite across the Circle, and through the Center, as the Line *BAC*, *Fig. 17*. It divides the Circle into two equal Parts, and is the longest

longest Line that can be drawn cross a Circle. Half that Line, as A C or A B, is called the Semi-diameter of the Circle.

28. An *Arch* of a Circle is a Part of the Circumference of the Circle, as in the Circle, *Fig. 17.* the Part C G of the Circumference is called the Arch C G of the Circle.

29. A *Sector* of a Circle, is a piece cut out by two Semi-diameters, and is bounded by them, or contained between them and an Arch of the Circle, as in *Fig. 17.* The Part G A C, bounded by the two Semi-diameters A G, and A C, and the Arch G C, is called a Sector of the Circle.

30. A *Segment* of a Circle, is a part cut off by a Right-line drawn any where cross the Circle; as the Part D E F of the Circle, *Fig. 17.* is a Segment cut off by the Line D F.

Note, The Word Circle is used promiscuously, sometimes signifying the Line drawn equally distant from the Center, as in the Definition of a Circle above: but it sometimes signifies all the Space contained within the Line, and of which that Line is the Bounds or Limits, and is so to be understood, when we speak of the Area, or superficial Content of a Circle, and then this Out-Line is called the Circumference.

31. An *Ellipsis* is a Circular Figure, but of different Diameters, as *Fig. 18.* the longest Diameter H I, being called the Transverse Diameter, and the shortest K L the Conjugate Diameter.

Axioms to the First Book of Euclid's Elements.

1. Things equal to one and the same Thing, are equal to one another.

2. If to equal Things are added equal Things, the whole will be equal.

3. If from equal Things, equal Things be taken away, the Remainder will be equal.

4. If equal Things be added to unequal Things, the whole will be unequal.

5. If equal Things, be taken from unequal Things, the Remainder will be unequal.

6. Things double to one and the same Thing, are equal between themselves.

7. Things which are half one and the same thing, are equal between themselves.

8. Things which mutually agree together, are equal to one another.

9. The whole is greater than its Parts.
10. Two Right-lines do not contain a Space.
11. All Right-angles are equal between themselves.
12. If a Right-line falling upon two other Right-lines, makes the inward Angles on the same thereof, both together, less than the Right-angles, those two Right-lines, infinitely produced, will meet each other on that Side where the Angles are less than Right ones.

Axioms to *Euclid's* Fifth Book of Elements are ; First, That Equimultiples of the same, or of equal Magnitudes, are equal to each other. And secondly, That those Magnitudes which have the same Equimultiple, or whose Equimultiples are equal, are equal to each other.

C H A P II.

G E O M E T R I C A L T H E O R E M S :

*Or, demonstrable Truths necessary to be understood
in Order to*

S U R V E Y I N G.

T H E O R. I.

ANY Right-line A G, standing upon the Middle of another Right-line B C, makes either two Right-angles, or two Angles whose Sum is equal to two Right-angles with the said Line B C. *Fig. 17. Euc. I. 13.*

For the Circle B D E F C G, contains four Right-angles (*Def. 24*) But the Diameter, B A C divides the Circle into two equal Parts (*Def. 26.*) therefore the part B G C is half a Circle, or two Right-angles ; and wherever a Line is drawn from the point A to the Periphery, it either divides the Semi-circle B H G C into two equal parts, as at H, and makes two Right-angles B A H and H A C, or into two unequal Parts, as at G, and makes two unequal Angles B A G and G A C ; but both together, equal to the whole Semi-circle, or two Right-angles ; which was to be proved.

Note, If more Lines than one be so drawn from the Point A, as A H and A G, the Sum of all the Angles B A H, H A G, and G A C taken together, shall be equal to two Right-angles.

T H E O R.

THEOR. II.

If two Right-lines BD and CE intersect one another, the opposite Angles BAC and EAD are equal, as also are the opposite Angles BAE and CAD. *Euclid. Lib. I. Prop. 15.*

For the Angles BAE and BAC, are equal to two Right-angles (*Theor. 1.*) also the Angles BAE and EAD are equal to two Right-angles (*Theor. 1.*) *Fig. 19.* Take away BAE common to both, there will remain BAC, equal to EAD, (*Ax. 3.*) which was to be proved.

THEOR. III.

If any Right-line GK cut two parallel Right-lines FH and LI, the alternate Angles Fnm and Inm are equal; and if the alternate Angles are equal, the lines are parallel. *Euclid. Lib. I. Prop. 29.*

For Fmn and Inm are equal, because FH and LI are parallel: But GmH is equal to Fmn (*Theor. 2.*) *Fig. 20.* therefore Inm and Fmn are equal, (*Ax. 1.*) which was to be proved.

THEOR. IV.

If any Side of a Right-lined Triangle be continued, the external Angle shall be equal to the two internal opposite Ones. *Eucl. I. 32.*

Let the Triangle be OPQ, continue the Side OQ to R; then is the external Angle PQR, equal to the two internal opposite Ones, OPQ and POQ;—draw QS parallel to OP; then is PQS equal to OPQ, and *Fig. 21.* SQR is equal to POQ; but PQS and SQR together, make the external Angle PQR; therefore the external Angle PQR is equal to both the internal opposite Angles OPQ and POQ; which was to be proved.

THEOR. V.

The three Angles of any Right-lined Triangle are equal to two Right-angles, or 180 Degrees. *Euclid. I. 32.*

For however the Line PQ fall upon the Line OR, it makes the two Angles PQO and PQR together, equal to two Right-Angles (*Theor. 1.*) but the Angles P and O together, are equal to the Angle PQR (*Theor. 4.*) therefore, *Fig. 21.* the Angles P, O, and PQO together, will be equal to two Right-angles, or 180 degrees, (*Ax. 2.*) which was to be proved.

Or

Or it may be thus demonstrated.

Let the Triangle be VXY , draw a Line to touch any of the Angles at V , which Line let be parallel to the opposite Side XY , as the Line TW : then is the angle TVX equal to VXY (*Theor. 3.*) but the Angle XVY with TVX and WVY together, are equal to two Right-angles (*Theor. 1.*) therefore the Angle XVY , together with the Angles X and Y , are equal to two Right-angles, (*Ax. 2.*) which was to be proved.

THEOR. VI.

Similar Triangles have their Sides proportionable; and on the contrary.

Triangles, whose Sides are equal or proportionable, have their Angles equal, which are subtended by the equal or proportionable Sides.

If in the Triangles OPQ and VXY , the Sides are proportionable; as OP to VX , so OQ to XY , and so PQ to VY ; then shall the opposite Angles Q be equal to Y , O equal to X , and V equal to P ; and Triangles, whose Angles are equal, have the Sides subtending those Angles equal or proportionable. *Euclid. Lib. VI. Prop. 4. and 5.*

THEOR. VIII.

In all four-sided Right-lined Figures, the four Angles taken together, are equal to four Right-angles.

In the Figure $ABCD$, draw the Diagonal AC , then is the Quadrangle divided into two Triangles, ABC and ADC , the three Angles of each of which Triangles taken together, are equal to two Right-angles (*Theor. 5.*) therefore the six Angles of the two Triangles (which contain the four Angles of the Quadrangle) are equal to four Right-angles; which was to be proved.

THEOR. VIII.

All the Angles of any Right-lined Figure (of how many Sides soever, and whether the Sides be equal or not) are equal to twice as many Right-angles, abating four, as there are Sides in the Figure.

Draw Lines from every Angle of the Figure, to any Point somewhere near the Middle of the Figure, so will the Figure be divided into as many Triangles as there are Sides in the Figure; but

but all the Angles of these Triangles taken together, amount to twice as many Right-angles as there are Triangles; because the three Angles of every Triangle are equal to two Right-angles (*Theor. 5.*) Also all the Angles about the Point within the Figure, how many soever, are equal to four Right-angles; because they fill up a Circle: These being taken away from the Sum of all the Angles of all the Triangles, there remains the Sum of all the Angles made by the Sides of the Figure.

Example. Let it be the Figure of six Sides $BCDEFG$; from any Point within, as at A , draw Lines to every Angle, as AB , AC , &c. So have you the six Triangles ABC , ACD , ADE , AEF , AFG , and AGB ; all the Angles of every one of them containing two Right-angles, viz. *Fig. 24.* Twelve Right-angles; but from twelve Right-angles, subtract four Right-angles (the Sum of all the Angles at A) there remains eight Right-angles, which is the sum of all the Angles ABC , ACB , ACD , ADC , &c. which then, supposing all the Lines AB , AC , AD , &c. to be taken away, you have the Sum of all the Angles at B , C , D , &c. viz. twice as many Right-angles, abating four, as is the Number of Sides.

THEOR. IX.

In Right-lined Triangles, equal Sides subtend equal Angles. *Euclid. Lib. 1. Prop. 5.* The greatest Side subtends the greatest Angle. *Euclid. Lib. 1. Prop. 19.* And consequently the least Side subtends the least Angle.

THEOR. X.

An Angle in a Semi-circle is a Right-angle; that is, if a Circle and its Diameter be drawn, and if Lines be drawn from each End of the Diameter, so as to meet one another any where at the Periphery, the Angle made by them shall be a Right-angle.

Let the Circle be BCD , the Center A , and the Diameter BAC ; and from the Extremities of the Diameter B and C , draw the Lines BD and CD to meet *Fig. 25.* any where in the Periphery, as at D : I say, the Angle BDC is a Right-angle.

From the Center A , draw AD ; then is AD equal to AB , and also to AC ; therefore the Angle B is equal to BDA , and the Angle ADC is equal to ACD (*Theor. 9.*) But the three Angles of the Triangle BDC , are equal to two Right-angles and the Angle B is equal to BDA , and C is equal to CDA there-

therefore BDA and CDA together, is equal to B and C together, equal to half of two Right-angles, viz. one Right-angle; which was to be proved.

Or it may be thus demonstrated:

The Angles CAD and DAB together, are equal to two Right-angles (*Theor.* 1.) But ADB and ABD together are equal to DAC (*Theor.* 4.) also ADC and ACD together, are equal to DAB (*Theor.* 4.) therefore ADC (equal to ACD) and ADB (equal to ABD) taken together, are equal to one Right-angle; which was to be proved.

Fig. 25.

THEOR. XI.

Parallelograms which stand upon the same (or an equal) Base, and between the same (or equal) Parallels, are equal: Thus the Parallelogram CDEF is equal to the parallelogram AB EF.

For AB is equal to EF; and also to CD, (*Def.* 18.) and BC common to both; then is AC equal to BD, (*Ax.* 2.) and AE equal to BF (*Def.* 18.) therefore the Triangles AEC and BFD are equal. Take away BHC, which is a Part of both, there will remain ABHE equal to HCDF (*Ax.* 3.) and EHF common to both, it makes AB EF equal to CDEF; which was to be demonstrated.

Fig. 26.

THEOR. XII.

If a Triangle have the same Base with a Parallelogram, and be between the same Parallels, it will be equal to half the Parallelogram.

In the Parallelogram IKLM, draw the Diagonal KL, it divides the Parallelogram into two equal Triangles: for the Side IL is equal to KM, and the Side IK is equal to LM, and KL is common to both; therefore the Triangles are equal, consequently each of them is equal to half the Parallelogram.—It will be the same in any Parallelogram, however the Diagonal be drawn.

Fig. 27.

THEOR. XIII.

All Triangles having the same or equal Bases, and between the same or equal Parallels, are equal.

It is proved (*Theor.* 11.) that Parallelograms standing upon the same Base, and between the same Parallels are equal, and

(Theor. 12.) that such Triangles are half the said Parallelograms; therefore Triangles standing upon the same Bases, and between the same Parallels, are equal, (Ax. 7.)

THEOR. XIV.

Parallelograms and Triangles that lie between the same Parallels are proportionable to their Bases; that is, as PL to LM, so the Parallelogram POIL, to the Parallelogram LIKM; and as LM to MN, so the Triangle LKM, to the Triangle MKN. *Euc. Lib. VI. Prob. 1.* Fig. 27.

THEOR. XV.

If a Line be drawn parallel to any Side of a Triangle, it cuts the other two Sides proportionally.

In the Triangle OPQ, draw TV parallel to the Side OQ, it shall cut the Sides PO and PQ; so that as PT to PO, so PV to PQ. Draw OV and TQ, and make the Triangles OQT and OQV which will be equal (Theor. 13.) therefore the Triangles OTV and TVQ are equal (Ax. 3.) But it will be as PTV to TOV, so PT to TO (Theor. 14.) and as PTV to TOV (because OTV is proved equal to TVQ) so PV to VQ; destroy the two first Terms (PTV and TOV) it will be as PT to TO, so PV to VQ; which was to be demonstrated. Fig. 21.

THEOR. XVI.

If in a Right-angled Triangle there be a Perpendicular let fall from the Right-angle upon the longest Side, it will divide the Triangle into two Triangles, similar and proportionable to the first, and to one another.

For the Triangles NOP and NQO are alike (by Theor. 6.) because NOP and NQO are both Right-angles, and N is an Angle common to both Triangles; and by the same Theorem NOP and OQP are alike, because both right-angled, and the Angle Q is common to both; therefore the Triangles NQO and OQP are alike by (Ax. 1.) and it will always be Fig. 28.

As the longer Leg in one Triangle
To its Hypothenuse,
So the longer Leg in the other
To its Hypothenuse,
And the same between any other similar Sides :

E 2

Hence,

Hence, as NQ (the longer Leg of the Triangle NQO)
 To NO (the Hypothenufe of the same :)
 So NO (the longer Leg of the Triangle NOP)
 To NP (the Hypothenufe of the Triangle NOP ;)
 Therefore the Square of NO , is equal to the Product of
 NQ , multiplied by NP .
 And by the same Rule,
 As QP (the shorter leg of the Triangle OQP)
 To OP (its Hypothenufe :)
 So OP (the shorter Leg of the Triangle NOP)
 To NP (its Hypothenufe ;)
 Therefore the Square of OP is equal to the Product of NP ;
 multiplied by QP , &c.

THEOR. XVII.

In all right-angled plain Triangles, the Square of the longest Side commonly called the Hypothenufe, is equal to the Sum of the Squares of the two short Legs.

Let the Triangle be ABC , right-angled at A ; and draw the Square $AHIC$, that each Side be equal to AC : Also with the other two Sides AB and BC , draw the Squares
Fig. 29. AF and BD ; draw AK perpendicular to BC , and continue it to L , parallel to BE .

The Product of CD (equal to BC) multiplied by KC produceth the Parallelogram $KCDL$, and is equal to the Square of AC , which is the Square $AHIC$ (*Theor. 16.*) and by the same Rule and Theorem, the Parallelogram $BKLE$, the Product of BE (equal to BC) multiplied by BK , is equal to the Square $ABFG$, the Square of AB ; but if $BKLE$ be equal to $ABFG$, and $KCDL$ be equal to $AHIC$, then the whole $BCDE$ (the Square of BC) is equal to both $AHIC$ and $ABFG$, the Squares of the Legs; which was to be proved.

CHAP. III.

GEOMETRICAL PROBLEMS:

*Or, Things to be performed in Order to the
Practice of*
SURVEYING.

PROB. I.

FROM a given Point in, or near, the Middle of a given Line to raise a Perpendicular.

Let the given Line be EG , and the given Point H , from whence it is required to raise the Perpendicular HF . Fig. 4.

Take any Extent in your Compasses, and with one Foot in the given Point H , make the marks E and G ; then open the Compasses to a little wider Extent, and with one Foot in E make the Arch F , and with the same Extent, and one Foot in G , cross the said Arch in F ; then from the said Crossing at F to the Point H , draw the Line HF , it will be the Perpendicular required.

For if you imagine Lines drawn from F to E , and from F to G ; then there are two Triangles formed, in which FE is equal to FG , and EH is equal to HG , and FH is common to both; therefore the Triangles are alike, and their Angles are equal (*Theor. 6.*) $\angle FHE$ to $\angle FGH$, $\angle FEH$ to $\angle FGH$, and $\angle FHE$ to $\angle FGH$: But $\angle FHE$ and $\angle FGH$ are equal to two Right-angles (*Theor. 1.*) and being equal between themselves, they are two Right-angles, and FH is perpendicular to EG ; which was to be proved.

PROB. II.

From a given Point F , over the Middle of the Line EG , to let fall a Perpendicular.

With any Extent more than the nearest Distance from the Point to the given Line, and less than the Extent from the said Point to the (nearest) End of the said Line, and placing one Foot in the Point F , make the Marks E and G . Divide the Space EG into two equal Parts in the Point H , making EH equal to HG ; draw a Line from th
E 3
Fig. 4.
given

given Point F to the Point H, it is the Perpendicular required. Demonstrated, as in *Prob. 1.*

PROB. III.

To raise a Perpendicular from the End of a given Line, suppose from the End N of the Line MN.

With any Extent (but best with little more than half the given Line) and one Foot in the given End N; turn the other End about, and place it any where above the Line, as in the Point P; and keeping it there, turn the other End about to cross the Line somewhere, as in M. Keep the Point still fixed in the Point P, and with the other make the Arch O over the Point N; lay a Ruler from the Mark at M, over the Point P, and where it cuts the Arch O, make a Mark; draw a Line from the Point O (where the Ruler cuts the Arch) to the given Point N, it is the Perpendicular required to be raised.

The Points M, N, and O are equally distant from the Point P by Construction; and the Points M, P, and O are in a Right-line, because O is found by laying a Scale over M and P. Therefore,

If upon the Center P, and with the Distance PO, PN, or PM (for they are equal) an Arch of a Circle was drawn from M to N and O, it would be a Semi-circle; and if the Line MPO was drawn, it would be its Diameter, and the Angle MNO would be an Angle in the Semi-circle, and consequently a Right-angle (by *Theor. 10.*) therefore NO is perpendicular to MN; which was to be demonstrated.

PROB. IV.

How to let fall a Perpendicular from a Point over a Line, when the Perpendicular will fall so near the End of the Line, that the Method for letting fall a Perpendicular from a Point over the Middle of a Line is impracticable.

Let the Line be RS, and the Point T be the Point from which the Perpendicular is to fall.

From the given Point T, draw the Line TR to cross the given Line RS, somewhere toward the contrary End, to which the Perpendicular is to fall, as in R; divide the Line TR equally in V, so that VR be equal to VT; then with the Extent VT or VR, and one Foot of the Compasses in V, with the other cross the Line RS in S. Lastly, from the given Point T to the Mark at S, draw the Line TS, it is the Perpendicular required.

Demonstrated as in *Prob. 3.*

PROB.

PROB. V.

To draw a Line parallel to a Right-line, at any given Distance from it.

Suppose the Right-line given at AB; take in your Compasses the Distance that is to be between the given Line, and the proposed parallel Line, and with one Foot of your Compasses in or near one End of the given Line, as at A, with the other describe the small Arch C; then with the same Extent, and one Foot in or near the other End of the Line, as at B, with the other draw the small Arch D; draw a Right-line just to touch the Extremity of the said Arches, it will be the Parallel required. Fig. 32.

PROB. VI.

To divide a Line given into two equal Parts, commonly called bisecting a Line.

Let the given Line be EF; take in your Compasses any Extent more than half, and (most conveniently) less than the whole Line, and with one Foot in one End of the Line as at F, draw the Arch GH; then with the same Extent, and one Foot in the other End at E, cross the said Arch in G and H: Lay a Scale from the crossing at G to that at H, and mark where it cuts the Line EF, as at O, for that Mark is the Middle of the Line EF. Fig. 33.

PROB. VII.

To make an Angle equal to an Angle given.

Let the given Angle be IKL, and let it be required to draw a Line, to make with the Line MN, an Angle equal to IKL.

With any Extent in your Compasses less than the Length of the shortest of the given Lines, and one Foot in the given angular Point K, sweep the arch PO; and with the same Extent, and one Foot in the Point where the other Angle is intended to be made, as at M, draw the Arch QR Fig. 34. at Pleasure; take in your Compasses the Extent OP, and keeping that Extent in your Compasses, set one Foot in Q, and set the other upwards to R. From M, and through R, draw the Right-line MR, continued at Pleasure, it shall make, at M, an Angle equal to the given Angle IKL, as required.

PROB. VIII.

To lay down an Angle to contain any (possible) given Number of Degrees : Suppose 48.

This requires the Help of a Line of Chords, and is thereby thus performed.

Draw the Line *ST* at Pleasure ; take the Chord of 60 Degrees (always) and with one Foot of the Compasses at the End of the Line where the Angle is to be made, as at *S*, with the other draw the Arch *VW* ; take in your Compasses *Fig. 35.* the Degrees proposed, which here is 48, from the Line of Chords, and with one Foot in *V*, extend the other upwards to *W* ; then through *W*, draw the Line *SW* continued ; it will make, with the Line *ST*, an Angle of 48 Degrees required.

PROB. IX.

Three Sides of a Triangle given, to lay down the Triangle, provided any two of the Sides be longer than the third.

Let the three Sides be the three Lines *A*, *B*, and *C*, and from them to form the Triangle *DEF*. *Fig. 36.*

Take the longest Side *C* in your Compasses, and with that Length draw the Line *DE* ; then take in your Compasses the next Side *B*, and with one Foot in *D*, draw the Arch *F* ; then take the shortest Side in your Compasses, and with one Foot in *E*, cross the aforesaid Arch *F* in *F*, and from that Crossing draw the Lines *FD*, and *FE*, and that is the Triangle required to be drawn.

PROB. X.

To bring any three Points (not situate in a Right-line) into the Circumference of a Circle.

Suppose the three Points be *G*, *H*, and *I* ; first with any Extent more than half, and less than the whole Distance *GH*, and one Foot in *G*, make the Arch *MON*, and *Fig. 37.* with the same Extent, and one Foot in *H*, draw the Arch *MPN*, to cross the former in *M* and *N*.

Also with the same Extent, or any other more than half *HI*, and one Foot in *H*, draw the Arch *LSK* ; and with this last Extent, and one Foot in *I*, draw the Arch *LXK*, to cross the former Arch in *L* and *K* ; then through the Crossings at *L* and *K*, draw a Line *KLO* : Also through the Crossings at *M* and *N*, draw the Line *NMO*, to cut the Line *KLO* in *O* ; then is *O* the Center of the Circle : If you place one Foot of the
Com-

Compasses in O, and extend the other to G, H, or I, that Extent will carry a Circle through all the three Points.

PROB. XI.

To divide a given Right-line into any Number of equal Parts.

Suppose I would divide the Line OP into five equal Parts.

First from P, draw the Line PQ of any convenient Length, and to make any Acute-angle with the Line OP. Fig. 38.

Also from O, draw the Line OR, to make the Angle POR, equal to the Angle QPO; for then will QP and OR be parallel (*Theor. 3.*) Set off four of any equal Parts (always one less than the Number of Parts into which the Line is to be divided) from P towards Q, and the same from O towards R, and draw Lines from 1 to 4, from 2 to 3, &c and where these Lines cross the Line OP shall be the Points that divide it into five equal Parts.

PROB. XII.

To make a true Geometrical Square, whose Side shall be equal to a Right-line given.

Let the given Line be ST, and it is required to make a Geometrical Square, whose Side shall be equal to it. Fig. 39.

Draw the Line YX equal to ST, and (by *Prob. 3.*) raise the Perpendicular YV from the Point Y, and set the Length of the Line ST, from Y to V; then with the same Length ST in your Compasses, and one Foot in V, make the Arch at W; and again, with the same Extent, and one Foot in X, cross the said Arch at W, and to the said Crossing, draw the Lines VW and XW, and the Square is finished.

PROB. XIII.

Having two Lines given, to find a third in a Geometrical Proportion, viz. the Square of the second equal to the Product of the first and third multiplied together: Or, as the first to the second, so the second to the third.

Suppose the two Lines be H and I, and it is required to find a Line in such Proportion, that as H to I, so I to the Line required. Fig. 40.

Draw the Line AC of any Length, and from one End of it, as A, draw the Line AB, continued to make any Acute-angle at Pleasure, with the Line AC, as the Angle BAC.

Take

Take the shorter of the two given Lines, as H, in your Compasses, and set from A to D; take also the longer of the two given Lines, as I, in your Compasses, and set it upon the Line AC, from A to E; and also on the Line AB, from A to F, draw the Line DE; and then from the Point F (by *Prob. V.*) draw FG parallel to DE; then it shall be

As AD (equal to H) to AE (equal to I:) So the said AE (equal to AF) to AG, the third Proportional required.

For DE is parallel to FG by Construction; therefore the Triangles ADE and AFG are proportionable. *Euc. Lib. VI. Prop. 2. viz.* As AD to AE, so AF to AG.

But AF is equal to AE by Construction: therefore, as AD to AE so AE to AG; consequently, as H to I, so is I to AG, the Line required.

PROB. XIV.

To divide a given Line into two Parts, which may be in such Proportion to each other, as two other given Lines.

Fig. 41. Let the whole Line be KL 35, to be divided in such Proportion, as the Line M 6, is to the Line N 9.

From either End of the given Line, as K, draw a Line KQ, to make any Angle at Pleasure with KL; then take in your Compasses the Line M, and set off upon the Line KQ, from K to P.

Set also the Line N from P to Q, and draw QL, and parallel to that draw PO from P to O; then is the Line QL divided in O, into two Parts that bear the same Proportion to each other, as the Line M to the Line N.

For QL and PR are parallel by Construction; therefore as KP to PQ, so KO to OL. *Euclid. Lib. VI. Prop. 2.* Also, by *Theor. 15.*

But KP is equal to the Line M, and PQ equal to N; therefore,

As M to KO, so N to OL; which was to be done.

Or in Numbers, the Line KL 35, is divided into two Numbers 14 and 21, in Proportion, as 6 to 9; for as 6 to 14, so 9 to 21.

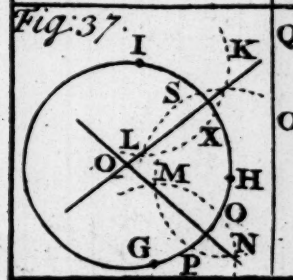
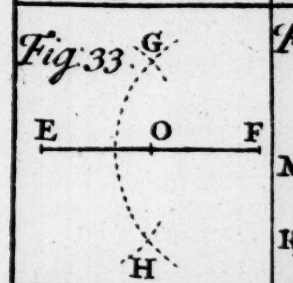
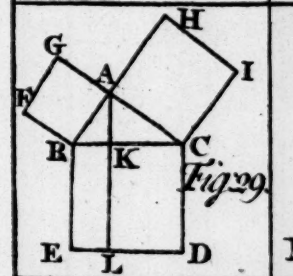
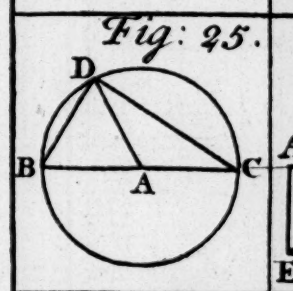
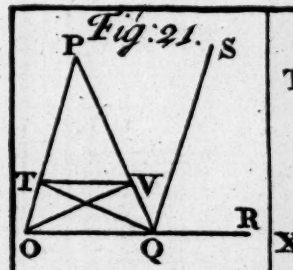
PROB. XV.

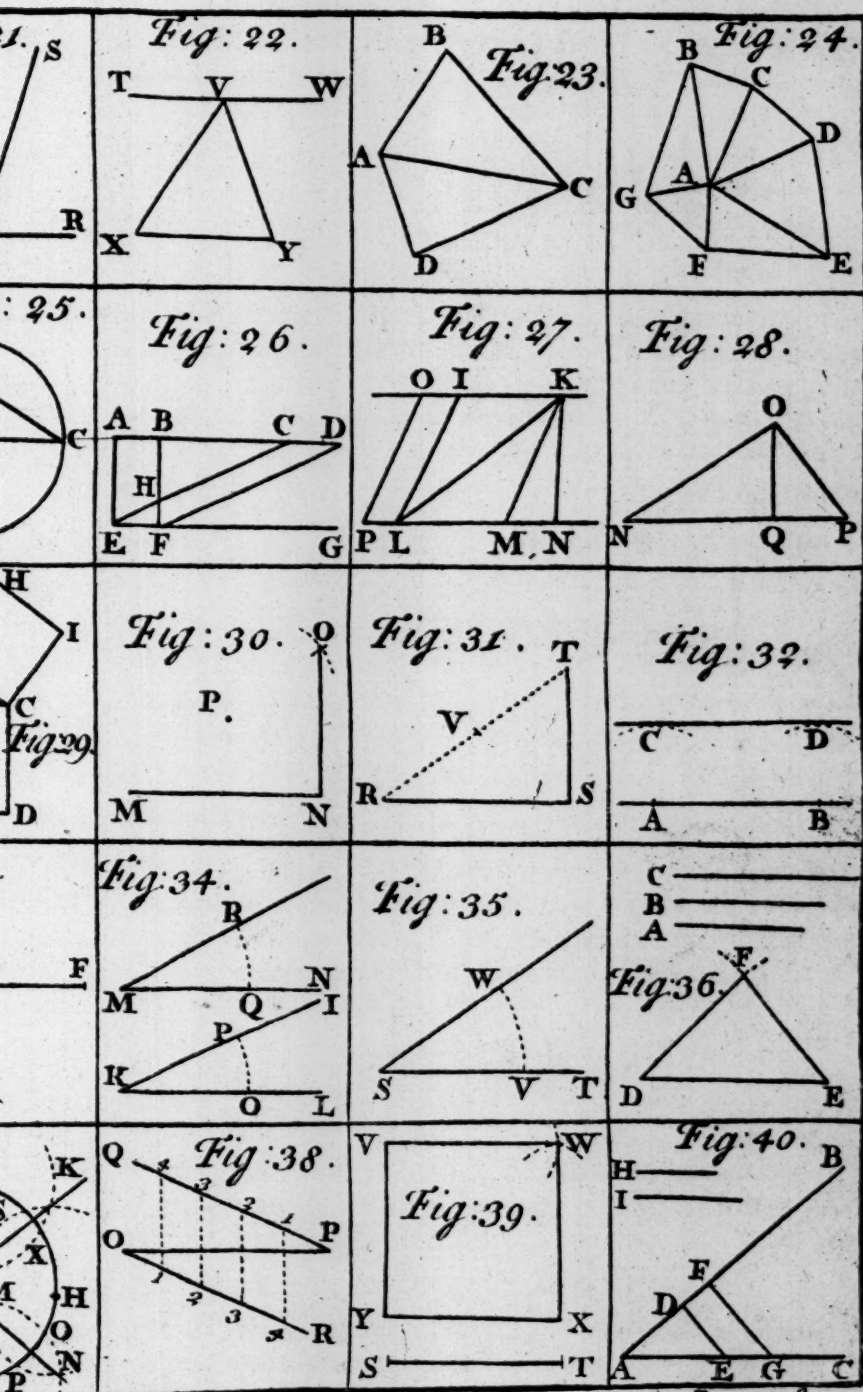
Three Lines being given, to find a fourth proportional; that is, as the first to the second, so the third to the fourth.

Fig. 24. Let the given Lines be G, H, and I, to find a fourth, that as G to H, so I to the fourth Line found.

Draw RS and RT continued, to make any Angle at Pleasure at R.

Set





Set the Line G from R to V, and set H from R to Y; and lastly, set L from R to X; then draw V Y, and parallel thereto, from X, draw the Line XZ, to cut RS in Z; then is RZ the fourth Line required. For,

As RV to RY, so RX to RZ. Demonstrated as *Prob. 13.* and 14.

N. B. In the foregoing Theorems and Problems, Problems the 13th, 14th, and 15th, quote *Euclid, Book VI. Prob. 2.*

<i>Theorem</i>	{	1, quotes <i>Book 1, Prob. 13.</i>	
		2, _____ 1, _____ 15.	
		3, _____ 1, _____ 29.	
		4, _____ 1, _____ 32.	
		5, _____ 1, _____ 32.	
		6, _____ 6, _____ 4, and 5.	
		9, _____ 1, _____ 5, and 19.	
		14, _____ 6, _____ 1.	

C H A P. IV.

Plain T R I G O N O M E T R Y.

WHAT the several Sorts of Angles and Triangles are: See *Definition 10, 11, 12, 13, 14, 15, 16, Chap. I.* And for laying down any Angle or Triangle, see *Prob. 7, 8, and 9.*

S E C T. I. *Axioms.*

There are four Axioms by which all right-lined Triangles are solved. The first Axiom is confined to right-angled Triangles, and the other three extend to either right-angled Triangles or oblique; in either of which, the Base signifies the longest Side; which, if it be a right-angled Triangle, is the Hypothenuse: See *Def. 10.*

A X I O M I.

In all right-angled plain Triangles, if one Leg be made Radius, the other Leg is the Tangent of its opposite Angle; and the Hypothenuse is the Secant of the same Angle; but if the Hypothenuse

pothenuse be made Radius, the Legs are the Sines of their opposite Angles ; that is,

As the Leg made Radius,
Is to the Radius or Sine of 90 Degrees ;
So the other Leg,
To the Tangent of its opposite Angle.

But as you are taught in the Golden-Rule, *Chap. VIII.* That the required Term is always the last or fourth Term, and must be always of the same Denomination with the second Term, we hence deduce this universal Rule.

Note, *If a Side be required, let an Angle be the first Term in the Proportion ; but if an Angle be required, begin with a Side ; that is, as GH to Radius, so HI, to the Tangent of the Angle, G : (See Fig. 43.) Or, as Radius to GH, so Tangent of the Angle G, to the Side IH.*

AXIOM II.

In all plain Triangles, right-angled or oblique, the Sides are proportionable to the Sines of their opposite Angles, by the second Part of *Axiom I* ; that is,

As the Sine of the Angle D,
To the Side EF :
So Sine of the Angle E,
To the Side DF.

Fig. 44.

Note, *The Angle E is obtuse (see Def. 7.) and the Rule to find the Sine of an Obtuse-angle is,*

Subtract the Degrees and Minutes of the Obtuse-angle from 180 Degrees, the Sine of the Remainder is the Sine of the Obtuse-angle required.

As, if I desire to know the Sine of 132 Degrees 35 Minutes, it is the Sine of 47 Degrees 25 Minutes, as in the Margin, viz. 9.86705.

180 : 00

132 : 25

47 : 25

AXIOM III.

In all right-lined Triangles,
As the Sum of Sides, including an Angle given,
To the Difference of the said two Sides ;

So

So the Tangent of half the Sum of the other two Angles,
To the Tangent of half their Difference.

Note, If one Angle be given, the Sum of the other two is also given, because the three are equal to 180 Degrees, by Theor. 5.

This half Difference, added to the half Sum, gives the greater of the unknown Angles ; and subtracted from it, leaves the less ; that is,

As the Sum of the Sides DE and EF,
To the Difference of the said Sides :
So the Tangent of half the Sum of the Angles D and F,
To the Tangent of half the Difference of the } *Fig. 44.*
said Angles.

AXIOM IV.

In all right-lined Triangles,
As the Base or longest Side DF,
To the Sum of the other two Sides DE, and } *Fig. 44.*
EF :
So the Difference of the said two Sides DE and EF,
To the Difference of the Segments of the Base.

Note, The Segments of the Base are DG and GF : The Base is divided into these two Segments in G, by a Perpendicular let fall from E to the Base DF, by Prob. 2.

SECT. II. Trigonometry Right-angled.

CASE I.

Given, the Hypothenuse, and one Acute-angle, to find a Leg.

Given { GI 540,
Angle G 33 45. } Required HI. *Fig. 43.*

CONSTRUCTION.

To lay down this Triangle, draw the Line GH at Pleasure ; and from one End of it, as G, draw another Line GI, to make the given Angle 33 45, with the Line GH, by *Prob. 8.* which will be the Line GI, upon which set 540 (the Side GI) from G to I, and from I let fall the Perpendicular IH, by *Prob. 4.* and then is GHI the Triangle required, and the Leg HI measured by the same Scale of equal Parts, from whence you took the Side GI, will be found to be 300.

All

All Trigonometrical Calculations are now generally performed by Logarithms, in which Addition performs the Office of Multiplication, and Subtraction performs Division; and as in the Golden-Rule direct, it is, as the first Term to the second, so the third to the fourth, so it is in Trigonometry; and therefore, whereas in the Golden-Rule, you multiply the two last numbers together, and divide the Product by the first, you must now observe the following Rules in this, and all other Calculations by Logarithms and Sines, or Tangents:

1. That all Sines or Tangents represent Angles; but all common Logarithms found in the Table of Logarithms, represent Sides.

2. Add the two last Numbers or Logarithms together, and from their Sum subtract the Logarithm of the first Term, the Remainder shall be the Logarithm of the fourth Term required, viz. Either the Logarithm of a Side, or the Sine or Tangent of an Angle.

3. If Radius be one of the two last Numbers given, you have only to subtract the first Number from the other of the two last, with 1 added on the Left-hand of it: For to add Radius to any Sine, is but to place 1 before that Sine. Thus, to add Radius 10.0000, to Sine of 25 Degrees 9.6260 will stand as you see, and may be readier done by setting 1 before 9.6260, and will stand thus without that Trouble, 19.6260.

4. If Radius be the first of the three given Terms, you need only add the two last together, and from their Sum, cut off the 1 towards the Left-hand, and the Remainder is the Logarithm, Sine, or Tangent required.

5. If none of the three given Terms be Radius, then instead of the first Term, take its Complement Arithmetical (which for Brevity, is commonly expressed Co. Ar.) by taking every Figure therein from 9, (or for more exactness, take the last Figure towards the Right-hand from 10) and set the Remainder for the first Term, and then add all the three Logarithms together, and from their Sum, cut off 1 towards the Left-hand, and the remaining Figures is the Logarithm, Sine, or Tangent required.

I shall explain these Rules by some Examples in the following Cases.

CALCULATION.

In the foregoing Triangle, Case I. Fig. 43.

Here is a Side required; therefore an Angle must be the first Term by the Rule in Ax. 1. therefore by Ax. 2.

As Sine of the Angle H 90°	- - - -	10.0000
To Side opposite G I 540:	- - - -	2.7324
So Sine of the Angle G 33° 45'	- - - -	9.7447
To Side opposite H I 330.	- - - -	1)2.4771

By

By Rule 4. above, add the two last Logarithms together, and from their Sum cut off 1 towards the Left-hand, the Remainder 2.4771 is the Logarithm of 300, the Side HI required.

CASE II. Fig. 43.

Given, the Angles and a Leg, to find the Hypotenuse.

Given $\left\{ \begin{array}{l} \text{Angle G } 33\ 45 \\ \text{Leg HI } 300 \end{array} \right\}$ Required GI.

The Angle I is 56 15. by Theor. 5. Therefore,

CONSTRUCTION.

Draw the Line HI 300, and at I, make the Angle HIG 56 15, by Prob. 8. and from H erect the Perpendicular HG, by Prob. 3. to cut IG in G; then is IHG the Triangle required.

CALCULATION, by Ax. 2.

As Sine of G 33 45	- - - - -	9.7447
To Side opposite HI 300	- - - - -	2.4771
So Radius or Sine of H 90 : 00	- - - - -	10.0900
To Side IG 540	- - - - -	2.7324

CASE III. Fig. 43.

Given, the Angles and a Leg, to find the other Leg.

Given $\left\{ \begin{array}{l} \text{Angle G } 33\ 45 \\ \text{Leg HI } 300 \end{array} \right\}$ Required Leg HG.

Construction, as in Case 2.

CALCULATION.

This may be performed by Axiom 1; by Axiom 2. it follows also, making the given Leg HI Radius, thus,

As Radius 90°	- - - - -	10.0000
To Leg HI 300	- - - - -	2.4771
To Tangent of Angle I 56 : 15	- - - - -	10.1751
To Leg GH 449	- - - - -	12.6522

CASE

CASE IV. *Fig. 43.*

Given the Hypothénuse and a Leg, to find an Angle.

Given { Hypothénuse IG 540 } Required Angle I.
 { Leg HG 449 }

CONSTRUCTION.

Draw GH 449, from G to H, and at H erect the Perpendicular HI; then take in your Compasses the Side IG, and setting one Foot in G, observe where the other Foot in turning about will cut the Perpendicular HI, which will be in I. From that Point to G, draw the Line IG, and your Triangle is finished.

CALCULATION, by *Ax. 2.*

As S IG 540	- - - - -	2.7324
To Radius	- - - - -	10.0000
So Side HG 449	- - - - -	2.6522

To Sine of Angle I 56 : 15	- - - - -	9.9198
----------------------------	-----------	--------

If the Angle G was required, subtract the Angle I 46 : 15 from 90 Degrees, and the Remainder 33 : 45, is the Angle G, by *Theor. 5.*

CASE V. *Fig. 43.*

Given, the Hypothénuse and a Leg, to find the other Leg.

Given { Hypothénuse IG 540 } Required Leg HI.
 { Leg HG 449 }

Construction, as in *Case 4.*

CALCULATION.

Find the Angle G 33 : 45, as in *Case 4.* and then by *Ax. 2.* find the Leg HI required.

As Radius,	- - - - -	10.0000
To Side IG 540	- - - - -	2.7324
So Sine of Angle G 33 : 45	- - - - -	9.7447
To Leg HI 300	- - - - -	1)2.4771

CASE

Fig. 1.

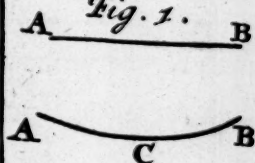


Fig: 5.

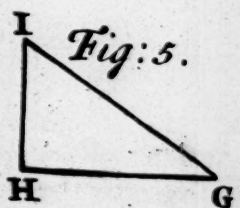


Fig: 9.

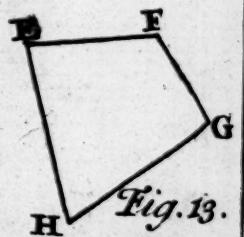
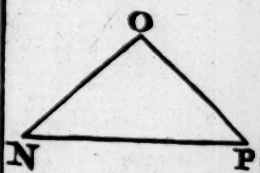
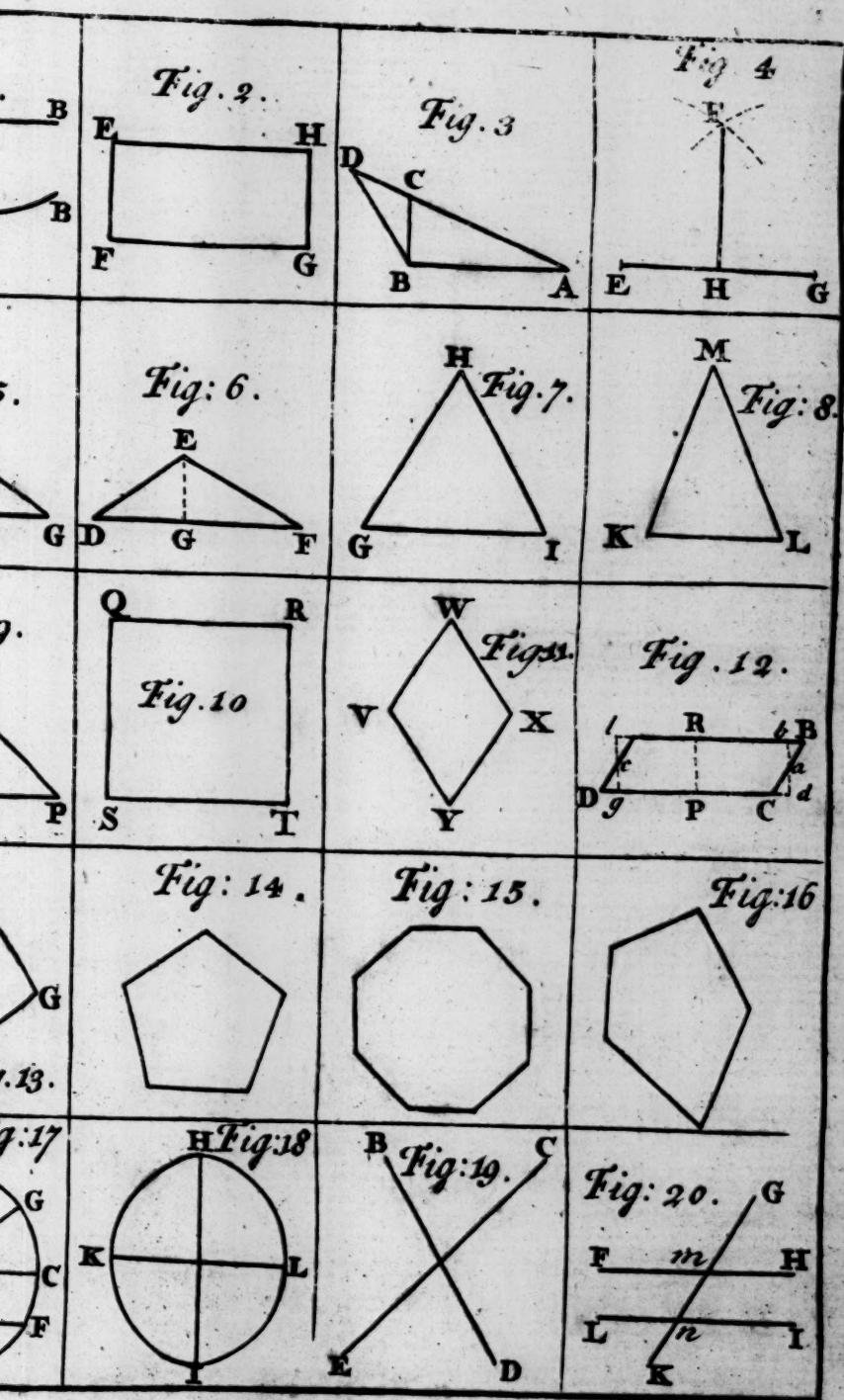


Fig. 13.

Fig: 17





CASE VI. *Fig. 43.*

The Legs given, to find the Angles.

iven $\left\{ \begin{array}{l} \text{Leg HI } 300 \\ \text{Leg GH } 449 \end{array} \right\}$ Required Angle G.

CONSTRUCTION.

Draw GH 449 from G to H, and at H erect the Perpendicular HI, upon which set 300 from H to I, and draw IG, it finishes the Triangle.

CALCULATION, by *Ax. 1.*

As Leg GH 449	- - - - -	2.6522
To Radius	- - - - -	10.0000
So Leg HI 300	- - - - -	2.4771

To Tangent of the Angle G 33:45 - - 9.8249

The Angle G being found, the Angle I is found, as in *Case 4.*

CASE VII. *Fig. 43.*

The Legs given to find the Hypothenufe.

Given $\left\{ \begin{array}{l} \text{Leg GH } 449 \\ \text{Leg HI } 300 \end{array} \right\}$ Required GI.

Construction, as in *Case 6.*

CALCULATION, by *Ax. 2.*

Find the Angle G, as in *Case 6.* and then to find the Hypothenufe.

As Sine of Angle G 33:45	- - - - -	9.7447
To Leg HI 300	- - - - -	2.4771
So Radius or Sine of H 90:00	- - - - -	10.0000
To the Hypothenufe GI 540	- - - - -	2.7324

SECT. III. *Of Oblique Plain Triangles.*CASE I. *Fig. 44.*

Two Angles, and a Side opposite to one of them, being given, to find the Side opposite to the other.

Given $\left\{ \begin{array}{l} \text{The Angle F } 30 : 0 \\ \text{The Angle D } 45 : 0 \\ \text{The Side D E } 290 \end{array} \right\}$ Required the Side E F.

CONSTRUCTION.

Draw D F at Pleasure, and from D draw D E, to make the given Angle of 45 with D F, by *Prob. VIII.* and set 290, from D to E; then because the Angles D and F together, make 75 Degrees, the Angle E is 105 Degrees, by *Theor. 5.* therefore from E draw E F, to make with E D an Angle of 105 Degrees, by *Prob. 8.* and then is D E F the Triangle required.

Note, *When one of the Sides of the Triangle that contains the proposed Angle, is not so long as your Radius or Chord of 60, you may continue the Side by a prick'd Line or Pencil, as here I have continued the Side E D to M.*

Note also, *When an Angle is obtuse, or more than 90 Degrees, you must take it off the Chords twice, because the Line of Chords goes but to 90 Degrees.*

Observe, *When the Sine of any Angle above 90 is required, subtract the Degree and Minute from 180, and the Sine of the Remainder is the Sine required: So the Sine of 105 is the Sine of 75, &c.*

CALCULATION, by *Ax. 2.*

As Sine of Angle F	30 : 0	- -	Co. Ar.	0.3010
To Side D E	290	- - - - -		2.4624
So Sine of Angle D	45 : 0	- - - - -		9.8495
To Side E F	410	- - - - -		1)2.6129

Here we have taken the Co. Ar. of the first Term, 9.9999 according to the Direction of *Rule V.* at the Beginning of this Chapter; the Sine of 30 being 9.6989, the Co. Ar. is 0.3010, as in the Operation annexed.

CASE

CASE II. Fig. 45.

Given, two Sides and an Angle opposite to one of them, to find the Angle opposite to the other.

Given $\left\{ \begin{array}{l} \text{Angle K } 45 : 0 \\ \text{Side LM } 410 \\ \text{Side KM } 560 \end{array} \right\}$ Required the Angle I.

This is called one of the ambiguous Cases, because it will admit of two Answers, for the given Sides and Angles may remain the same, and yet the required Angle L may be either acute or obtuse.

CONSTRUCTION.

Draw KM 560, and from K draw KL; continued, to make at K, an Angle of 45, with KM, by *Prob. 8.* take in your Compasses the given Side LM 410, and with one Foot in M, turn the other about, it will cross the Line KL, continued, in L and L; then if the Angle L be supposed obtuse, the lower L is the Angle required; but if acute, the uppermost L is the Angle, and the prick'd Line LM is the third Side of the Triangle.

CALCULATION. by Ax. 2.

As Side LM 410 - - - Co. Ar. - - 7.3872
To Sine of the Angle K 45 : 0 - - - 9.8494
So Side KM 560 - - - - - 2.7482

To Sine of the Angle L - - - - - 1)9.9848

The Angle L is 74 : 58, if it be supposed acute, or 105 : 2, if it be obtuse.

180 : 00
74 : 58

105 : 02

CASE III. Fig. 45.

Given, two Sides, and an Angle opposite to one of them, to find the third Side.

Given $\left\{ \begin{array}{l} \text{Side LM } 410 \\ \text{Side KM } 560 \\ \text{Angle K } 45 : 0 \end{array} \right\}$ Required the Side KL.

F 2

Here

Here again, as before, it mst be known, whether the Angle L be obtuse or acute; for if the Angle L be acute, it makes the Side K L longer than if it was obtuse, as appears by the Figure.

Construction is the same as in *Case 2*.

CALCULATION.

Find the Angle L, as in *Case 2*. and the Angle K being given 45 : 0, the Angle M is found by *Theor. 5*. and then by *Axiom 2*.

If the Angle L be acute,

As Sine of the Angle L 74 : 58	Co. Ar. -	0.0151
To Side K M 560	- - - - -	2.7482
So Sine of the Angle M 60 : 2	- - - - -	9.9376
To the Side K L 502	- - - - -	1)2.7009

But if the Angle L be obtuse, viz 105 : 2, the Angle M will be but 29 : 58; and then, as before by *Ax. 2*.

As Sine of the Angle L 105 : 2	Co. Ar. -	0.0151
To Side K M 560	- - - - -	2.7482
So Sine of the Angle M 29 : 58	- - - - -	9.6985
To Side K L 290	- - - - -	1)2.4618

CASE IV. Fig. 46.

Given, two Sides, and the Angle contained between them, to find either of the other Angles, or consequently both.

Given $\left\{ \begin{array}{l} \text{Side OP } 410 \\ \text{Side NP } 560 \\ \text{Angle P } 30 : 0 \end{array} \right\}$ Required the Angle N.

CONSTRUCTION.

Draw the Line NP 560, and from P draw PO, to make the given Angle of 30 Degrees at P, by *Prob. 8*. and set 410 from P to O; then from O to N, draw the Line ON, and the Triangle is finished.

CAL-

CALCULATION.

This Case is performed by *Axiom* the third.

Side NP	- - - - -	560		- - - - -	180.0
Side OP	- - - - -	410		Angle given	- - - 30.3
Sum	- - - - -	970		Sum of the other two Angles	150.0
Difference	- - - - -	150		Half Sum	- - - 75.0

As the Sum of the Sides 970 Co. Ar. - - 7.0132
 To their Difference 150 - - - - - 2.1761
 So Tangent of $\frac{1}{2}$ Sum of the unknown } 10.5719
 Angles 75 : 0 - - - - - }
 To Tangent of $\frac{1}{2}$ their Difference 29 : 59 - 9.7612

Half Sum of the unknown Angles - - - - 75.00
 Half their Difference - - - - - 29.59
 Added together, gives the Angle O - - - 104.59
 Subtracted, leaves the Angle N - - - - 45.01

CASE V. Fig. 46.

Given { Side OP 410
 Side NP 560 } Required the Side ON.
 Angle P 30 : 0 }

Answer, ON = 290.

Construction, as in Case 4.

CALCULATION.

Find the Angle O and N, as in Case 4. and then find the Side ON required, by *Ax.* 2.

As Sine of Angle N 45 : 1 Co. Ar. - 0.1504
 To Side OP 410 - - - - - 2.6127
 So Sine of Angle O 104 : 59 - - - - 9.6989
 To Side NP 560 - - - - - 1)2.7480
 As Sine of the Angle N 45 ; 1 Co. Ar. - 0.1504
 To Side OP 410 - - - - - 2.6127
 So Sine of Angle P 30 : 00 - - - - - 9.6989
 To Side NO - - - - - 1)2.4620

TRIGONOMETRY.

CASE VL Fig. 47.

Three Sides given to find an Angle.

Given $\left\{ \begin{array}{l} \text{Side RS } 290 \\ \text{Side RT } 560 \\ \text{Side ST } 410 \end{array} \right\}$ Required Angle R.

CONSTRUCTION.

This is laid down by *Prob. 9.*CALCULATION. by *Ax. 4.*

Side ST	- - - - -	410
Side RS	- - - - -	290
Sum of the Sides	- - - - -	<u>700</u>
Difference of the Sides	- - - - -	120
Base RT	- - - - -	560

As the Base RT 560	- Co. Ar.	- -	7.2518
To the Sum of the Sides 700	- - - -	- -	2.8451
So the Difference of the Sides 120	- - - -	- -	<u>2.0791</u>
To the Difference of the Segments RW	- - - -	- -	} 1)2.1760
and WT 150	- - - -	- -	
Half the Base	- - - -	- -	280
Half Difference of the Segments	- - - -	- -	<u>75</u>
Sum is the Segment WT	- - - -	- -	355
Difference is the Segment RW	- - - -	- -	205

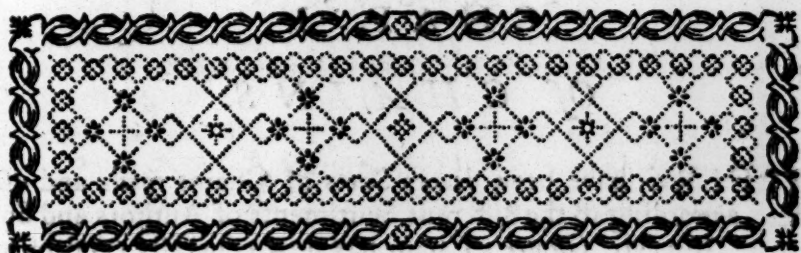
Then by *Case 4.* of right-angled plain Triangles,By *AXIOM II.*

As RS 290	- - - - -	- - - -	2.4624
To Radius	- - - - -	- - - -	10.0000
So RW 205	- - - - -	- - - -	<u>2.3117</u>
To Sine of the Angle RS W	45 : 0	- - - -	9.8493

The Angle RS W 45 subtracted from 90, leaves also 45 the Angle at R required.

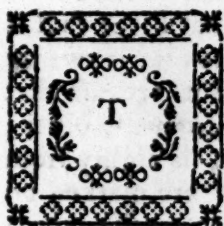
And by the same Method, the Angle at T may be found 30 Degrees.

PART



P A R T III.

The DESCRIPTION and Use of such INSTRUMENTS as are useful and necessary in the Art of SURVEYING.



THE whole Practice of actual Surveying; namely, that Part of the Art which must necessarily be performed in the Field, or upon the Spot; together with all Instruments peculiarly applicable to that Part of the Work, may be reduced to these two Heads or Uses.

First, For measuring Distances; and for that Purpose is the Surveying-wheel and Chains of several Sorts, &c.

Secondly, For taking the Quantity of any Angle; and for that Use is the Circumferentor; the Theodolite; the Semi-circle; the Plain-table, &c.

But there are other Instruments also of Use for plotting a Survey upon Paper from the Field-book, or upon the Spot, or for more readily casting up, or performing Multiplication or Division; and these are Protractors, Scales of equal Parts, of several Sorts: Also Compasses, Gunter's Scale, the Sliding-rule, &c. Of which in Order.

C H A P. I.

Of CHAINS.

THERE have been divers Sorts of Chains made and used according to the different Sentiments of Authors and Surveyors: But Mr. *Gunter's* Chain has now generally (and in my Opinion justly too) gained the Ascendant of the rest, being commodiously contrived for measuring, and expeditious for casting up what is measured by it.

It is in Length twenty-two Yards; that is, four Pole, at five Yards and a half, or sixteen Foot and a half to the Pole: So that every square Chain, *viz.* every Piece of Ground a Chain in Breadth, and a Chain in Length, contains 16 square Pole; and there being a hundred and sixty square Pole in an Acre, ten square Chains, to wit, ten Chains in Length, and one in Breadth, (or any Superficies equal to that) is just an Acre.

This Chain is divided into a Hundred Links of equal Length, and these are sometimes marked at every ten Links with Pieces of Brass, numbered 10, 20, 30, &c. namely, that Piece that is ten Links from one End is marked 10, and that that is twenty Links from the same End is marked 20, &c. But because that Brass that is ten Links from one End, is ninety from the other; and that which is twenty Links from one End, is eighty from the other, &c. therefore these Pieces of Brass are marked on both Sides, *viz.* that with 10 on one Side, has 90 on the other; and that with 40 on one Side, has 60 on the other, &c. So that which End soever of the Chain goes foremost, you may, by the Help of these Pieces of Brass, readily know how many Links you are from the End of the Chain; for the Middle of the Chain being marked with a Ring, it is easy to know in all the rest, which Figure to make use of, by observing which Side of the Middle of the Chain you are on, which the better to distinguish amongst long Grass, or Wood, or the like, it is best to have a red or white Rag tied to the Ring at the Middle of the Chain.

The Chain is thus divided into 100 Links, and marked at every 10, for the Conveniency of working by Decimals, which work like whole Numbers, and prevent the Trouble of Vulgar Fractions.

The Way of using this Chain is with two Persons, one at each End of the Chain. The foremost (or, as it is commonly called, he

he that leads the Chain) takes in his Hand ten Sticks (by some called Arrows), of ten or twelve Inches long. The softer the Ground, or the longer the Grass, the longer the Sticks ought to be; and if they measure along the Side of a straight Hedge, let him that leads the Chain, keep equally distant from the Hedge, and three or four Yards from it; and when the hindmost Man is at the Point from whence they are to begin to measure, let him hold the End of the Chain there, till he that leads the Chain, having the End of the Chain hung upon one or two of the Fingers of his Right-hand, and one of his sharp Sticks in the same Hand (keeping the rest in his Left-hand) pull the Chain tight, and stick down the Stick that he has in his Right-hand.

This done, let him run forward with the Chain, till the hindmost Man come at the Stick, and take up that, till he that leads the Chain, stick down another, and so forward, till all the ten Sticks be spent, if your Space be so long.

Then let the Man that leads the Chain have all the Sticks again, and proceed as before: And thus, by the Number of Sticks spent, or the Number of Times spending them all, you are helped to remember how many Chains you have measured; so when the hindmost Man has got two, three, or four of the Sticks in his Hand, there is just so many Chains measured, from the Place where you begun, to the Place where he took up the last Pin or Stick.

If you have renewed the Sticks once or more, there is so many Times ten Chains measured; and thus, if you have renewed three Times, and the hindmost Man have again got 7 Sticks in his Hand, you have measured thirty-seven Chains; and so in other Cases.

As for odd Links, the Brass Pieces fixed to the Chain, help in counting them; for suppose you have renewed your Sticks three Times, and the Foreman have again put down four Sticks, but has not quite a Chain between the last Stick and Hedge he is to measure to, let him pull the End of the Chain to the Hedge, and hold it there, till the hindmost Man (laying down the Chain straight) come to the last Pin or Stick, and observe which Brass Piece is next to the last Stick towards the Hedge, and how many Links it is from the Stick; as for Instance, suppose it is the Brass Piece marked 70 on one Side, and thirty on the other; and if I see the Middle of the Chain (marked with a Rag for that Purpose) between me and the Hedge, I know it is 70 Links, and if the Brass Piece be 6 Links from the Stick towards the Hedge, that makes 76 Links; so that the whole Length that I have measured, is 37 Chains and 76 Links.

But

But you need not distinguish Chains and Links, but write them like Decimals 37.76; or if you will 3776, for 37 Chains is 3700 Links, to which add 76 Links, the Sum 3776 is the whole Number of Links that you have measured, and may be put down, either 37.76, or 3776; but it is best to put them down in the Field-book in Chains and Links 37.76.

In Order to carry a Chain straight at a great Distance; some that lead the Chain, only fix their Eye upon some Object in that Part of the Hedge they are to go to, and go towards it; but even that is very subject to Error.

The best Way to be exact is, while one stands at the Point they are to begin to measure at, to send another towards the Object they intend to measure to; and when he comes within twenty or thirty Yards of it, and if he is in a Right-line between the Person so standing and the said Object, let him set up a Stick with a white Paper or Handkerchief at the Top of it, and so come back; but if he is not in a Right-line when he comes so near, let the Party so standing, wave him to the Right or to the Left, till he settles between him and the said Object, and then put up his white Mark as before.

And then, in leading the Chain, let him keep the said white Mark, and the Object both in one; that is, in a Right-line with himself, and he will be sure to describe a Right-line with his Chain, and carry it straight to the said white Mark, and it can be no Difficulty to keep straight for the other twenty or thirty Yards.

Sometimes it happens in the Country, that there are some small Parcels of Land to be measured or set out, where there is no Chain to be had to do it with. In such a Necessity, you may supply the Place of a Chain with a Rope or Cord: but then great Regard ought to be had to the Weather, for wet Weather will make it shrink and grow shorter, and very hot Weather will make it grow longer than usual; but if Necessity oblige it, it is best to have a Rope or Cord half worn, and use it in Weather that is fair but cloudy. You may cut it to four Pole, or two Pole, and may measure the odd Links by a Stick cut to the Length of ten Links, and marked with a Notch at the Length of every Link; and it may with Care serve very well to measure, or set out Land for mowing, and the like, where better Opportunities cannot be had.

CHAP. II.

Of the SURVEYING-WHEEL.

THIS Instrument consists of a Wheel made of Wood, but neatly shod or lined about the Edge with Iron, something like the Wheel of a Chariot, to prevent its wearing.

The Wheel is fixed upon a short Axis, whose Ends play in two Pieces of Wood, which go out forward from the Body of the Instrument for that Purpose, between which the Wheel moves.

In the Body of the Instrument is a Machine of Clock-work acted by the Wheel, by means of an Iron Rod, which lies along in a Groove from the End of the Axis of the Wheel to the Clock-work, and so contrived with Wheels and Pinions, that when the Wheel is run forward upon the Ground (by two Handles at the after Part of the Instrument made for that Purpose) it turns two Hands, which are visible on the upper Side of the Instrument, which Hands, by Figures they point at as they go about, shew how far you have gone with the Wheel.

To describe all the Clock-work within the Instrument, with the Numbers of the Wheels and Pinions, would be of little Use to the Learner; but the Curious who desire it, may see the whole Description in *Bion's Use of Instruments*, sold by Mr. Senex.

The Wheel (including the Thickness of the Iron about it) is two Foot seven Inches and a half in Diameter; so designed that its Circumference may be eight Foot three Inches, or half a Pole; so that once about of the Wheel is half a Pole, eight Times about is a Chain, eighty Times about is a Furlong, and six hundred and forty Times about is a Mile.

There are two Hands turn about on the Top, or upper Side of the Instrument; one of them goes round once in a hundred and sixty Pole, or half a Mile, its End pointing round a Circle that is divided into a hundred and sixty equal Divisions, numbered at every ten, and passes one of them at every Pole the Wheel goes.

The other Hand goes once about in twelve Miles, and moves in a Circle divided into twelve equal Parts round it, and marked I. II. III. &c. to XII. and subdivided into Halves and Quarters, exactly like the Dial-plate of a Clock.

The Use of the Wheel is to shew, only by driving it before you, how many Miles, Furlongs, Poles, or the like, you have gone,

which,

which, without any Charge of Memory, or trouble of writing down, is easily discovered by the two Hands; as for Instance, the Hand that goes about once in half a Mile, or a hundred and sixty Pole, points out by the small Divisions every Pole you go, and when it has made a Revolution, the other Hand has passed half of one of the twelve Divisions that are upon its Circle: And hence,

Running your Wheel along, suppose in the Road, and having run by Computation two or three Miles, you look upon the two Indexes, and you find the Hand which moves in the Circle numbered I. to XII. to stand between II. and the Mark that is half between II. and III. and the other Hand at 100; I conclude by the first Index, or Hand, that I have gone between two Miles, and two and a half; and by looking at the other, which points at 100, I find I have gone two Miles 100 Pole.

But if the first Index, or Hand, had been between II. and a half, and III. then the other shews how much you have gone above two Miles and a half.

Example. If the first Hand be more than half past II. and the other at 90, I know I have gone two Miles and a half, and ninety Pole, which (because eighty Pole is a Quarter of a Mile) is two Miles, and three Quarters, and ten Pole.

With these Instructions, they that understand the Use of the Hour Hand and Minute Hand of a common Clock, cannot be ignorant in this.

But there are these Cautions to be observed in the Use of the Wheel.

First, That before you begin Work, you put the two Hands to the Beginning of their Revolution; namely one of them to XII. the other to 160, and then count as before.

Secondly, Avoid as much as possible, running the Wheel into Holes or Ditches in the Road; for as the Wheel will incline to sink to the Bottom, you thereby measure the Concave, instead of the horizontal Surface, and make your Distance greater than it ought to be.

Thirdly, Endeavour at a convenient Distance to foresee when such Holes or Ditches are in your Way, and by little and little avoid them; for if you do not see them before you come at them, going round the Edges of them will encrease your Distance, as much, or perhaps more, than going through them, and cause you to contract as bad an Error, or worse.

Fourthly, In Surveying the Roads, avoid (as much as the Ground will admit) crossing the Lanes, for that also encreases the Distance, even in measuring with a Chain; but in the Cautions,

tions above, the Error is not so conspicuous in the Measure by the Chain, because it will stretch over a small Hole, and measure pretty near the horizontal Distance; however, all these Errors ought to be watched against, and the rather, because they all tend one Way; namely, to make the Distance more than it is; whereas, if one made it more, and the other less than it should be, one might help to compensate the other, and the Error would be more tolerable.

Hence the Wheel is not a proper Instrument for measuring of uneven, woody, or marshy Grounds, or other Places, where the above-mentioned Inconveniences occur.

The Use of the Wheel is (by the Help of the Theodolite, Circumferentor, or Plain-table, adapted to the Use of either of them) to plot Roads, Rivers, or the like, thus:

Fix your Wheel and Instrument at the Place where you intend to begin, whether upon a Road or River, &c. and call that your first Station. Then place some Person or Mark at as great a Distance from you along the Road or River as you see it straight, and by your Instrument take the Bearing of the said Mark, which set down.

Also run the Wheel from that first Station to this, which we shall now call the second Station, and find the Distance; which also set down.

Then place your Assistant or a Mark at the third Station; that is, as far as you can see it straight from the second Station, and take your Bearing and Distance as before directed; which also set down as before, and so proceed to the fourth, fifth, and sixth Stations, &c. and you have the Road or River surveyed; and how to protract it upon Paper or Parchment, shall be shewn in the fourth Book.

Note, When you survey a Road, River, &c. by the Help of the Needle, Care ought to be taken to allow for the Variation; for although the Compass will, without any Regard to the Variation, give you all the Angles, and the Position of the Stations, with Respect to one another; yet they will differ from the true Position, with Respect to the true Meridian, by as much as the Variation is, if that is not regarded and allowed for.

The Directions above being but a Specimen of the Use of the Wheel, I shall shew how to survey any curving or bending Road or River, where the Parts of it are neither straight, nor yet make Angles with one another, or however otherwise irregular; but this you shall have in the fourth Book of Practical Surveying, under its proper Head of Surveying a Road or River.

CHAP. III.

Of the THEODOLITE.

IT will be needless to spend as much Time in describing all the Parts and Divisions of every Instrument, as if every one that intends to learn Surveying, were first to make his own Instruments; I rather think it sufficient to give such a general Description of them, as may enable the most ignorant, that desire to learn, to know and distinguish the several Instruments when he sees them; and to them that know but so much, and are willing to learn more, a plain Account of the Use of them will be of more advantage than such a large and particular Description.

The Theodolite is a very useful Instrument in Surveying; they are most commonly made of Brass, and consist of a Circle of twelve or fourteen Inches Diameter. The Limb is divided into 360 Degrees, and these Degrees are again subdivided into smaller Parts, as the Magnitude of the Instrument will admit of, sometimes by equal Subdivisions, and sometimes by Diagonals drawn from the outermost to the innermost concentrick Circle of the Limb.

There is in the Middle of the Brass Circle a Box, and a Needle touch'd with a Loadstone; the Center or Cap of the Needle being in the Center of the Circle; as is also the Center of the Index upon which it turns, to take any Angle, as Occasion requires.

On the Back, or under Side of it, there is a Ball and Socket screwed on, whereby to fix it upon the Head of the three-legged Staff; so that in Time of Use, it may be turned about, or fixed in a horizontal Position, without removing the Legs of the Staff.

The Use of the Theodolite is for taking Angles, as the Chain is for measuring Distances. The Use of it in taking an Angle is this.

Fig. 48. Suppose ABC was a Wall, which made an Angle at B, whose Quantity I desire to know.

Place your Theodolite at B, and when you have laid the Index upon the Diameter of it, and fixed it there, turn the Instrument about, till you can see the Mark A, or the further End of the Wall, through the Sights; then screwing the Socket fast, to keep

keep the Instrument fixed, turn the Index 'till you can through the Sight see the Point C, and then see what Degrees are cut by the Index, and that is the Quantity of the Angle ABC required.

Observe at which Object, A or C, you look first, that you may have the increasing Degrees, for the moving Index.

Note, In this, and all other Cases of this Kind, when 'tis said, Place your Instrument at B, it is not meant that you are obliged to place it in the Point B, that being solid Wall, and inaccessible; but you are to understand by that, to place it as near the Point B as you can conveniently, so as to have Room for the Instrument, and for your Body to move round it; as suppose, in the Corner of the Field at D: Only always observe;

Either set up Marks at E and F, Points just as far distant from the Lines AB and BC, as the Point D is, or else by Judgment direct your Eye in a Line parallel to the Wall you are observing, and as far distant from it as your Instrument is, and then, instead of observing by the Lines BA and BC, you observe by the Lines DE and DF; and they being parallel to the former, the Angles will be the same, FDE equal to ABC.

Observe the same in any other Instrument for taking Angles.

Also in taking any Angles on the Out-side of a Field or Wood (when you cannot get into it, or when being in, you cannot take your Observation by reason of the great Quantity of Wood or Bushes) observe the same Rule, and place your Instrument at G instead of B, and look along the Lines GH and GI, parallel to BC and BA, and your Angle will be true.

CHAP. IV.

Of the CIRCUMFERENTOR.

THE Circumferentor is composed of two Parts, a Circle and a long Index; the Index twelve or fourteen Inches, or more or less at Pleasure, and the Circle in Proportion, commonly having its Diameter about equal to half the Length of the Index.

There is on the Back-side of the Instrument, a Socket screw'd on, in order to place it upon the three-legged Staff, in the same Manner as the Theodolite is fixed, when it is to be made Use of.

The Index is divided into several Scales of equal Parts, of six, eight, ten, twelve, or more, to an Inch, and serves for a Scale
to

to a Protractor; the Instrument itself being applicable to that Purpose.

In the Bottom of the Box or Circle, there is a Card pasted, divided into thirty-two Points of the Compass, and on the Edge thereof, divided into 360 Degrees, if the Instrument be large enough to admit of it, so that you can count either by the Points of the Compass, or Degrees from the Meridian; and upon one of these Divisions, there is a Flower-de-luce, which with the Center of the Compass, is placed exactly over the Line that goes up the Middle of the Index, and that End of the Index towards which the Flower-de-luce stands, is always to be placed towards you, or next the Eye, in Time of Observation.

In the Center of this Circle or Box (which is exactly in the Middle of the Index) there is a small sharp Brass Point, upon which there is suspended a Needle touched with a Loadstone, with a Glass over it, to preserve it from the Injury of the Weather, and the Violence of the Air, which so secured, will posite itself North and South, except so much as the Variation is, as I have hinted in *Chap. II.*

There are two Vanes or Sights, one belonging to each End of the Index, through which you are to look at the Object proposed for Observation.

This Instrument is fixed upon the three-legged Staff, by Help of the Ball and Socket, as the Theodolite, and other horizontal Instruments are.

The Use of the Circumferentor, is to measure the Quantity of any Angle in the Field, in order to protract the same; and it is thus performed.

Suppose BA and BC are two Walls or Hedges, which partly enclose the Field ABCD, and I am willing to know what Angle the Wall or Hedge, BA, makes with that BC; set up the Instrument at B, and fixing the Flower-de-luce of the Chart towards you (always) look first along the Hedge BA, and observe what Degree the Needle points at, which suppose 30; then turn the Instrument about upon the Socket, till (still keeping the Flower-de-luce towards you) you can through the Sights, look along the Hedge BC to see the Point C; and then observe what Degree the Needle points at, which suppose 125; take the former observed Number of Degrees 30, out of the last 125, there remains 95, the Angle required.

Note, If in two Observations to find an Angle, the Needle points in one of them on one Side of 360; and in the other on the other Side, add what one wants of 360 to what the other is past it; the Sum is the Angle required.

Example.

Example. Suppose in the Angle ABC above mentioned, (Fig. 23.) I look along the Side BA, and find the Needle Point at 320 Degrees; and in looking along the Hedge BC, I find it point at 55. The first Observation 320, wants 40, or is 40 less than 360; that 40 added to 55, the Sum 95 is the Degrees of the Angle required.

The like Rule and method is to be observed in taking any Angle by the Circumferentor, as shall be shewn more at large in the Fourth Part.

CHAP. V.

Of the SEMICIRCLE.

THIS Instrument is commonly made of Brass or Wood; the Radius or Semi-diameter, from four to eight Inches. It is (as its Name imports) a half Circle, with a fixed Sight at each End of the whole Diameter; and at the middle of the said whole Diameter, which is also the Center of the Semi-circle, there is a moveable Index turning upon a Pin, whose fiducial Edge points to the Graduations of the Limb of the Semi-circle, which is graduated like the Theodolite.

Upon each end of the moveable Index, there is a Sight or Vane, through both which the Sight is to be taken, along the Hedge or Wall, or to any Object.

There is also a Compass fixed to it, sometimes in the void Space, between the whole Diameter and the Limb; but I think it is much more convenient, to have it fixed upon the Center, so as that the Center of the Compass may just be in, or over the Center of the Semi-circle, and upon the Middle of the Index, for then it answers the End of the Theodolite and Circumferentor, as well as of a Semi-circle.

The Use of the Semi-circle is so near the same with the Theodolite, that I shall, for the present, refer the Reader to that for Instruction, and shall suspend any farther Directions about it, till I come to speak of the Use of it, in the practical Part of Surveying, in the latter Part of this Book.

C H A P. VI.

Of the P L A I N - T A B L E.

THE Plain-table is, in my Opinion, the most universally useful, of all the Instruments of Surveying, for it will supply the Place of both Theodolite, Circumferentor, and Semi-circle; for by it we can, without the Help of a Protractor (the Description and Use of which shall be inserted hereafter) plot any small Enclosures upon the Spot, as we are surveying them.

The Plain-table, is a Parallelogram of Oak, or other Wood conveniently fine, and should be so dry, and of such a competent Thickness, as to be out of Danger of warping.

Plain-Tables are made about fifteen Inches long, and twelve Inches broad; or whether larger or smaller, they ought to retain about that Proportion of their Length to their Breadth.

The larger Sort are composed of three Boards, which may, upon Occasion, be taken asunder, for conveniency of Carriage.

There is belonging to it a Box-frame, consisting of six Parts, and as many Joints, so contrived that when it is open it is square, and the inside Dimensions of the Box-frame are equal to the outside Dimensions of the Table; so that it may be put on, and taken off at Pleasure.

This Frame serves to hold the Table together, and must be contrived to go easily on with either Side of the Frame upwards, because it also serves to fasten a Sheet of Paper upon the Table, by forcing down the Frame, and squeezing in all the Edges of the Paper, pulling it so tight, as to lie firm and plain upon the Table, and fit to be drawn upon.

On this Frame are also the several Divisions necessary; as first, on both Sides of the Frame, are the Inches, each divided into ten equal Parts, and proper Figures at every Inch, which serve for the more expeditious ruling Parallels or Squares upon the Paper, and for shifting the Paper, when the Work requires more than one Sheet.

There is on one Side of the Frame 360 Degrees, which, tho' not equally divided, because the Frame is Square, yet they are equal Divisions, with respect to the small Hole that is in the Middle of the Table, and is the Center of these 360 Degrees, and would be equal Divisions in a Circle, if described about the said small Hole; as a Center.

These Degrees are subdivided as small as the Magnitude of the Table will admit ; and at each tenth Degree, are two Numbers, one the Number of Degrees, and the other its Complement to 360 : They are thus numbered about both Ways, to avoid Trouble of counting in the taking an Angle.

There is also another Center-hole in the Table, exactly in the Middle, between the two Ends, and about one Quarter of its Breadth from one Edge of it. To this Center-hole, and on the other side of the Box-frame, there is 180 Degrees of a Semi-circle adjusted ; each of these Degrees is subdivided into 30 Minutes, and numbered 10, 20, 30, &c. both Ways to 180, for the same Reason as before ; and of these, that Side of the Frame that expresses all the 360 Degrees, supplies the Place of a Theodolite, and the other of 180 Degrees that of a Semi-circle.

There is a Box with a Card pasted to it, and Needle touched with a Load-stone ; this Needle plays upon a Pin, in the Center of the Box. The Box is let into one Side of the Table, with a Kind of a Dove-tail Joint, and fastened with a Screw, that it may be put on, and taken off at Pleasure. This Box (besides its rendering the Plain-table capable of answering the End of a Circumferentor) is very useful for placing the Instrument in the same Position every remove.

There is also an Index belongs to the Plain-Table ; it is a Brass Ruler and ought to be as long as the Diagonal of the Plain-table, and near two Inches broad, and Thickness proportionable ; it has a slope Edge, like that of a *Gunter's Scale*, and that is called the *Fiducial-Edge*. There are two Sights (at each End one) screwed perpendicularly upon the Index, the Sights are exactly both of a Height, and equally distant from the Fiducial-edge. Upon the plain Side of the Index are Scales of equal Parts, of several Sizes, with Diagonal-scales, and a Line of Chords.

On the under Side of the Table, there is a Brass Spangle and Socket, fastened to the Table with three Screws, in order to fix it upon the Head of the three-legged Staff ; which is performed, as you are already taught in the Use of the Theodolite, Chapter the Third.

Of the USE of the PLAIN-TABLE.

The Plain-table, as above described, will supply the Place, and answer the End of a Theodolite, Circumferentor, or Semi-circle ; but what is here meant by the Use of it is, as looking upon it as a Plain-table only, and those Properties peculiar to it, that other Instruments are not capable of.

When you intend to make Use of the Plain-table, take off the Box-frame, and spread a clean white Sheet of Paper over the Table, something longer and broader than the Table is, and put on the Frame upon it, so that the Edges of the Paper may be squeezed in between the Frame and the Edge of the Table, taking Care, in pressing the Frame on, to keep the Paper straight and smooth upon the Table; then put the Box, with the Needle in it, in its Place, having the Meridian-line of the Card parallel to the Meridian of the Table, which is a Right-line drawn from the Beginning of the Degrees to the End of them, and through the Center of the Table. Then screw the Socket upon the Head of the Staff, and lay the Index, with the Sights fixed upon it in their Places, upon the Table; and then is the Instrument fit for Use.

How to take an Angle with the PLAIN-TABLE.

Suppose the Lines AB and BC be two Walls meeting at B, and making there the Angle B, and you would know by the Plain-table, how much the Angle is; place your Instrument at B, or rather, with the Reason and Caution before given, place it at D; and then turn the instrument about upon the Socket, till the North End of the Needle be just directly over the Meridian-line in the Card, and there screw the Table fast; then assign a Point at Pleasure upon the Table, with this Consideration, that it be so near that Part of the Table that stands next the Angle, as to give convenient Room for the Index to lie, and the Lines to be drawn over the Plain of the Table, and applying the Edge of the Index to that Point, turn it about upon it, till through the Sight of it, you can see the Mark set up at F; and by the Edge of the Ruler, draw the Line DF; then keeping the Table fast, and the Index at the aforesaid Point in the Table, turn it about till you can, through the Sights, see the Point E in the Field; and then upon the Paper, by the Edge of the Ruler, draw the Line DE; then shall the Angle FDE (whose containing Sides FD and DE, are parallel to the Walls or Hedges of the Field AB and BC) be the Angle required.

But if you are to take all the Angles of a Figure, or Field, of three or four or any greater Number of Sides; as suppose the Figure BCDEFG, 'tis no Matter at which Angle you begin; but suppose you would begin at the Angle E, and by looking through the Sights from E to F, and from E to D, find the Angle FED, as above directed:

This done, measure the Length of the Side ED in Chains and Links, or what other Measure you propose, and taking that off from some of the Scales of equal Parts, put upon the Index for that Purpose, and set it from E to D: Then remove your Instrument to D, and again placing the Needle directly over the Meridian Line, lay the Edge of your Index at the End of the Line ED, in the Point D, and turn the Index about, till you can, through the Sight, see the Point or Mark C; and along the Side of the Index, draw the line DC, and measuring the Length of it in the Field, take it from the same Scale of equal Parts, from which you took the Line ED, and set it from D to C, as above directed.

Note, This Method of placing the Needle over the Meridian Line, and proceeding as above, is exactly true, if performed with Care; and if the Needle is well touched and traverses extraordinary well, and no Loadstone, Iron or Steel, be near it to affect it; but lest any of those Difficulties should occur, and draw you into an Error insensibly, it is safest, after having taken the first Angle, as suppose at E, and having removed your Instrument to D, and placed the Needle over the Meridian Line, as above directed, lay the Index upon the last drawn Line DE upon the Table, and looking through the Sights back from D, if you then see the Mark at E, the Table is fixed right, and the Compass stands true; but if not, you must turn the Table a little about, till (the Edge of the Index lying still exactly upon the Line ED) you can from D see the Mark E, and there fix your Table fast, and then proceed to look towards C, as above directed; and this is a surer Way of taking an Angle, than by trusting to the Compass only, and is properly called Back-sight and Fore-sight.

When you have taken all the Angles in the Field proposed, of how many Sides soever, you may examine your Work, and find generally whether it is right or no, by adding all the Angles together; and if their Sum amounts to twice as many Right-angles less by four, as there are Angles in the Polygon, the Work is probably right. See Theor. 8.

By this Method you may proceed to take all the Angles in a Field, and by measuring and laying down the Sides, as before directed, you have all the Plot of the Field upon your Plain-table: And by the same Rule, if you have a small Estate or a Farm to survey, which consists of several Enclosures, and would do it to such a small Scale, as to bring it into one Sheet; take Care to begin at the same Part of your Sheet, that you begin at

of the Farm; that is, if you begin the Work at the South-West Part of the Farm, begin upon the Paper at the South-West Part of the Plain-table, &c. and then in laying down all the Hedges of the Field in which you begun, you will have laid down all or a Part of one of the Hedges of the contiguous Field (and perhaps of more than one) pass over into that next Field, and proceed as before; and if you are careful to take all the Angles and Distances exactly right, you will, when you have finished your Work, have the exact Figure of the Estate or Farm, upon the Paper on the Plain-table.

But if the Estate or Farm be so large, that with the Scale you use, it will not come all into one Sheet, which commonly happens, you must, when one Sheet is full, take it off the Plain-table, and lay another clean white Sheet on; and this is called shifting of Paper, and is thus performed:

Suppose B C D E represent the Sheet of Paper laid upon the Plain-table, and upon that is intended to be drawn the Plot of the Field F G H I K L; and suppose I take my first Station at L, *Fig. 49.* and so proceed to F, and so to G, by the Directions already given; but intending to proceed to H I, find there is not Room upon the Sheet that is upon the Plain-table, to draw the Line G H. In this Case, draw it to the Edge of the Paper, and then remove the Instrument to the first Station at L, and proceed the contrary way to K; but when you should draw the Line K I, you find again, that the Paper upon the Instrument will not contain it; but draw it to the Edge of the Table.

Then, by the Help of the equal Divisions upon the Edge of the Table, which are intended for that purpose, draw upon the Paper, the Line P Q, parallel to the Edge of the Paper C D, and observe where it cuts the Line or Hedge of the Field I K, as in P. Through the Point P, draw P R parallel to the Edge of the Table D E; and then in some blank Part of the Sheet, mark it with N^o 1. or the Letter A, or some Mark that may distinguish it to be the first Sheet, and then take it off and lay it by.

Then put another clean Sheet upon the Table, in the same Manner, as you were directed in the laying on of the first, and towards the contrary Edge of the Instrument, draw a Line parallel to the said Edge, as the Line S T, and upon *Fig. 50.* that Line, lay the Line P Q of the first Sheet, which may be best done, by folding the first Sheet exactly in the same P Q, and then apply it, as above, with this Consideration, that you take such a Point as the Line S T of the second Sheet, for the crossing at P in the first Sheet, as to allow for the

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Tendency of the Field, or that the Lines representing the Hedges may not run out at the Bottom of the Sheet, and yet not come near the Top, which may be easily avoided, by considering the general Situation of the Ground.

But it frequently happens that Hedges are curving; and although the Rules before given for laying down Hedges, or other Bounds of Ground, when they consist of Right-lines, are sufficient; yet in curving Hedges or Walls, the following Rules are to be observed.

In the Field ABHIKCD, the Hedge or Wall, BHIKC is a Curve: When I go to the Angle A, I take the Angle BAD, as before directed, and likewise the Angle ADC; but coming to the Point C, and finding no Angle between C and B, only a continual Curve, place the Instrument at C and observe to B and D, to find the Angle BCD; only instead of the Hedge or Wall, draw the Right-line BGFE C, and draw it upon the Plain-table, as before taught. *Fig. 51.*

Then to account for the Curvity of the Hedge or Wall, BHIKC, measure from C to E, and from E to F, and from F to G, and from G to B, and lay down the Distance CE, EF, FG, and GB, from the same Scale that you laid down the Sides BA, AD, and DC; and then from E, F, and G, raise Perpendiculars to K, I, and H, and, with your Chain, measure EK, FI, and GH, and set these off from E, F, and G, to the Points K, I, and H; and so through the Points B, H, I, K, C, draw with a steady Hand, the Curve BHIKC, for the curving Hedge or Wall, and then you have laid down the true Form of the Field ABHIKCD.

I shall not here enlarge any further upon the Use of the Plain-table, because I shall have Occasion to speak more of it, with Respect to the casting up any Quantity of Ground laid down as before directed, when I come to the Practical Part of Surveying.

CHAP. VII.

Of the Description and Use of SCALES.

SCALES in general may be distinguished into two Sorts, namely, Scales of equal Parts, and those of unequal Parts:

Those of equal Parts are for protracting Lines, and are made of different Sizes, some of 10, 15, 20, 30, or 36, Parts to an Inch; and may be used as if every Part accounted for 10, and then 10 is 100, 15 is 150, &c. and in these Scales the first Division is commonly subdivided into 10 Parts; so that if you reckon the first 1 to be 10; 2, 20, &c. then extending from 2 back to 0, it is 20; but if you extend back from 2 to the fifth Subdivision, it is 25; and by the same Rule, if you would take off 36, fix one Point of the Compasses in 3, and extend the other back to the sixth small Division, accounting from the Cypher, and that Extent represents 36, whether Chains, Yards, &c. and the same Rule is to be used for setting off any other Number.

But if you account the Figures Hundreds, then the small or Subdivisions are Tens; as for Instance: If you would set off 460, set one Foot of your Compasses in 4, and the other in the sixth small Division on the other Side of the Cypher, or 0, and that Extent shall represent 460 (which is the same as represents 46) only now every large Division; where the Figures are placed, is accounted 100; as 1 is 100, 2 is 200, 3 is 300, &c. and then every small Division at the Beginning of the Line is reckoned 10; and in this Case, if you have any odds above even Tens, it must be computed between the small Divisions; as suppose it is required to set off 635, set one Foot in 6, and extend the other to the third small Division on the other Side of the Cypher; that is, at the Beginning of the Line, as if you were to set off 63, and that is 630; but considering that you are to set off 635, extend that moveable Point to half-way between the third and fourth small Division; for as from six to the third small Division, is 630; and from 6 to the fourth small Division is 640, it is easy to conceive, that from 6, to half between the third and fourth, is 635; and if it is required to set 637, you are to compute as near as you can, seven Tenths of the Distance, from the third to the fourth small Division and from 6 to that, is 637 required.

But there are other Sorts of Scales to measure equal Parts, which are commonly called Diagonal Scales, and, as I suppose there are few that undertake to learn the Art of Surveying, but what are furnished with a common Plain-scale, or a *Gunter's* Scale; and the Diagonal-scale being the same upon both, and also the Description of that, serving as a Description of other Diagonal Scales, I shall apply the Directions to these.

The Plain-scale it made twelve Inches long, but *Gunter's* Scale is twenty-four; they are both made square at one Edge, and sloping at the other. The Plain-scale has the Diagonal-scale on one Side, and the Line of Chords, Rhumbs, Sines, Tangents,

Tangents, &c. on one Side (which are all Scales of unequal Divisions.) *Gunter's Scale* hath all the abovementioned Lines and Scales on one Side, that the Plain-scale has upon both, and on the other Side of it, there is a Line of artificial Numbers, Lines, Tangents, &c. which shall be described and explained hereafter in this Chapter.

The Diagonal-scale (so called, because of the Diagonal or Slope-lines used therein, to distinguish more nicely the small Subdivisions) is numbered both Ways, with 1, 2, 3, &c. on one Edge, at every Inch, and proceeds to 10, and on the other Edge, at every half Inch, and proceeds to 19. The larger Scale serves where you have 1, 10, or 100 Parts to an Inch, the lesser where you have 1, 10, or 100 Parts to half an Inch. In shewing the Use of one, I shall have shewn the Use of both; both being the same.

If you intend to let every Division, where the Figures are, to stand for one, in what you are going to lay down, you only set one Foot of your Compasses in the first Line of the Scale (not in Number 1. but in the Line that parts the large Divisions from the Diagonal, and is the beginning of both) and extend the other to the Number proposed, that Extent shall represent the Space required. — Thus from the said Line to 3 is 3, and from the said Line to 9 is 9, &c.

But if every Figure in the large Division stands for 10, you call the first 1, 10; the 2, 20; &c. and then every small Division in the Out-line of the Diagonal stands for 1; that is, the first is 1, the second 2, &c. and in the common Scales, they are frequently marked at every other Division, as 2, 4, 6, 8, the last Line being 10; and then if you would set off 32, set one Foot in 3, on the outermost Line, and the other to 2, or the second in the small Divisions upon the same Line, and that Extent shall represent 32.

But if every whole Division be to represent 100, then the Extent from the first Line to 1, is 100, and from the said Line to 2, is 200, &c. every small Division is 10, and then the Extent of 320, is the same of 32.

But as the Diagonals are contrived to run so much sloping, that the Bottom of one is just as far distant from any of the large Divisions, as the Top of the next, and these Diagonals being equally divided into 10 Parts, by Lines drawn parallel to the Edge of the Ruller, it will follow, that tracing any of these Lines cross the Ruler, from the Top of the Diagonal to the Bottom, every parallel Line that crosses it, is just one Tenth of one of these small Divisions further from the first Line, than at the last Crossing; because in crossing all the 10 parallel Lines, it is one
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of the said small Division further off; and the small Divisions being thus artificially divided, each into 10, the great ones are divided into 100, and by that Means, any Number consisting of three Figures may be measured; and is thus performed.

Suppose you would set off 468; the second Figure being 6, count along the Top of the small Divisions, to the sixth Division, which is commonly marked 6; and because the last Figure of your Number is 8, trace that Line cross to the eighth of the Lines that are drawn parallel to the Edge of the Scale, (these Lines being also marked at the End of the Scale, with every other Number, as 2, 4, 6, 8,) and fixing one Point of the Compasses where these two Lines cross each other, bring the other Point to the Place where the Line, in which the Compass Point stands, crosses the Line marked 4 in the great Divisions (because 4 is the first Figure or Place of Hundreds in your Number) and that is the true Extent of 468, upon the Diagonal Scale.

But besides those Scales of equal Parts, there are also Scales of unequal Parts, of which the most useful and pertinent for our present Purpose, are the Line of Chords for Projection, and the Line of Numbers for Calculation.

Of the LINE of CHORDS.

The Line of Chords (commonly marked *Cho.* at the Beginning of it) is a Line of unequal Parts, numbered 10, 20, 30, 40, &c. to 90, the Divisions growing smaller, as the Numbers increase. There is commonly a Brass Point at the Beginning of the Line, and another at 60 Degrees; because the Chord of 60 Degrees being equal to the Radius, or Semi-diameter of a Circle, that Extent (commonly called the sweep of 60) is frequently taken in the Compasses to sweep the Circle with, and the Line of Chords, according to the unequal Divisions upon it, are the Measures of their respective Angles; and this may be thought sufficient for a Description of the Line of Chords.

How a Chord is to be applied to the measuring or projecting a given Angle, which is the chief Use for which it is intended, is taught in *Prob. 8.* to which Definition and Problem the Reader may have Recourse for the Use and Application of the Line of Chords.

Of the LINE of NUMBERS.

The Line of Numbers, which is one of the Lines upon *Gunter's Scale* is often (by Mechanics, and such as use common Two-foot Rules, upon which the Line of Numbers is inscribed) called *Gunter's Line*; but upon *Gunter's Scale*, where it is amongst artificial Sines and Tangents, &c. it is distinguished by the Name of the Line of Numbers, and is commonly marked at the Right-hand, with *Numb.* or *Num.* It is numbered from the Left-hand, with 1, 2, 3, &c. to 10, to the Middle of the Scale, and from thence to the Right-hand, with 2, 3, 4, &c. to 10 again; the Divisions are unequal, decreasing from the Left to the Right as the Numbers increase, and the second Row of Figures keep exactly the same Order and Distance as the first, that is, the Space from 1 to 2 in the second Line, is the same as the Space from 1 to 2 in the first Line; and the Space from 2 to 3, in one, equal to the same in the other; but the Space between 1 and 2, is not equal to that between 2 and 3, but larger; the Space between 1 and 2, being equal to that between 2 and 4, because 1 bears the same Proportion to 2, as 2 does to 4, and as 3 to 6, &c. and every two Numbers that bear the same Proportion between themselves, are at the same Distance upon the Scale; and hence the Proportion of any two Numbers being given, the Proportion of a Fourth to a Third may be found by the Extent of a Pair of Compasses.

Therefore it is, that the greater the Numbers, the closer they are ranked upon the Line of Numbers; because every Number included between any two Numbers, though never so large, are expressed between them upon the Line, as well as all Numbers included between any two smaller, in the same Proportion; as for Instance, 60 bears the same Proportion to 15, as 4 to 1; therefore all the Numbers between 15 and 60, are contained in the same Space upon the Line, as the Numbers that are between 1 and 4.

The Figures on the Line of Numbers may be read, either for their proper Value, or for 10 Times, 100 Times, &c. of their Value; and the second Row is always 10 Times the Value of the first: Thus if 1 at the Beginning be called 1, the 2 is called 2, the 7 is 7, &c. and so the 9 is 9, and 1 in the Middle is 10, and the 2 in the second Row is 20, 3 is 30, 4 is 40, &c. and the 10 at the Right-hand is 100; and then the Numbers between 10 and 20 are supplied by the first subdivisions, as the next large Subdivision towards the Right-hand of the 1 in the Middle

Middle is 11, the second 12, where a small Brass Point is commonly fixed) and then the fifth, which for Distinction is longer than the rest is 15, and so on to 16, 17, 18, 19, and then the 2 in the second Line is 20, and accounting 21, 22, 23, &c. upon the larger Divisions, between 20 and 30; the 3 will be 30, the 4 will be 40, &c. and the 10 at the End will be 100.

But if your Numbers run higher, and that the 1 at the Beginning be 10, then 1 in the Middle is 100, and 10 at the End is 1000; because then the first Row goes on 10, 20, 30, &c. to 100, which is the 1 in the Middle and the second Row is 100, 200, 300, &c. to the End.

When the Figures in the second Row are Hundreds, then the larger Subdivisions are Tens, and the Subdivisions between them are Units.

Thus, if I would find 126, the 1 in the Middle is 100, and two larger Subdivisions further (which is where the Brass Nail is) is 120; but because I want 126, I must count six of the small Subdivisions further; and that is the Point that represents 126.

This Line is only intended for Proportion, all Numbers equally distant upon the Line, bearing exactly the same Proportion to each other; and upon that Foundation it is, that Multiplication, Division, and the Rule of Three, may be performed upon the Line of Numbers: because,

In Multiplication as 1 to the Multiplier, so the Multiplicand to the Product.

In Division, as the Divisor to 1, so the Dividend to the Quotient.

In the Rule of Three, as the first Term to the second, so the third to the fourth.

First of MULTIPLICATION.

Extend your Compasses from 1, to the Multiplier; the same Extent applied the same Way, will reach from the Multiplicand to the Product.

Example Suppose I should multiply 7 by 5, extend the Compasses from 1 to 5, the same Extent will reach from 7 to 35, the Product required.

And the like in any other two Numbers, is observed on the Line according to the foregoing Directions; as suppose it is required to find the Product of 16, multiplied by 6, extend the Compasses from 1 to 6, the same Extent shall reach from 16 to 96.

But

But suppose you are to multiply 65 by 9, extend from 1 to 9, the same Extent will reach from 65, to beyond the End of the Line, if 9 in the first Row be reckoned 9, and 6 in the second Part of the Line be called 60 as before directed: but to remedy this, having got the Extent from 1 to 9 in the first Part of the Line, you may then call the 6 in the first Part of the Line 60; and the fifth Subdivision from thence towards 7, will be 55, in which Point placing one Foot of the Compasses, the other will fall in the last Part of the Line at 8 Divisions and a half beyond 5; and, as before observed, if you call the 6 in the first Part of the Line 60, the 5 in the latter Part is 500; and the eight small Divisions is 80, and the half Division being a Tenth is 5; so that the Product is 585.

By the same Rule you may multiply any Number upon the Line, if not so large as to be impossible to be distinguished, by Reason of the Smallness of the Divisions; and this will answer all Cases in Multiplication, and ought to be rather understood, because in any Survey, the casting up is performed by Multiplication only, and the Line of Numbers is the rather useful in Surveying by Gunter's Chain, because both work by Decimals.

As for Multiplication of Decimals, none can be at a Loss that can read Decimals upon the Scale, as I shall explain in an Example or two.

Suppose I would multiply 5.7 by 8.3, extend from 1 to 5.7 (which is the same that may stand for 57) with the same Extent, and one Foot in 8.3, the other will reach to 47.31, the Product required.

This Way of casting up by Gunter's Line is of great Use in giving a general Guess of the Content of any Parcel of Ground very expeditiously, when there is not Time for Calculation by the Pen; and it may also be of Use to prevent a Mistake by misplacing or miscalling a Figure; for although it will not exactly discover the minute Truth; yet it may serve to find an Error, for that may as easily happen in a Figure or Place towards the Left-hand, as towards the Right, if a Mistake is committed in multiplying or adding the Sums together; or in copying, which this Instrument will easily discover.

C H A P. VIII.

Of the PROTRACTOR.

THE common Protractor consists of two Parts, a Scale and a Semi-circle, the former serving only as a common Scale of equal Parts to plot Sides, and the Semicircle to project Angles.

It may be made of Brass, Wood, &c. but if it is to be but of a small Size, it is necessary it should be of Brass; the Diameter of the Semi-circle about four Inches, and the Breadth of the Scale about one and a half; but that may be altered at discretion, according to every one's Fancy.

Upon the Middle of one Edge of the Scale there is a Point, and upon that is the Center upon which the Semi-circle is drawn. This Semi-circle contains three concentric half Circles, at such a Distance within each other, that the two Spaces between them will contain Figures, each Quadrant of the Semi-circle being divided into 90 equal Parts or Degrees, and consequently the Semi-circle is divided into 180 Degrees.

The outermost is numbered from the Left to the Right with 10, 20, 30, &c. to 180, and the innermost is numbered the same Way from 180 to 360.

The Line in which the Center of the Semi-circle is, is the Meridian, in which placing that End North, where the Degrees begin to be numbered upon the Semi-circle, and the other South; the straight Edge of the Protractor, applied to the Center and to the Edge of the Protractor, where the Figures and Divisions are shewn, the Degrees from the North-easterly to the East 90 Degrees; and from thence southerly to the South 90 more, or 180 in all; and if you would continue to reckon from the North, quite round to the North again, or 360 Degrees, keep your Protractor in the same Position as before; and when the straight Edge of the Ruler comes off from the Semi-circle at the South End of the Arch, the other End of the Ruler will go on at the North End of it only; then begin to number on the inner Row of Figures in the Semi-circle, and go on to 190, 200, &c. as Occasion requires, towards 360.

But if you would reckon your Angles from the South Westward, keep still your Protractor as before, only begin again to reckon upon the outer Semi-circle and Figures, because the South Part of the Index makes the same Angle with the Meridian

dian (although it is quite off the Semi-circle) that the North End doth, and therefore in counting the Degrees upon one you have the other, and are as well supplied with a Semi-circle, and with as little Trouble as if it were a whole Circle.

There ought to be a Line of Chords upon the Parallelogram or Scale; but it being spoke of before, we shall refer our Reader to that Description.

CHAP. IX.

The Construction and Description of the SECTOR.

THE Sector is an Instrument of almost universal Use, being applicable to Arithmetic, Trigonometry, Navigation, Astronomy, and several other Parts of the Mathematics; as shall be shewn in its proper Place.

It is an Instrument made of Brass, Silver, or Wood (according as the Artist is willing to be at the Expence:) It consists of two Parts, fixed together by a Joint; and is 12, 9, or 6 Inches in Length, when shut; or may be made of any other Length at Pleasure.

The Construction of the Sector depends upon the Fourth Proposition of the Sixth Book of *Euclid*, viz. *That Equiangular Triangles are similar*, that is, *have the Sides opposite to the equal Angles proportionable*: which being granted, it will be allowed that every Ifofceles Triangle having the same Vertical Angle, will also have their Sides proportional to their Bases; that is, if an Ifofceles Triangle be divided into several Segments, by Lines drawn parallel to the Base (and consequently parallel to one another) the Angles at the Base being thereby equal, [*Euclid. Lib. VI. Prop. 2.*] the Sides will be proportional to the Bases, viz. as PT to TV , so PO to OQ . Fig. 21.

As the Sector is divided both equally and unequally from the Center down each Limb (as I shall hereafter describe) it will follow, that as the Extent between the Center and any Number, to the Extent between that Number and its Collateral on the other Limb (the Sector at any Opening) so the Extent between the Center and any other Number, to the Extent between that and its Collateral Number, the Sector being kept at the same Opening

Opening as before ; and if these Extents are taken upon the equal Parts, or Line of Lines, it shall produce a Plain Geometrical Proportion : if upon the Sines, Tangents, &c. it shall produce a Proportion in Sines or Tangents from whence it was taken ; of which (having first described the Lines upon the Sector) I shall give an Example or two.

The most useful Lines upon the Sector (considered as a Sector, and the Lines proceeding from the Center thereof) are, 1. The Line of Lines. 2. The Chords. 3. The Sines. 4. The Tangents. 5. The Secants.

1. The Line of Lines is marked [*L.*] or in some [*Lin.*] It is divided into ten equal Parts, numbered 1, 2, 3, &c. to 10, on each Limb of the Sector, but on the same Side, and every one of these 10 Divisions are subdivided into 10, which makes in all 100 ; and if the Figures be read 1, 2, 3, &c. the Subdivisions are Tenths : Thus the third small Division after 3, is 3.3, viz. $3\frac{3}{10}$, or 3 and 3 Tenths ; and the seventh small Division after 5, in 5 and 7 Tenths, &c.

But if you have Occasion for any Number between 10 and 100 ; the first 1 must be called 10, and the 2, 20, &c. and the Subdivision is to be reckoned 1, and then the Figure 7 signifies 70 ; and the fourth Division after 8 is 84, &c. and by the same Rule you may find any Number tho' above 100 ; as suppose you would find 575, the fifth large Division where the 5 is, is 500 ; the seventh Subdivision from that is 70 ; and the small Subdivision is 5, which is the Point that represents 575 ; and by the same Method any other Number may be found.

2. The Line of Chords is the same as upon the common Scales, only to a Radius of near six Inches, which is larger than the Radius of the common Scales ; they are projected from a Quadrant, or Quarter of a Circle, divided into 90 equal Parts ; the nearest Extent from the Beginning of these Divisions to every Division being transferred to a Right-line, is the Line of Chords mentioned ; the Circle being drawn with that Extent of the Compasses that you intend the Length of the Line or Chord of 60 to be.

The Use of this Line of Chords is to project any circular Figure in Astronomy, &c. to any Radius ; for as upon the common Scales the Chord of 60 is the Extent from the Beginning of the Line to 60, so here upon the Sector the Chord of 60 may be taken of any Extent ; for suppose you would project any Thing in Astronomy, or the like, to an Inch Radius, take an Inch in your Compasses, and with one Foot in the Brads Point at the Chord of 60 upon one Limb of the Sector, open the Sector till the other Foot (kept the former Extent) will fall in the Brads Point

Point at 60, on the other Limb of the Sector; then (the Sector kept at that Opening) the Extent from any Degree on one Limb of the Sector, to its collateral Degree on the other Limb, shall be the Chord of that Degree to the said Radius: As if you desire to set off the Chord of 35 to an Inch Radius; the Sector prepared as before, set the Compasses from 35 Deg. on one Limb, to 35 Deg. on the other Limb, shall be the Chord of 35 Deg. upon a Circle of an Inch Radius or Semi-diameter.

Note, *The Chord of 60, and the Sine of 90, and the Tangent of 45, are equal to Radius; the Secants begin at Radius, and proceed to Infinite.*

3. The Line of Sines, commonly marked [Sin.] or [S.] is the natural Sines upon the common Scales, and is also projected from a Quarter of a Circle, divided into 90 equal Parts: The Perpendiculars let fall from the equal Divisions of the Limb of the Quadrant to one Side of it, and parallel to the other, being Sines to that Radius.

The Sines are numbered from the Center with 10, 20, 30, &c. to 90, which is placed at the Point at the End of the Sector (because it is the Radius of the Sines) and in this (as in the Chords) the Sine of any Arch is found to any Radius by taking in your Compasses the said Radius, and placing one Foot of your Compasses in 90 of the Sines on one Limb of the Sector, bring the Sector to such an Opening, that the other Point may fall in 90 of the Sines on the other Limb of the Sector; then keeping the Sector at that Opening, place one Foot of your Compasses in any Degree on the Line of Sines on one Limb of the Sector, and the other Foot to its collateral Sine on the other Limb, that Extent is the Sine of the Degree by whose Extent it was taken.

4. The Line of Tangents (because the Tangent of 45 Deg. is Radius) is numbered from the Center with 10, 20, 30, 40, 45, which last is at the End of the Sector, and is Radius; but the Tangents above 45 Deg. are described within the other, and commonly marked with small Letters [tan.] or [t.] If you would find the Extent of any Tangent under 45, take the Radius of your Circle in the Compass, and set the Sector to such an Opening, that one Foot of the Compasses being placed in Tangent 45, at the End of the Scale, on one Limb of the Sector, the other Foot may fall in Tangent 45, at the End of the Sector, on the other Limb thereof, and letting the Sector stand so, the Extent from any Degree of the Line of Tangents on one Side of the Sector, to its collateral Degree on the other Side, is the Tan-

H

gent

gent of the Degree required; but to find the Extent of the Tangent above 45 Degrees, set the Radius of your Circle from 45 to 45 on the small Tangents, and letting the Sector stand so, extend the Compasses from 50, 60, &c. on one Limb of the Sector, to the Collateral on the other Limb; that Extent is the Tangent of the Degree by whose Extent it was taken.

The Tangents are also projected from a Quadrant or Quarter of a Circle, by drawing a Line from the End of one Limb of the Quadrant, and perpendicular to it, which will just touch the Limb or Circle of the Quadrant; a Ruler laid from the Center of the Quadrant, over the equal Divisions upon the Limb of it, shall cut the said Tangent-line, in the Tangent of the Degree, over which the Ruler was laid.

5. The Line of Secants marked [*Sec.*] numbered 10, 20, 30, &c. to about 75: They are also projected from a Quadrant, and are the Lines drawn from the Center over the several Degrees upon the Limb of the Quadrant, till they cut the Tangent Line.

To find the Secant of any Degree, to any Radius by the Sector, take in the Compasses, the Radius of your Circle, and open the Sector; till placing one Foot of the Compasses in the Secant of 0 Degrees, on the Beginning of the Secants on one Limb, the other will fall in the same on the other Limb; and keeping the Sector at that Opening, the Compasses extended from any Degree on one Limb, to its Collateral on the other, that Extent is the Secant of the Degree, from which the Extent was taken to the Radius proposed.

There are also many other Lines that may be described upon a Sector, as the artificial Numbers, Sines, and Tangents; also the Lines of Hours, Inclinations, Chords, and Latitudes for Dialling; also other Lines or Tables for Gauging, Mensuration, &c. as every Man's particular Business requires; but the above-mentioned being of most universal Use, and peculiar to the Instrument as a Sector (because they, and only they, respect its Center) I shall shew the Uses of them, as they may be applied to our present Purpose.

The USE of the SECTOR.

I shall only touch upon Geometrical Proportion, or the Rule of Three, which indeed is a great Part of Arithmetic; and in performing this, observe this Method, which indeed is also to be observed in the Proportions of Trigonometry, or any Operation performed by Sines or Tangents, as well as the Line of Lines.

R U L E.

R U L E.

Take the second Term in your Compasses, and placing one Foot of your Compasses in the first Term, on one Limb of the Sector, open the Sector, so that the other Foot of the Compasses will fall in the said first Term on the other Limb of the Sector; then keeping the Sector at the same Opening, place one Foot of the Compasses in the third Term, on one Limb, extend the other to the said third (or Collateral) Term on the other Limb; that Extent measured on the Line in which the Answer is to be (whether Number, Sine, or Tangent) shall give the Answer to the Question.

Example. If 40 Pound of Tobacco cost 2 Pound 10 Shillings, what will 60 Pound cost?

Reduce the Money to Shillings, and the Question will stand thus:

If 40 lb. 50s. what 60 lb?

The last Term bring always the same Denomination with the second, it will here be Shillings; take the second Term 50 in your Compasses *viz.* extend the Compasses upon the Line of Lines, from the Center to 50, and with that Extent, and one Foot in 40, the first Term, open the Sector till the other Foot fall in 40 on the Line of Lines on the other Limb; then keeping the Sector at that Opening, extend the Compasses from the third Term 60 on one Limb, to the same on the other; that Extent measured on the Line of Lines, from the Center of the Sector, will reach to 75, *viz.* 75 s. or 3 l. 15 s. the Answer to the Question.

And as the Rule of Proportion is worked by the Extent before mentioned, so Multiplication is performed by making 1 the first Term in the Proportion, and saying, as 1 to the Multiplier, so the Multiplicand to the Product; but that Proportion being already explained in the Use of the Line of Numbers, and the Manner of extending upon the Sector, being just now described, I shall not need to enlarge further upon it.

But one chief Case in which the Sector is preferable to any other Instrument is, that the Line of Lines may be brought to any possible Scale of equal Parts, of how many Chains, Yards, or Feet, &c. whatsoever, to an Inch, or any other measure that the Sector will reach; as for Instance, suppose I would, by the Sector, form a Scale of 36 Parts to an Inch. Take an Inch in your Compasses, and with one Foot in 36, in the Line

of Lines, on one Leg of the Sector, open the Sector so that (the Compasses kept at the same Extent of one Inch) the other Foot may fall in 36, on the other Leg of the Sector; keep the Sector at that Opening lying by you, during the Time that you are using that Scale of 36 to an Inch, and you may take off any Number under 100 at one Extent.

Example. Suppose I would take off 18, which is just half of 36, set one Foot of the Compasses in 18, on one Leg of the Sector, and the other Foot in 18 on the other Leg just against it, that Extent shall contain just 18 such Parts as there is 36 to an Inch, which will be just half an Inch; for as the Divisions upon the Line of Lines are equal, and the two Legs of the Sector (at any Opening whatsoever) together with a Line supposed to be drawn from any Number upon one Leg, to the same Number on the other, make an Ilosceles Triangle, the Bases shall be proportionable to the Length of the Sides. *Euclid. Lib. VI. Prob. 4.*

And hence, whatever Number of Parts be proposed in an Inch, take an Inch in your Compasses, and setting one Foot on the Number of Parts proposed in an Inch on one Leg, set the Sector at such an Opening, that the other Point of the Compass may just fall upon the same Number on the other Leg, and then is your Sector set to the proposed Number of Parts to an Inch.

Again, suppose you are to reduce a Draught from one Scale to another, in any Proportion assigned; as for Instance, if you would reduce the Triangle GHI, and make another in Proportion to it as 4 to 7.

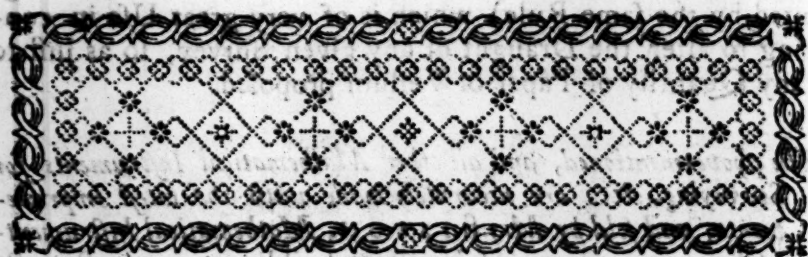
Take any Side of the given Triangle, as suppose GH, in your Compasses, and set that Extent from 7 on one Leg of the Sector, to 7 on the other; then keeping the Sector at that Opening, set one Foot of the Compasses in 4 upon one Leg, open the other to 4 in the other Leg, and with that Extent, make the Line OR of the second Triangle; then take the Line HI of the greater Triangle in your Compasses, and open the Sector, so that the Extent may reach from 7 to 7 as before, and keeping the Sector at that Opening, extend from 4 to 4, and set that Extent from O, to make the Arch at P; then taking GI in your Compasses, open the Sector till that Extent reach just from 7 to 7, and keeping the Sector at the same Opening, extend from 4 to 4; and with that Extent, and one Foot in R, cross the Arch P in P; then draw the Lines OP and RP, and you have finished the Triangle RPO, whose Sides bear the same Proportion to the Sides of the Triangle GHI, as 4 to 7.

By

By the same Rule may any right-lined Figure, of how many Sides soever, or how irregular soever, be reduced in any given Proportion to the Figure it was reduced from (or it may be augmented by the same Rule) which is of very great Use in contriving to alter the Draught of any given Survey, so as just to fill any Quantity of Paper or Vellum proposed.

The above-mentioned, and all other Mathematical Instruments for Surveying, &c. are accurately made with the latest Improvements, and sold by Mr. S I S S O N, Mathematical Instrument-maker, at the Corner of Beaufort-buildings, in the Strand, London.





P A R T II.

The PRACTICE of SURVEYING, shewing, to measure a Plot, and divide any Parcel of Ground, regular or irregular, accessible or inaccessible; with the Varieties of taking Heights and Distances.



General R U L E for measuring Superficies.



ALL Superficies that are to be measured, are either Squares or Right-angled Parallelograms, or supposed to be reduced to such, and then the universal Rule in measuring is,

Multiply the Length by the Breadth, and the Product is the Content in such Measure as you measured by, whether Chains, Yards, Feet, Inches, &c.

CHAP. I.

Of reducing Superficies, of whatsoever Form, to Squares or Parallelograms.

SECT. I.

Of a TRIANGLE. Fig. 52.

FIRST, a Right-angled Triangle, ACF , is reduced to a Parallelogram $BCFE$, by bisecting the Base AC in B , and by the Perpendicular BE , cut off that Part of the Triangle ABD , and apply it to that Part of the Parallelogram FDE , which is equal to it, because AB is equal to EF , and DE equal to BD , and DF equal to AD ; therefore the Triangle ABD , is equal to the Triangle FED ; but the Trapezium $BCFD$, being a Part of the Triangle ACF , and also of the Parallelogram $BCEF$, the Parallelogram $BCFE$ is equal to the Triangle ACF . *Euclid. Lib. 1. Ax. 2.*

Secondly, in Oblique Triangles.

Fig. 47.

By the same Rule the Oblique Triangles are reduced to Squares or Parallelograms, by first dividing the Oblique Triangle into two Right-angled Triangles, by letting fall a Perpendicular from the verticle Angle, as in *Fig. 47.* where the Oblique Triangle RST is divided into two Right-angled Triangles, by the Perpendicular SW , let fall from the vertical Angle S , to the Base RT , thereby constituting the Right-angled Triangles RWS and TWS ; each of which being reduced to a Parallelogram by the Directions beforementioned, and the half of each Base being the Base of the Parallelogram, to which it is reduced; and the half of each Base, RW and WT in one Sum, being equal to half the whole Base, RT , it will follow,

That multiplying half the Base by the whole Perpendicular, shall produce the Area of any Oblique Triangle, as well as of a Right-angled one; and consequently multiplying the whole Base by half the Perpendicular will produce the same.

A Trapezium is divided into two Triangles, by a Diagonal drawn between the opposite Angles; as the Trapezium $ABCD$ is divided into two Triangles ABC and ADC , by the Diagonal AC ; and then the Areas of these two Triangles, found as before, and added together, is the Area of the Trapezium required. *Fig. 23.*

Any irregular Polygon is to be divided into Triangles, and when those Triangles are measured as above, the Sum of the Areas when added together, is the Area of the Polygon required; and the same Method is to be used to find the Area of a Rhombus.

Note, Any irregular Polygon may be divided into a Number of Triangles less by two than the Number of Sides contained in it; as for Instance, a Polygon of seven Sides may be divided into five Triangles, &c.

The Area of a Rhomboides is found by multiplying one of the Sides by the Length of a Perpendicular drawn from one Side to the other: Thus the Area of the Rhomboides $ABCD$ is found by multiplying DC (or AB , which is equal) by RP ; *Fig. 12.* for the Illustration of which, draw bd perpendicular to AB , so as to cut BC in the Middle as at e , and join dC ; also draw the Perpendicular gf to cut AD in the Middle, as at e , then is Dge , equal to Aef , and Bba equal to adC ; take away from the Rhomboides the Triangles Dge , and Bba , and add their Equals Cda , and Afe ; then is the Rhomboides $ABCD$ reduced to the Parallelogram $fgdb$, whose Breadth fg or bd being equal to RP , viz. the Breadth of the Rhomboides, and fg or gd being equal to AB , or DC , it is to be measured by multiplying the Length AB , by the Perpendicular RP , to find the Area.

A regular Polygon may be divided into as many Isosceles Triangles, as there are Sides in the Polygon, by letting fall Perpendiculars from the Center to the Middle of each Side; but for the more expeditious casting up the Measure of it, the Sides being equal, and the Perpendiculars also equal, multiply the Sum of the Sides, by half one of the Perpendiculars, the Product is the Area required; or multiply half the Sum of the Sides by the whole Perpendicular, it will produce the same.

From this Principle is founded that Method of Measuring a Circle, by multiplying half the Circumference, by half the Diameter: for the more Sides any regular Polygon contains, the nearer its Area is to that of the Circle, in which it is inscribed; and therefore if we suppose a Circle to have inscribed in it a regular Polygon of an infinite Number of Sides, and consequently, each Side infinitely small, it will be divided into an infinite Number of Isosceles Triangles, whose Bases coincide with the Periphery of the Circle, which is the Sum of the Bases of all these Isosceles Triangles, and the Semi-diameter is the common Perpendicular; but as multiplying half the Base of any Triangle by its whole Perpendicular, gives the Area; so multiplying half the

Sum of the Bases of all those equal Iſosceles Triangles (*viz.* half the Circumference of the Circle) by the common Perpendicular, which is the Semi-diameter of the Circle, ſhall give the Area of all the Triangles, which is the Area of the Circle: Therefore,—

Note, *Multiplying half the Circumference of a Circle by half its Diameter, the Product is the Area or ſuperficial Content of the ſaid Circle.*

There is another Method of finding the ſuperficial Area of a Circle, which is done by ſquaring the Diameter, and multiplying that Sum by 11, and dividing the Product by 14; the laſt Quotient gives the Area, it being deduced from this *Theorem*.

Note, *The Area of any Circle inſcribed in a Square, is to the Area of the Square in which it is inſcribed, as 11 to 14; as I ſhall juſt now prove, if we allow the following Proportion, between the Diameter and the Circumference, to ground the Theorem upon.*

This Manner of Operation is deduced from that commonly allowed Proportion of the Diameter of a Circle to its Circumference, as 7 to 22, which may better be illuſtrated by an Inſtance in Numbers: As for Example.

Let the Diameter of a Circle be 14; then, according to the Proportion above, the Content of a Square, whoſe Side is 14, equal to the Diameter of the Circle, is 196.

But allowing the Proportion above, as 7 to 22, &c. then the Circumference of that Circle, whoſe Diameter is 14, will be 44; but if you multiply 22, which is half the Circumference 44, by 7, which is half the Diameter 14, the Product 154, is the Area of the Circle; therefore it will always from this Principle be—

$$\begin{array}{r} 14 \\ 14 \\ \hline 56 \\ 14 \\ \hline 196 \\ 22 = \frac{1}{2} \\ \hline 7 \end{array}$$

As 196 to 154, ſo the Area of a Square, to the Area of a Circle, whoſe Diameter is equal to the Side of the ſaid Square.

But as 196 to 154, ſo 14 to 11; therefore it being moſt convenient to work with the leaſt Numbers, it is as 14 to 11, ſo the Area of the Square to the Area of the Circle above-mentioned.

$$\begin{array}{r} 154 \\ 14 \\ \hline 616 \\ 154 \\ \hline 196 \overline{) 2156} \quad (11 \\ 196 \\ \hline 00 \end{array}$$

C H A P

CHAP. II.

SECT. I.

How to take the Plot of a Field at one Station in some Place within, or towards the Middle of the Field.

I Shall mention in two Instruments, which are most useful and most in Practice, namely, the Plain-table, and the Theodolite; the latter of which, as to its Use, differing so little from the Semi-circle and Circumferentor, that what serves as a Description and Use of it, may, in Practice, in some Measure, serve for the other with a little Alteration.

First, By the Plain-table.

Plant the Plain-table any where towards the Middle of the Field, so as you may from thence see all the Angles of the Field; let the Sheet of white Paper be put upon it as is already taught in the Use of the Plain-table; and if the Angles of the Field be at such a Distance, that you cannot perfectly perceive them from your Station, put Marks of white Paper or Linen at each Angle, or cause your Assistant to stand close up in the Angle that you are observing, and so cause him to remove from one Angle to another, as you have Occasion to make your Observations.

Then upon your Sheet of Paper, that is, upon the Plain-table, make a Point or Mark to represent the Point in the Field upon which the Plain-table stands; and laying the Edge of your Index upon that Point, and keeping it there, turn it about so, that through both Sights you may see one of the Angles; and from the Point along the Edge of the Index, draw a Line, which done, measure the Distance between the Plain-table and the said Angle, in Chains and Links; and taking that Number of Chains and Links from any equal Parts upon the Index, suitable to the Scale you would lay down your Field in, set it from the Point upon the Paper along the Line, and there make a Mark; then (keeping your Plain-table fixed just as it was) do so by the next Angle, and so by every Angle in the Field, setting off all your Distances upon their respective Lines, and making Marks at the End of every Distance;

then draw Lines from Mark to Mark quite round the Field, these shall represent the Hedges of the Field to be laid down.

Example. Suppose the Figure P, Q, R, S, T, be a Field to be plotted; place your Plain-table any where in the Field, so as that you may see all the Angles, by the Help of white Papers or Handkerchiefs set up at the Angles for that Purpose, as before directed; then assuming a Point, O, upon the Plain-table, as near as you can, in such a Proportion, towards the Middle of the Paper, as the Plain-table is placed in towards the Middle of the Field, which done, lay the Scale over the Point O, and turning it so, (still keeping it upon the Point O) that you can, through the Sights, see the Angle P; then, by the Edge of the Scale, draw the Line O P, and measure the Distance from O, to the Angle of the Field at P, and measure the Line O P, which suppose to be 7 Chains 35 Links, which set upon the Line O P, from O to P.

Then keeping your Table exactly fixed, as it was, turn the Index so as that it laying upon the Point O, you may, through the Sights, see the Angle Q; and then, by the Edge of the Index, draw the Line O Q, measure in the Field the nearest Distance from O to Q, which let be 5 Chains 42 Links; set 5 42, upon the Line O Q, from O to Q.

Then keeping the Plain-table always fixed, turn to the Angle R, and having seen the Angle R, through the Sights, from O draw the Line O R, and measure, and supposing it 5 Chains 15 Links, set that upon the Line O R, from O to R.

By the same Rules draw the Lines O S, and O T, and set off the Distances, when measured, from O to S and from O to T; and from the Points P, Q, R, S, T, draw the Lines P Q, Q R, R S, S T, and T P, these Lines (if the Hedges at straight) are the Hedges of the Field, and then is the Field protracted.

If you have done your Work exactly, the Sides T P, P Q, &c. may be measured by the Scale, from which you protracted the rest; but if you have met with Interruption by Ponds or Trees, or the like, that you distrust the Protraction; you may measure the Sides, or some of them, to see how they agree with the Scheme, for your own Satisfaction.

line, as above, direct your Sights from O to the Angle Q, and observe how many Degrees and Minutes it cuts; which suppose it be 67 Deg. 30 Min. place Q in the first Column of your Field-book, and 67 Deg. in the second Column, and 30 Min. in the third, the Column of Minutes; then measure the Distance from the Instrument to the Angle Q, which let be 5 Chains 42 Links; place 5 in the Column of Chains, and 42 in the Column of Links, and you have done with the Angle Q.

Then proceed to the next Angle at P, and direct your Sights from O to P, and mind what Angle is cut by the Index, as suppose 96 Degrees 00 Minutes, which put in the Column of Degrees and Minutes; and then measure the Distance from the Instrument to the Angle at P, which suppose to be 7 Chains 35 Links, place them also in their proper Columns; and proceed in the same Method with the Angles T, S, and R, and place the Angles and Distances as before directed, the Field-book will appear as follows.

R.	Deg.	Min.	Chains.	Links.	Remarks.
Q.	67	30	5	42	
P.	96	00	7	35	
T.	205	30	7	21	
S.	290	20	6	28	
R.	346	40	5	15	

Having thus taken your Angles and Distances, and placed them in this Order, in the Field-book, you may proceed to the Surveying of other Fields, &c. and commit them all to the Field-book in the same manner; and from thence at any Opportunity protract them upon Paper, which is to be thus done.

Make a Point in the Sheet of Paper to represent that Point in the Field where your Instrument stood, as at O, and upon it, as a Center, with the Chord of 60 Degrees describe the Circle *abcdefg*, and draw the Meridian or North and South Line *VW*.

Then have Recourse to your Field-book, and see what Angle the Line *OQ* makes at O, with the North and South Line *VW*, which you see in the Table or Field-book is 67 Deg. 30 Min. set 67. 30 off the Chords from *b* to *c*, and through *c* draw the Line *OQ*, and set off upon that Line the Distance to Q, which you see in the Field-book is 5 Chains 42 Links from O to Q.

Then subtract 67 Deg. 30 Min. from 96 Degrees, there remains 28 Deg. 30 Min. Set 28. 30 off the Chords from *c* to *d*, and

and through *d* draw the Line *OP*, upon which set 7 Chains 33 Links from *O* to *P*; then taking 96 Deg. 00 Min. from 205 Deg. 30 Min. there will remain 109 Deg. 30 Min. which take from the Chords, and set from *d* to *f*, and through *f* draw *OT*, and upon it set 7 Chains 21 Links from *O* to *T*.

Note, It often happens, as in this last Case, that the Angle is more than 90 degrees; and when it happens so, it cannot be set off all at once, therefore to set off 109 Degrees 30 Minutes, it is best to do it at twice, setting off 60 Degrees first, and then 49 Degrees 30 Minutes; and, indeed, that is more exact than to set off first 90, and then 19 Degrees 30 Minutes, because the Degrees on the Chords grow so small near 90 Degrees, that a small Error cannot be so well perceived there, as at the Beginning or Middle of the Line.

In the like Manner take 205 Degrees 30 Minutes from 205 Degrees 20 Minutes, there remains 84 Degrees 50 Minutes which set from *fg*, and through *g* draw *OS*, 6 Chains 28 Links from *O* to *S*. And, by the same Method, taking 290, 20 from 346, 40, and setting the Chord of the Remainder 56, 20 from *g* to *a*, through *a* draw the Line *OR*, 5 Chains 15 Links.

And lastly, for the Proof of your Work, take the last Number 346, 40 from 360, if the Remainder 13 Degrees 20 Minutes do not reach just from *a* to the North and South Line at *b*, there is some Error in laying down; but if that Remainder reach just to the North and South Line, it is probable the Work is right, for every Circle is 360 Degrees.

S E C T. III.

To take the Plot of a Field at one Station, in any Corner or Angle thereof.

PLACE your Instrument in the Corner or Angle proposed, as suppose at the Angle *H*; and then (whether you use a Theodolite, or that Side of the Plain-table that *Fig. 55.* may be used as a Theodolite) turn the Instrument about till the Needle lie just over the Meridian-line of the Card, and then is your Instrument fit for Observation.

Then suppose your Field be in the Form of the Figure *AB CDEFGH*, make and rule your Field-book as before di-

rected,

rected, and in the first Column place the Letters representing the Angles, and rule the other Columns, as in the foregoing Example of a Field-book.

Having placed your Instrument in this Position, turn the Index, till through the Sights you see the Angle A, and observe what Degree it cuts upon the Limb of the Instrument; which suppose to be 22 Degrees 15 Minutes; place that in the Columns of Degrees and Minutes against A, and measure the Distance from your Station to the Angle A, which let be 8 Chains 46 Links; place them in the Columns of Chains and Links against A, as you see in the following Table, and then is your first Observation finished.

Proceed next to direct your Sights to the Angle B, and observe again what Degree and Minute is cut by the Index; which suppose to be 42 Degrees 45 Minutes, and the Distance from the Station let be 15 Chains 21 Links; put the Degrees, Minutes, Links, and Chains, in their proper Columns, as you see in the following Table.

In the same Manner take the Angles and Distances to the other Corners of the Field, as C, D, E, F, and G, in which we will suppose the Angles with the Meridian to be found by the same Manner of Observation, as also their Distances from your Station, to be as in the following Table.

R.	Deg.	Min.	Chains.	Links
A	22	15	8	46
B	42	45	15	21
C	66	30	16	64
D	86	45	16	23
E	122	30	16	68
F	130	15	5	22
G	162	00	7	73

To protract this Field from this Table, draw the Meridian or North and South Line N S on the Sheet of Paper you intend to protract it upon, always observing to draw it towards that Side of the Sheet of Paper that lies towards that Point of the Compass that your Station was in the Field, and with the Chord of 60 Degrees, and one Foot of the Compasses in some Point in the North and South Line, representing the Point of your Station, as at H, draw a Circle N i k l m n o p S; then look in the Table for the Observation to A, which you find to be 22 Degrees 15 Minutes, set the Chord of 22 Degrees 15 Minutes from N to i, and through i draw the Line H A, upon which set the given Distance, which you find in the Table, to wit,

wit, 8 Chains 46 Links from H to A; then set off the second Angle mentioned in the Table, which is 42 45, which take off the Line of Chords, and set from N to *k*, and through *k*, draw the Line H *k* B, upon which set 15 Chains 21 Links from H to B: In the same Manner set the Chord of the third Angle C 66 30 from N to *l*; and through *l* draw the Line H C, and set off upon that, according to the Directions against C, in the Table, 16 Chains 64 Links: And so proceed to set off the Chord of 86 45 from N to *m*, and through *m* draw H D 16 Chains 23 Links; proceed in the same Manner with all the rest, and draw the Lines H E, H F, H G, with their Distances from H, as in the Table; and then drawing the Lines A B, B C, C D, D E, E F, and F G, which with the Lines G H, and H A, enclose the Field, and give the Shape of it in its just Proportion, from whence it may be divided into Triangles, in Order to find its Content in Acres, Roods, and Perches.

Note, In plotting any Field, or any Estate that lies contiguous, take Care to set off all the Boundary or dividing Lines from one and the same Scale of equal Parts.

If you protract by the Plain-table, and use it as a Plain-table, it is to be done upon the Sheet of Paper fixed thereupon, as has been taught in the Use of that Instrument.

CH A P III.

To plot a Field at two Stations by the Theodolite, or by the Plain-table used as a Theodolite.

S E C T. I.

To plot a Field at two Stations near the Middle thereof.

YOU must chuse your Stations on some Eminence or rising Ground, from whence you can see all the Angles or remarkable Places in the Hedges or Bounds of the Field, and take Care to have your two Stations as far asunder as conveniently you can

can, for the nearer they are together, the more Danger there is of contracting an Error; and chuse them, as much as possible, to have the Ground tollerably plain between them, that their Distance may be the truer taken, for it is upon the true Distance between the two Stations, that the Truth of the Work depends, and without which, it is impossible to avoid an Error.

Let the Field be ABCDEFGH, and the Stations I and K, which suppose to be 5 Chains 60 Links asunder. *Fig. 56.*

To perform this by the PLAIN-TABLE.

Fix your Instrument in either Station (suppose in I) and turn it about till the Needle hang just over the Meridian-line of the Card; and then having a Sheet of Paper fixed upon the Plain-table, mark a Point in it, to represent the Station you are at (as I), and from that Point lay your Index, directing the Sights to the other Station K, and by the Side of the Index draw a Line IK; then measure the Distance between the Stations, which suppose, as above, to be 5 Chains 60 Links; set that upon the Line IK from I to K, and then are your two Stations marked down.

Note, It will be convenient to draw the Meridian-line NS through I, your first Station, which will always lie parallel to the Edge of the Table, when the Needle is directly over the Meridian-line.

Then keeping the Index fixed upon the Point I, turn it about till you can, through the Sights, see the Angle B, and keeping it there, draw the Line IB at pleasure along the Side of the Index.

Then keeping the Index still upon the Point I, turn it till you can, through the Sights, see the Angle C, and draw the Line IC, and so proceed in all the rest, keeping the Instrument fixed as before; turn the Index about, till (keeping it always upon the Point I) you can through the Sights, see the Points or Angles of the Field DEFGHA, and by the Side of the Index draw the Lines ID, IE, IF, IG, IH and IA, by the same Method that you draw IB and IC.

This done, remove your Instrument to the other Station K, and fixing it with the Needle over the Meridian-line, as before, lay the Index upon the Point K, and turn it about till you can, through the Sights, see the Angle B; and then by the Side of the Index, draw the Line KB; it will cross the former Line IB in B; then turn the Index about (keeping always the Edge of it upon the Point K (till you can, through the Sights, see the

the Angle C, and draw the Line K C, to cross the former Line I C in C; then turning it further till you can, through the Sights, see the Point D, and draw the Line K D, and in the same Manner draw the Lines K E, K F, K G, K H, and K A, and they will cross the former Lines I B, I C, &c. in the Points B, C, D, E, F, G, H, and A; then from one of these Crossings to the next, draw the Lines A B, B C, C D, D E, E F, F G, G H, and H A, and these shall represent the Field, with the Bounds thereof, in its just Proportion, if it is done carefully and exactly

Note, You need not in this Case measure any of the Distances from either Station I or K, to any of the Angles A, B, C, &c. Nor need any Distance be measured, but the Distance between the Stations I and K, the Intersections of the Lines I B, I C, &c. with the Lines K B, K C, &c. determining the Distances, if rightly done.

To take the same Plot of a Field at two Stations, by the
THEODOLITE.

Fix your Theodolite in one of the Stations, suppose at I, and if your two Stations are not directly North and South from each other (as in this Case they are supposed not to be) turn the Theodolite about, till the Needle be directly over the Meridian-line of the Card; and then turn the Index, till you can, through the Sights, see the other Station K, and observe what Degree is cut by the Index, which suppose be 31 Degrees 30 Minutes, and then measure the Distance I K, which let be 5 Chains 60 Links, both which set down at the Head of the Table or Field-book thus:

I K 31 Deg. 30 Min. ——— 5 Chains 60 Links.

Next turn the Index to the Angle B (keeping always the Needle over the Meridian-line and observe what Degree is cut by the Index, which suppose to be 61 Degrees 30 Minutes; and then proceed to C, turning the Index till you see the Angle C through the Sights, and mind what Degree is cut by the Index, which suppose to be 120 Degrees 30 Minutes, and so in the rest; and when you have found what Degree is cut by the Index, in looking to the several Angles D, E, F, G, and A, put them down in a little Table. Then go to the Station K, and placing your Instrument as before, turn the Sights to the Angle A, and observe the Degree cut by the Index, which suppose 28 Degrees

Degrees 00 Minutes; then turn to the Angle B, and suppose the Index cut 93 Degrees 20 Minutes, and so find the Degrees cut by observing to C, D, E, &c. and place them in a Table, as here you see.

I K 31. Deg. 30 Min. — 5 Chains 60 Links.					
Station I.			Station K.		
	Deg.	Min.		Deg.	Min.
A	28	30		28	00
B	61	30		93	20
C	120	30		169	30
D	148	15		194	00
E	211	30		211	30
F	274	00		240	00
G	350	30		306	30
H	12	00		334	30

Then to plot the Field from the Numbers thus found in the Table.

Prepare a clean Sheet of Paper, and draw thereon the North and South Line N S, and from the Point I draw I K, to make an Angle of 31 Deg. 30 Min. with the Line N S (as it is expressed in the Head of the Table, and drawn in the Figure) and upon that set the Distances between the Stations 5 Chains 60 Links from I to K. *Fig. 56.*

Then according to the Numbers in the Table lay down the other Angles of the Field, as first by the Degrees and Minutes in the two Columns under Station I, as first, against A you find 28 Deg. 30 Min. therefore draw the Line I A, to make an Angle 28 Deg. 30 Min. with the North and South Line N S (you need not regard the Length of the Line I A, but draw it at pleasure.) Then draw I B to make an Angle of 61 Degrees 30 Minutes with the said Meridian-line N S, as in the Table; then proceed to draw the Line I C, which because we see in the Table makes an Angle of 120 Degrees 30 Minutes, with the Meridian-line N S, draw it to make that Angle with the said Line, and so proceed to draw all the given Lines I D, I E, I F, I G, I H, to make the Angles set down in the Tables.

Note, All these Angles are to be made by the Help of the Line of Chords, according to the Eighth Geometrical Problem, Part II. Chap. III. Page 56.

Note also, That as the Angles go round to any thing less than 360 Degrees in this Way of protracting, it is best, after having drawn the two Circles of which I and K are the Centers; and when the Number of Degrees to be laid down is more than 90, set off the Chord of 60 as often as you can, and then the Odds, as in drawing the Line IF you see in the Table, it lies 274 Degrees and 00 Minutes from the Meridian-line NS; set off, therefore, four Times the Chord of 60, which is 240 from a to b; and then because 240 wants 34 of 274, set 34 of the Chords from b to c, and through c the Line IF is to be drawn.

When you have finished all the Lines to be drawn from I, draw another Meridian-line parallel to the former; but to pass through the Point K (by Prob. V.) and look in the Table for the Figures under Station K, where you find A is 28 Degrees 00 Minutes; set the Chord of that upon the Circle from the Meridian-line at d to e, and through e draw KA to cut IA in A; then looking for B in the Table, I find 93 Degrees 20 Minutes, set the Chord of that from d to g, and through g draw the Line CB to cross IB in B; and so, according to the foregoing Directions, draw the Lines KC, KD, KE, KF, KG, and KH to cross the former Lines in the Points C, D, E, F, G, and H. Lastly, between each of these Crossings, as A, B, &c. draw the Lines AB, BC, CD, DE, EF, FG, GH, and HA, they shall exactly represent the Hedges or Bounds of the Field.

Note, It is best in protracting to draw all the Lines with a black Lead Pencil that are within the Field, as the Lines IK, IA, also KA, KB, KC, &c. because they are of no Use, but to find the Points A, B, C, &c. by the Help of which, the Lines AB, BC, &c. the Bounds of the Field, are drawn; and then when you have done with the Lines within the Field, you may rub them out with a Bit of Bread, and then the Plot with only the Out-lines of it remains clear of Confusion, and ready to have Lines drawn in it, to divide it into Triangles, in order to be measured and cast up, which Lines ought also to be drawn with a black Lead Pencil, to be taken out as soon as the measuring is done, that nothing but Hedges, Walls, or other Bounds or Things necessary may remain.

S E C T. II.

To take the Plot of a Field at two Stations near the Middle, from neither of which all the Angles can be seen, only some two of them may be seen from both Stations, as the Angles A and F (Fig. 57.) from the Stations L and V.

LET the Field proposed be ABCDEFGHIK, and let the two Stations be V and L, from both of which you can see the Angles or Points A and F. Fig. 57.

Place your Instruments at one of the Stations at V, and by the Help of the Needle, used according to the Directions given before, find the Meridian or North and South Line N S, and then,

I. By the Plain-table.

Keeping the Plain-table fixed at this Point V, and laying the Index over the Point V, in the Sheet of Paper that represents the Point V, that is your Station in the Field; direct the Edge of the Index so, that you can through the Sights, see the Station L, and by the Edge of the Index, draw the Line V L; then measure the Distance V L, which suppose to be 9 Chains 30 Links, set 9, 30 off the Scale you intend to plot the Field by, from V to L; then keeping the Plain-table fixed as before, direct the Edge of the Index from V to F, so that you can, through the Sights, see the Point or Angle F, and then by the Edge of the Index draw the Line V F; then measure in the Field, the Distance in Chains and Links, from the Station V to the Angle F, suppose to be 6 Chains 35 Links, set 6, 35 (off the same Scale you used for the Line V L) from V to F; then keeping your Plain-table fixed in the same Place, direct the Sights from V to G; and draw V G, which let be 7 Chains 55 Links, which set off the same Scale from V

to G; proceed in the same Manner	Chains. Links.
with the Lines V H, V I, V K, and V	V H - 8 : 68
A, whose Lengths suppose to be as in	V I - 8 : 00
the Margin, and then you have found	V K - 8 : 56
the true Position of the Points of the	V A - 6 : 25
Field F G H I K A, visible from the	
Station V.	

Then proceed to the Station L, and, as before directed, find the Meridian-line N S, which, if exactly found, will be parallel

to the former found at V; and therefore a very safe Way for finding it is, having placed your Instrument at the Station I, in the Field; lay the Index along the Line V L, which you drew when at the Station V, and keeping the Index upon that Line, turn the Plain-table about, till you can, through the Sights of the Index, see the Station V, that you last came from, and there screw the Instrument fast, and keep it till you have finished all the rest of your Observations.

Through L draw the Meridian-line N S, parallel to the other Line N S, drawn through V, and it shall be the true Meridian-line required; or if by the Compass you find the true Meridian-line last drawn, it will be parallel to the other; and the Angles N L V, and S V L, will be equal. *Euclid. Lib. I. Prop. 29.*

Then proceed as in the foregoing Directions, viz. lay the Index upon L, and turn it about till you can, through the Sights, see the Point A, and along the Edge of the Index draw the Line L A; then measure the Distance from L to A, in the Field, which suppose to be 6 Chains 30 Links, set 6, 30 (still off the same Scale) from L to A.

Then keeping the Edge of the Index upon L, turn it about till you can, through the Sights, see the Point of the Field B; and by the Edge of it draw the Line L B, which suppose to be 8

	Chains.	Links.	do the same by the Angles C, D, E,
LC	- 6	: 00	and F, drawing the several Lines, and
LD	- 3	: 8	setting off their respective Distances
LE	- 7	: 85	LC, LD, LE, and LF, as in the
LF	- 8	: 45	Margin; and then the Field is all pro-

tracted, and is found to be exactly in the Form of the Figure ABCDEFGHIK, and is ready to be divided into Triangles, or reduced to a proper Form to be cast up, as shall be shewed hereafter.

Note, To find the true Meridian-line by the Compass, particular Care ought to be had, to allow for the Variation, if you intend to place the Field in its true Position; but if you use it only for a Direction from Angle to Angle, without any Respect to the general Position of the Field, the Variation need not be regarded.

How to plot the same Field by the SEMI-CIRCLE.

Chuse your two Stations, as before, from both which some two Angles in the Field will be visible, as in the foregoing Figure, where, from both the Stations V and L, you may see the Angles A and F.

Placing

Placing your Instrument at V, find the Meridian-line N S, as before taught, by bringing the Needle over the Meridian-line of the Card, and there screw it fast.

Then turn the Index about, till you can, through the Sights, see the Angle I, and observe what Degree the Index cuts, which, suppose to be 24 Degrees 30 Minutes, set that in the Table against V I in the Column of Degrees and Minutes; then measure the Distance V I, which suppose to be (as in the last Example) 8 Chains 00 Links, set that in the last Column of the Table under *Ch. Lin.*

	Deg.	Min.	Ch.	Lin.
V I	24	: 30	8	: 00
V K	62	: 00	8	: 56
V A	146	: 45	6	: 25
V F	241	: 30	6	: 35
V G	302	: 15	7	: 55
V H	344	: 30	8	: 64
L V	5	: 30	9	: 30
L A	50	: 45	6	: 30
L B	95	: 45	8	: 26
L C	153	: 00	6	: 00
L D	208	: 45	3	: 8
L E	242	: 30	7	: 85
L F	331	: 00	8	: 45

The Angle and Distance between the Stations.

Then take your Observation to the Angle K through the Sights of the Index, and observe what Degree it cuts upon the Verge of the Instrument; as suppose 62 Degrees 00 Minutes, put that down in the Column of Angles under Degrees and Minutes; and then measure the Distance V K 8 Chains 56 Links, which put against V K, in the last Columns under *Chains and Links*, as you see in the Table.

In the same Manner take the Angle that V A makes with the Meridian-line, which is 146 Degrees 45 Minutes, put that down in the Table in the Column of Degrees and Minutes, and the Distance V A, 6 Chains 25 Links in the last Column; and so proceed to take the Angles and Distance of the Lines

V, F, V, G, V, H, and put them all down in the Table, as you see done:

Then remove your Instrument to the Station L, and having found the Meridian-line N S that passes thro' that Station, and screwed the Instrument fast, as before directed, turn the Index till you can, through the Sights, see the first Station V, and then observing the Index to cut 5 Degrees 30 Minutes on the Limb of the Instrument, I place 5 : 30 in the Column of Degrees and Minutes, and put the Distance L V, which, by measuring, I find to be 9 Chains 30 Links, in the last Column of Distances.

Then turn the Index so that you may, through the Sights, see the Angle at A ; which done, observe what Degree of the Instrument is cut by the Index, which suppose to be 50 Degrees 45 Minutes, place that against L A in the Table, and measure the Distance L A, which being 6 Chains 30 Links, place that in the last Column of the Table against L A, as before taught.

Your next Observation is to the Point B, to which turning the Index, and finding it to cut upon the Verge of the Instrument 95 Degrees 45 Minutes, and measuring the Distance L B, 8 Chains 26 Links, put them in their proper Columns, as you see in the Table, and proceed in the same Manner with all the rest of the Angles and Distances in the Field, and the Table will appear as above.

How to plot the Field by the THEODOLITE.

Having placed your Instrument at the Station V, and brought the Needle over the Meridian-line, and thereby found the Meridian, or North and South Line of the Field N V S, screw the Instrument fast, and turn the Index about, till you can, through the Sights, see the Point I, and observe what Degree and Minute is cut by the Index, which suppose 24 Degrees 30 Minutes; place 24 : 30 in your Table, against I, in the Column of Degrees and Minutes, and then measure the Distance from the Station V to the Angle I, which suppose 8 Chains 00 Links, set that in the Table also against I, in the Column of Chains and Links; and then keeping the Instrument fast, turn the Index till you can, through the Sights, see the Angle K; and then suppose the Index cut 62 Degrees 00 Minutes, put 62 Degrees 00 Minutes in the Column of Degrees and Minutes against K, and then with your Chain measure the Distance V K, which suppose to be 8 Chains 56 Links, set that against K in the

the Column of Chains and Links, as you did before in the Angle I.

Then turn the Index to the Angle A, and by the same Method of observing what Degree and Minute is cut by the Index, when you can, through the Sights, see the Angle A, which suppose to be 146 Degrees 45 Minutes, set that in the Column of Degrees and Minutes against A, and measure the Distance V A, which suppose to be 6 Chains 25 Links, put that in its proper Place in the Table against A, and in the Column of Chains and Links, as before directed in the other Angles.

The next Angle of the Field that you can see from the Station V, is supposed to be F, to which turning the Index, till you can, through the Sights, see the said Angle F; let it be supposed to cut 241 Degrees 30 Minutes, and the Distance V F, measured as before, let be 6 Chains 35 Links, put each in their proper Columns in the Table, and proceed to turn the Index to G.

Then suppose, seeing G through the Sights of the Index, and the Degrees cut by the Index to be 302, 15, and the Distance V G, when measured, to be 7 Chains 55 Links, put them in their proper Places against G in the Table; proceed to the last Angle H, which, when you see through the Sights, suppose the Index cut 344, 30, and the Distance V H, when measured, let be 8 Chains 64 Links; place these Degrees and Minutes, and also the Chains and Links, each in their proper Columns against H, as you see in the Table.

Then remove the Instrument to the Station L, and having fixed it as before, with the Needle over the Meridian-line, and then screwed it fast, turn the Index till you can, through the Sights, see the former Station V, and observe the Degrees cut by the Index, which suppose 5 Degrees 30 Minutes, and the Distance V L to be 9 Chains 30 Links, put that in a Space by itself in the Middle of the Table; and then take all the Angles and Distances from the said second Station L, as you did before from the Station V, supposing the Index turned to A cut 50 Degrees 45 Minutes, and the Distance L A is 6 Chains 30 Links: Also the Index turned to B cut 95 Degrees 45 Minutes, and the Distance L B 8 Chains 26 Links. Let the Index turned to C cut 153 Degrees 00 Minutes, and the Distance L C be just 6 Chains; then suppose the Index turned to D cut 208 Degrees 45 Minutes, and the Distance L D be 3 Chains 8 Links: Also let E cut 242 Degrees 30 Minutes, and the Distance L E be 7 Chains 85 Links; and lastly, let the Index turned to F cut 331 Degrees 00 Minutes, and the Distance

L F

LF be 8 Chains 45 Links, bringing that also into the Table, and the whole will appear as follows.

	<i>Deg.</i>	<i>Min.</i>	<i>Cha.</i>	<i>Links.</i>
I	24	30	8	00
K	62	00	8	56
A	146	45	6	25
F	241	30	6	35
G	302	15	7	55
H	344	30	8	64
The Angle N L V is 5 <i>Deg</i> 30 <i>Min.</i> and the Distance is 9 <i>Chains</i> 30 <i>Links</i> between the Stations.				
A	50	45	6	30
B	95	45	8	26
C	153	00	6	00
D	208	45	3	8
E	242	30	7	85
F	331	00	8	45

First Station V.

Second Station L.

I might have added the Performance of the foregoing by the Circumferentor, but the Operations are so much alike, that he who understands one, cannot well be ignorant of the other.

How to plot the foregoing Field upon Paper, after the Dimensions have been taken and placed in a Field-book, by the foregoing Directions.

It is the best Way when the Weather is precarious or inclined to Moisture, to work by the Theodolite; because, with the Plain-table, by which you may survey and protract all at once, the Sheet is in danger of being wetted and spoiled; and the Method by the Theodolite being performed, and the Angles and Distances brought into a Field-book, as already taught, the Work may be protracted at home, or in any convenient Place where you may be secured from the Injuries of the Weather, and may be done at any Time after the Angles and Distances

stances are taken, or at any Distance from the Place, as well as upon the Spot, and is to be performed as follows.

Note, The Sector being an Instrument of such universal Use, and yet so portable, I shall first shew the Way of Protracting from the Field-book by that Instrument, without any other, only a Pair of Compasses.

You must first determine the Dimensions of the Paper or Space that you intend to plot your Field upon, which suppose to be four Inches the longest Way, and Breadth proportionable, besides a Margin, if required.

Then consider the longest Dimensions of the Field, which may be computed exact enough for this Purpose, by adding together the Distances IV, VL, and LD; which found in the Table and added together, their Sum is 20 Chains 38 Links; therefore you are to contrive for a Scale, that the said 20 Chains 38 Links may come into the four Inches, the Length of your Paper, or something less than the said four Inches, rather than more; which is best done by reducing all the abovesaid 20 Chains 38 Links into Links, and divide that by 4, the Quotient of that Division is the Number of Links to an Inch; which, if it be a whole Number of Chains without any odd Links, is the Scale required; but if you find odd Links come out in the Division, as it commonly happens, reject the odd Links, and fix your Scale to the remaining Number of Chains to an Inch; except the odd Links amount to very near another Chain.

In the Example above, the Sum of the Lines IV, VL, and LD is 20 Chains 38 Links, which reduced to Links, is 2038 Links; that divided by 4, the Length of the Paper in Inches, the Quotient is 509 Links, or 5 Chains 9 Links, and consequently you may fix your Scale at five Chains to an Inch.

If you think to plot by the Sector, and take your Scale from thence, take the Distance between the two Stations V and L, 9 Chains 30 Links, or 930 Links, and having first drawn the Meridian-line V, and at V made the Angle SVL, then finding the Line VL 930 Links, as above, take such an Extent in your Compasses as may be supposed to be allowed for the Extent VL, in Proportion to what the other Lines are, and keeping that Extent in your Compasses, place one Foot in 930 (the Number of Lines between V and L) open the Sector (not the Compasses) till the other Foot fall in the same Number, viz, 930 on the other Leg of the Sector, and keeping the Sector at that Extent, you have a Scale fitted to the whole Plot.

Example. Suppose you would afterwards set off four Chains 50 Links (which is four Chains and a half) keep the Sector always at the same Angle, and set one Foot of your Compasses in 4.5 on one Leg, and extend the other till it come just to the 4.5 on the other Leg; that Extent to that Scale shall be just 4 Chains 5 Tenths, or 4 Chains 50 Links required, &c.

The Scale being thus fixed: proceed to plotting thus:

Draw one of the Meridian-lines, suppose NVS , and make the Angle NVI 24 Degrees 30 Minutes, and from V draw VI , upon which set off, according to the Table, 8 Chains 00 Links from V to I ; then by the Help of the Chords, make the Angle NVK 62 Degrees 00 Minutes, and look in the Table for the Length VK 8 Chains 56 Links, which set off the same Scale from V to K , and so proceed to the Angles and Distances of the rest, *viz.* Find what Angle the Line VA makes with the Meridian-line NV , and also the Distance VA in Chains and Links, as in the Table; and so find the Angles and Distances of the Lines VF , VG , and VH , and laying down both the Angles and Distances, as above proposed, you will have found the Points I , K , A , F , G , and H .

Then for the next Station, look in the Table for the Angle SVL , 5 Degrees 30 Minutes, at which Angle draw the Line VL , and set off the Distance VL 9 Chains 30 Links; and then remove to the Station L , and draw the Meridian-line NLS , through L , and proceed as before, *viz.*

In your Plot set off the Angle NLA 50 Deg. 45 Min. and the Distance LA 6 Chains 30 Links from L to A , to find the Point A .

In the same Manner find the Points B , C , D , E , and F , by the Table, and then draw the Lines FG , GH , HL , IK , KA , AB , BC , CD , DE , and DF , which shall represent the true Proportion of the Field, and the Contents ready to be cast up, as shall hereafter be directed.

S E C T.

S E C T. III.

To plot a Field at two Stations in any two Angles thereof, from both which all the Angles may be seen.

IN this Case you have no Occasion to find any Distance but that between the two Stations, which being obtained, the Work is to be performed as follows.

By the Plain-Table. *Fig. 58.*

If you would only take the Plot of the Field as it lies, without any regard to the Position of it with Respect to the Meridian, place the Instrument in one of the Angles of Station, suppose at A, and place the Index with the Sight parallel to the Edge of the Table, and letting it stand so, turn the Table about till you can, through the Sights, see the Station B, and there screw the Instrument fast, and keep it so during your Observations to all the Angles from the Station A, and by the Side of the Index draw the Line A B; then measure the Distance A B, which suppose 7 Chains 35 Links, take that from the Scale you intend to plot from, and set upon the Line A B, from A to B.

Then keeping the Instrument fast, lay one Part of the Index upon the Point A, and turn it about till you can, through the Sights, see the Angle C, and by the Side or Edge of the Index draw A C: Again, with the Index still upon A, turn it about till you can, through the Sights, see the Angle D, and by the Edge of the Index draw the Line A D.

Proceed in the same Manner to draw the Lines A E, A F, and A G, drawing them at pleasure upon the Paper, without any regard to measuring the Length of them.

Then remove the Instrument to the Angle B, and having placed it as before, lay the Index again parallel to the Edge of the Table, and then turn the Table about till you can, through the Sights, see the Angle of Station A, and there screw the Instrument fast.

Keeping the Edge of the Index upon the Point B, turn it about till you can, through the Sights, see the Angle G, and by the Side of the Index draw the Line B G to cut A G in G.

Then in the same Manner direct your Sight from B to F, and draw the Line B F to cut A F in F ; and in the same Manner directing the Index from B to the several Angles E, D, and C, till you can see them through the Sights, draw the Lines B E, B D, and B C, to cut the former Lines A E, A D, and A C, in the Points E, C, and D ; and through the Intersections G, F, &c. draw the Lines A G, G F, F E, E D, D C, and C B ; these, with the stationary Distance, or Line A B, enclose the Field, and represent it in its true Proportion ; and if any of the Hedges, as F E or E D, &c. or any Distance cross the Field, as B F, or A E, &c. be taken in your Compasses, and applied to the same Scale upon which the Line A B was measured, it will give the true Length of the Hedge or Distance so required, as if it was measured with a Chain, if the Plotting be exactly performed.

N. B. *By this Method the principal Places in a Survey of a County may be placed in a Map, if you make Choice of two Eminencies, whose Distance you can measure, and from both which, you can see several others, whether Churches, Castles, Hills, Gentlemens Seats, and whatever else is remarkable : But more of this when we come to treat of County Surveying.*

To plot the Field (Fig. 58.) from the two Stations A and B, by the THEODOLITE.

First prepare a Table with only two Columns, the first for the Letters representing the Angles, and the other for Degrees and Minutes (for you need not in this Case have regard to any Distance, but the stationary Distance A B) and then

Place your Instrument at either Station (it matters not which first) as suppose at A, and turning the Index to 00 Deg. 00 Min. turn the Body of the Instrument about, till you can, through the Sights, see the other Station B, and there screw the Instrument fast.

This done, turn the Index about till you can, through the Sights, see the Angle G, and observe what Angle it makes, or what Degree it cuts upon the Theodolite ; which suppose to be 62 Degrees 30 Minutes ; put G in the first Column of the Table belonging to the first Station, and against it put in the other Column 62 Degrees 30 Minutes, as you see below.

Then turn the Index till you can, through the Sights, see the Angle F, and observe the Degree cut, which let be 88 Deg. 30 Min.

30 Min. place that in the Table against F, as you see; and by the same Method turn the Index to E, D, and C, observing each Angle, and setting them down against their respective Letters in the Table, they will stand, as you see against the first Station in the Table.

Then proceed to the second Station B, and again fixing the Index upon 00 Degrees 00 Minutes of the outer Circle, turn the Instrument about till you can, through the Sights, see the first Station A, and there screw it fast.

This done, turn the Index till you can, through the Sights, see the Angle G, and observe the Degrees cut by the Index, which suppose to be 20 Deg. 30 Min. set that in the Table against the second Station, as you see in the following Table, and so proceed to observe the Angles F, E, D, and C, and place each Degree against its respective Letter, and the Table will appear as follows.

		Deg.	Min.
First Station A -	G	62	: 30
	F	88	: 30
	E	122	: 15
	D	133	: 45
	C	163	: 30
Stationary Distance, 7 C. 35 L.			
Second Station B	G	20	: 30
	F	40	: 00
	E	45	: 10
	D	68	: 15
	C	131	: 00

From the Table above, to lay down the Draught of the Field upon Paper.

Having proportioned your Scale to the Paper or Plan that you intend to plot upon, draw the Line A B near the Edge of the Paper (because it is one of the Out-sides of the Field :) And if you intend to plot by the Sector, continue the Line B A to H, and upon A, as a Center, draw the Circle H L K, to what Radius your Draught will admit of.

Then

Then look in your Table for the first Observation G, against which you find 62 Deg. 30 Min. set 62 Deg. 30 Min. from H to *m*, and through *m* draw the Line A *m* G at pleasure, and so go on to the Angle F, which you find in the Table, is 88 Deg. 30 Min. set 88 ; 30 from H to L, and draw the Line A L F.

In the same Manner draw the Lines A E, A D, A C, through their respective Degrees in the Circle H *m* L K, as found in the Table, and then you have done with the Station A.

Then with the Chord of 60 Degrees, and one Foot of the Compasses in B, draw the Arch I *n* C, and looking in the Table for G, in the second Station, you find 20 Degrees 30 Minutes; set the Chord of 20 Deg. 30 Min. from I to *n*, and through *n* draw the Line B G, to cut the former Line A G in G; then the next Angle being F, against which in the Table stand 40 Degrees 00 Minutes. set 40 Deg. 00 Min. of the Chords from I upon the Circle, and through the Place where the Point falls, draw B F.

The next being E, found in the Table 45 Deg. 10 Min. set 45 Deg. 10 Min. from I to o, and through o draw B E, to cut A E in E; and by the same Method draw the Lines B D, and B C to intersect A D and A C, in the Points D and C; and from each Intersection to the next, draw the Lines B C, C D, D E, E F, F G, and G A, and these are the Bounds of the Field.



C H A P. IV.

To plot a Field by going round it, which is necessary in Wood-Land or Copses, &c. where the foregoing Method, by taking Sights, and measuring Distances within the Enclosure, is impracticable.

S E C T. I.

To perform this by observing at every Angle.

I. By the Plain-table.

SUPPOSE a Field in Form of the Figure SHIKLMNOP was to be measured, and you would begin at the Corner H.

Fig. 59.

Having determined your Scale, either amongst the Scales of the Index, or upon the Sector, as directed in the foregoing Chapter, fix your Instrument at H, and turn it about till the Needle be just over the Meridian-line of the Card, and there screw the Instrument fast; and parallel to that Side of the Table that lies farthest from the Field, draw the pricked Line HG, for a Meridian-line, in which let H be South and G North, and then the Angle H is the westernmost Angle of the Field.

Note, If you begin at the Eastermost Angle in the Field, the Meridian-line must be on the East Side of the Field; as if H was the Eastermost Angle, then in the Line HG, H would be North, and G South; but if you intend to begin at an Angle at the North or South Part of the Field, as at K or L, or at O, you must consider to draw the Meridian-line proportionably near the Middle of the Plain-table, if you intend to bring all the Enclosure into one Sheet.

But suppose H be the South-westward Angle in the Field, consider, as all the Field has mostly Northing from you, assume a Point in the Southermost of your Meridian-line, as at H, to represent the Angle H in the Field, and at that Angle place your Instrument, with the Needle over the Meridian-line, and there screw it fast.

K

Then

Then if you cannot certainly see where the Angle is, by reason of Bushes or Branches of Trees leaning out of the Hedge, &c. let your Assistant go and set up a Mark at the said Angle, and laying the Index upon H, turn the other Part of it about, till you can, through the Sights, see the Angle or Mark I, and by the Side of the Index draw the Line HI; then with your Chain measure the distance HI, which suppose 7 Chains 90 Links; set 7 : 90 off your Scale from H to I, and also write upon the Line 7 : 90, or in a Book or Paper a-part, write down HI 7 : 90.

Take up your Instrument, and where it stood set up a Mark that you can see from I, and then remove your Instrument to I, and turning it about till the Needle lie over the Meridian-line of the Compass, screw it fast, and laying the Index upon the Point I, the Station you are at, turn the other Part about, till you can, through the Sights, see the Angle K, and by the Side of the Index draw the Line IK, and measuring the Distance IK, which suppose to be 6 Chains 25 Links, set 6 : 25 off your Scale from I to K, or write it down in Paper under the former HI, as you see in the Margin.

Note, *Left your Needle should not traverse freely, and by that Means some Error might be contracted, the safest Way is, at every Remove to your next Station, to lay the Index upon the Line drawn between it and the last, and then turn the Body of the Instrument about till you can, through the Sights, see the last Station, and there screw it fast, for then you are sure you are right.*

Example. *Removing from H to I, leave a Mark at H, and setting up the Instrument at I, lay the Index upon the Line HI, and turn the Instrument about till you can, through the Sights, see the Mark H, for then you are sure the Meridian-line of the Table is parallel to the Meridian-line found at the Mark H.*

When you have finished your Observation at I, remove to K, and taking your back Sight to I, as just now directed, screw the Instrument fast, and laying the Index upon K, turn it, till you can, through the Sights, see the point L (helping yourself with a Mark, as before directed, if it cannot otherwise be seen) and by the Edge of the Instrument draw the Line KL, which measure, and suppose it be found 4 Chains 50 Links, set 4 : 50, off your Scale from K to L, and set also the Figures 4 : 50, on the Line KL; or put them down in Writing KL 4 : 50, under the other, as above directed.

Then,

Then, leaving a Mark at K, remove your Instrument to L, and by the Help of the Needle or Back-sight, fix your instrument as before, and laying the Index upon L, turn the Index, till you can, through the Sights, see the Corner M (or a Mark put up at it, if there is Occasion) and draw the Line L M, and measure the Distance L M 8 Chains 00 Links, which set off the same Scale from L to M, also upon the Line L M 8 : 00 or write them in a Table as before.

It will be needless to repeat the Operation for all the rest of the Angles, the Method being exactly the same; and when you have in the same Manner taken all the Angles and Sides of the Field, it will appear upon the Plain-table, as *Fig. 59.* and the Table of Angles and Distances will appear as in the Margin, and is ready to be divided into Triangles, and measured and cast up, as shall hereafter be directed.

	Ch.	Lin.
HI	7	: 90
IK	6	: 25
KL	4	: 50
LM	8	: 00
MN	6	: 15
NO	6	: 10
OP	4	: 55
PH	3	: 00

To plot the same Field by the THEODOLITE.

Suppose you would begin at the Angle or Corner H, as before Place your Instrument at the said Corner or Angle, and set the Index at 00 Degrees 00 Minutes, and then turn the Instrument about with that End of the Index forward, or towards the Angle I, that lies upon 00 Degrees 00 Minutes, till you can, through the Sights, see the said Angle I, and there fix the Instrument fast, and turn the Index about, till you can, through the Sights, see the Corner P, and then mark the Degrees cut by the Index, which is 99 Deg. 30 Min. and measure the Distance H I 7 Chains 90 Links, both which put down in the Table or Field-book, as before taught, and as we shall instance in this Example. *Fig. 56.*

Then remove your Instrument to I, and placing the Index upon the Beginning of Degrees, as before, turn the Instrument about, till with the Beginning of Degrees towards K, you can, through the Sights, see the Point K, and there screw the Instrument fast.

Turn the Index about till you can, through the Sights, see the Point H, and see what Degrees the Index cuts, which suppose to be 166 : 45, place that in your Table, or Field book,

and measure the Distance I K, 6 Chains 25 Links; which also set down as you see hereafter in the Table.

Next remove your Instrument to K, and fixing the Index upon 00 Degrees, as before, direct it forward to L, turning the Instrument till you can, through the Sights, see the Corner L, and there screwing it fast, turn the Index till you can, through the Sights, see the Point I, and finding the Index cut 111 : 30, measure also the Distance K L 4 Chains 50 Links; both which put in the Table in their proper Places.

In the same Manner proceed with the Angle L, which, by Observation, suppose you find to be 122 : 45, and the Distance L M 8 Chains 00 Links, also the Angle M 139 : 15, and the Side M N 6 Chains 15 Links, and the Angle N 65 : 30, and the Side N O 6 Chains 10 Links: These found, as has before been directed in finding the foregoing Angles and Sides, place also in their proper Places in the Table.

But when you come to an Angle that bendeth into the Field, as the Angle O, it is to be reckoned upon the Degrees above 180, by first looking forward to P, as before, and then turning the Index to N, it will (if you turn it according to the Succession of Figures upon the Instrument) be turned a great deal more than half round, and will in this Example rest upon 284 Deg. 00 Min. Or if you measure the external Angle of the Field, P O N 76 : 00, and subtract it from 360 Degrees, there will remain 284 : 00, as above.

Having found the Angle O, measure the Side O P, which suppose 4 Chains 55 Links: also the Angle P 90 : 45, and the Side P H, 3 Chains 00 Links; you have been round the Field, and your Work is finished: And the Method of finding all the Angles and Sides, being the same as those already laid down in finding the Angles H, I, and K, and the Sides H I, I K, and K L, &c. any further Repetition is needless.

The Field-book to the foregoing Work.

	Deg.	Min.		Ch.	Lin.
H	99	: 30	H I	7	: 90
I	166	: 45	I K	6	: 25
K	111	: 30	K L	4	: 50
L	122	: 45	L M	8	: 00
M	139	: 15	M N	6	: 15
N	65	: 30	N O	6	: 10
O	284	: 00	O P	4	: 55
P	90	: 45	P H	3	: 00
1080 : 00			Sum of all the Angles.		

The

The Truth of the Work may be examined by *Theor. VIII.* where it is proved, *That all the Angles of any right-lined Figure, of how many Sides soever, are equal to twice as many Right-angles, abating four, as there are Sides of the Figure.*

This Figure has eight Sides, twice 8 is 16, but abating 4 there remains 12, the Number of Right-angles: So that all the Angles in the Figure added together, should be equal but to 12 Right-angles, viz. 12 Times 90 Deg. is 1080 Degrees, agreeing with the Sum of the Angles in the Table, and proving the Work to be right.

How to plot this Field by Help of the Table or Field-book.

Having adjusted a Scale to the Size of the Paper you intend to plot upon, draw a Line along one Side of your Paper, and not far from the Edge of it, and in that Line assume a Point, to represent the Place where you intend to begin, as suppose at H, and let the Line be H I; look in the Table or Field-book against H, and you find 99 : 30; therefore having drawn H I, and assumed the Point H, draw from H the Line H P at Pleasure, to make an Angle of 99 Deg. 30 Min. with H I; then look in the Right-hand Part of the Table for H I, and you find against it 7 Chains 90 Links; take 7 Chains 90 Links off the Scale, and set from H to I.

Then look in your Table for I, you find against it 166 Deg. 45 Min. therefore draw the Line I K, to make at I an Angle of 166 Deg. 45 Min. with the Line H I (the Way to do it is taught in the eighth Geometrical Problem, *Page 56.* of this Book;) and finding in the other Part of the Table that I K is 6 Chains 25 Links, set 6 : 25 from your Scale upon the Line I K, from I to K.

In the same Manner make the Angle K 111 : 30, as is found in the Table against K, and make K L 4 Chains 50 Links, as found in the other Part of the Table; and so proceed to the rest of the Points L, M, N, O, and P, laying down all the Angles by the same Method, and all the Sides L M, M N, N O, O P, and P H, from the same Scale of equal Parts; only *Note,*

When you find an Angle bending into the Field, as the Angle O 284 Degrees, it is best, after having laid down the Side N O, and found the Point O, to take the inward Angle 284 from 360, and with the Remainder 76 Degrees, make the external Angle N O P, which will also form the internal Angle O 284, and be less liable to Error.

S E C T. II.

To plot a plain Field only by Measuring one Line in the Field.

Fig. 60. **L** E T it be required to plot the Field EFGHIK, and you would plot it

By the Plain-table.

Chuse any Station toward the Middle of the Field, so as you may from thence see all the Angles of the Field, as suppose L; there place your Instrument, and turning it about till the Needle lie just over the Meridian-line, there screw it fast.

Then some where near the Middle of the Sheet of Paper that is upon the Plain-table, chuse a Point L to represent your Point of Station in the Field, and through L, parallel to the Side of the Instrument, draw the Meridian-line NS.

Lay the Index upon the Table on the Point L, and turn it about till you can, through the Sights, see the Angle E; and by the Side of the Index draw the Line LE continued at Pleasure; then measure in the Field with your Chain the Distance LE, which suppose to be 7 Chains 35 Links; take 7 : 35 from the Scale you intend to plot by, and set upon the Line LE, from L to E, and there make a Mark, rejecting the rest of the Line LE, if there is any more of it; this Line, LE, is all that you have to measure in the whole Field.

Keeping your Instrument still screwed fast, and the Index upon L, turn it about till you can, through the Sights, see the Angle F (or a Mark set up at it, if you cannot otherwise exactly discern it) and by the Side of the Index draw the Line LF at Pleasure.

Then turn the Index to G, laying one Part upon L, and turning the other about till you can, through the Sights, see the Point G, and along by the Edge of the Index draw the Line LG.

In the same Manner keeping the Index upon L, as a Center, turn it about till you can, through the Sights, see the several Angles H, I, and K, and draw the Lines LH, LI, and LK, continued at Pleasure.

Note, It is best to draw all the Lines that are drawn from L, as LE, LF, &c. with a Black-lead Pencil, that when the Out-Lines of the Field are drawn, these in-side Lines may be rubbed out with a Bit of Bread, they being of no Use after the Bounds of the Field are described.

This

This done, remove your Instrument to E (the Angle that you measured to) and bringing the Needle over the Meridian-line, which may be most exactly done by laying the Index upon the Line L E, and turning the Instrument about till you can, through the Sights, see the first Station L, and there screw it fast.

Then placing your Index upon the Mark E, turn it about till you can, through the Sights, see the Point or Angle F of the Field; and by the Edge of the Index draw the Line E F, to cut the former Line L F in F.

Remove your Instrument to F, and laying the Index upon the last drawn Line E F, turn the Instrument about till you can, through the Sights, see the last Station E (then will the Needle be over the Meridian-line) there screw the Instrument fast, and laying the Index upon F, turn it about till you can, through the Sights, see the Point G; and from F by the Side of the Index draw the Line F G, to cut L G in G.

Then remove your Instrument to G, and laying the Index upon the Line G F, turn the Instrument about till you can, through the Sights, see the Point F, and there screw it fast; and laying the Index upon G, turn it till you can, through the Sights, see the Point H, or the Angle in the Field represented by H, and from G along the Edge of the Index, draw the Line G H, till it cut the Line L H in H.

In the same Manner removing your Instrument to H, and rectifying it, by bringing the Needle over the Meridian-line, and seeing the last Station, G, by the Back-sight, as before directed, screw it fast, and laying the Index upon the Point H of the Draught, turn it till you can, through the Sights, see the Corner or Angle I of the Field, and by the Edge of the Index draw H I, to cut the former Line L I in I.

The next Angle K is an inward Angle, but the Way of laying it down by the Plain-table, is exactly the same; for having removed your Instrument to I, lay the Index again upon the Line H I, and turn the Instrument about till you can, through the Sights, see the Angle H, screw it fast, and laying the Index upon I, turn it till through the Sights you can see the Point K; and although here, being within the Field, you lose Sight of the Hedge or Wall K E, yet draw the Line I K, as before, till it cut the former Line L K in K.

Lastly, remove to K, and having fixed your Instrument by the Needle and Back-sight as before directed, and laying the Index upon the Point K of the Draught, turn it till you can, through the Sights, see the Point E of the Field, and by the Index draw the Line K E, to cut L E in E, and then you have encompassed the Field; and this Way, if exactly performed, is

as mathematically true as any other, and more expeditious; and if there be never so many Angles, and whether bending inwards or outwards, the Method for plotting them is the same: And when you have drawn all the Out-lines EF, FG, &c. with Ink, you may rub out all the Lines LF, LG, &c. only taking care before you rub them out, to adjust your Scale by the Line EL 7 Chains 35 Links, which is best done by the Sector, viz. Take in your Compasses the Line EL, and setting one Foot in 7 : 35 on the Line of Lines, on one Leg of the Sector, open the Sector till the other Foot of the Compasses will just fall in the same Member 7 : 35, on the other Leg of the Sector kept carefully at that Opening, which is a Scale for all the rest of the Lines, either real, as Hedges, &c. or imaginary, as Bases and Perpendiculars, &c. When the Field is divided into Triangles in order for Measuring, yet the Sector kept as above, is a Scale for them all.

Example. Suppose I would know the Length of the Hedge or Wall HI, set the Sector as above directed, and then take the Extent HI, in your Compasses, it will reach from 9 : 35 on one Leg of the Sector, to 9 : 35 on the other, which shews that HI is 9 Chains 35 Links; and herein lies the Excellency of Gunter's Chain, because both arithmetically and instrumentally, it works by Decimals.

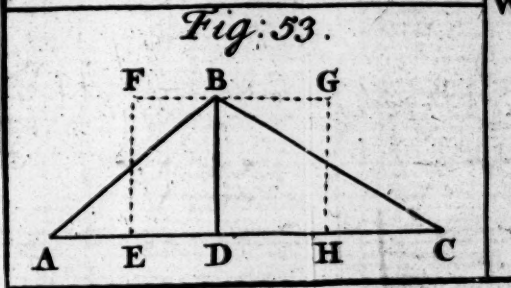
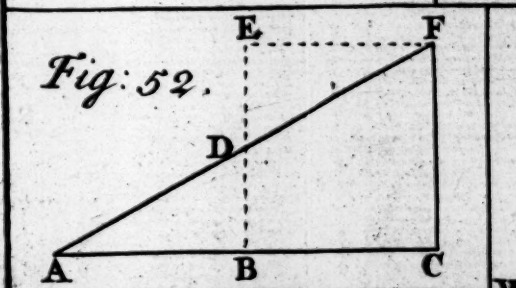
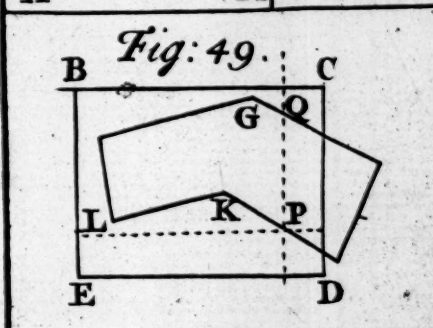
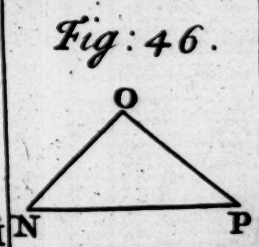
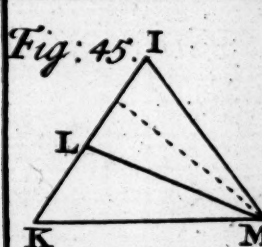
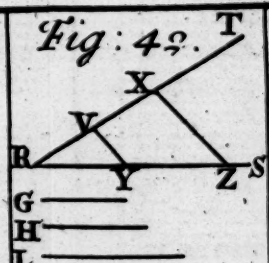
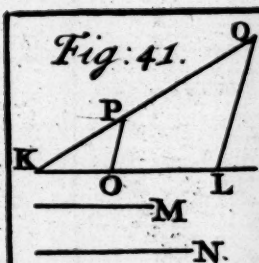
To perform the same by the THEODOLITE.

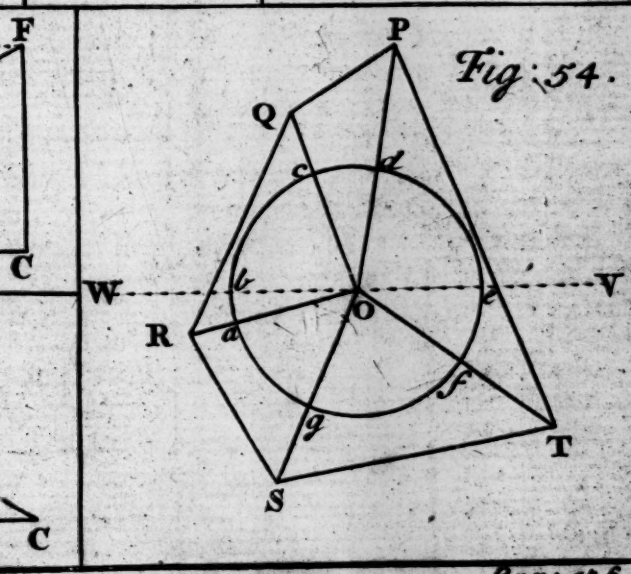
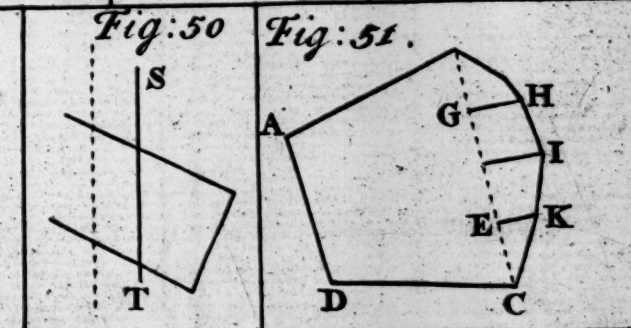
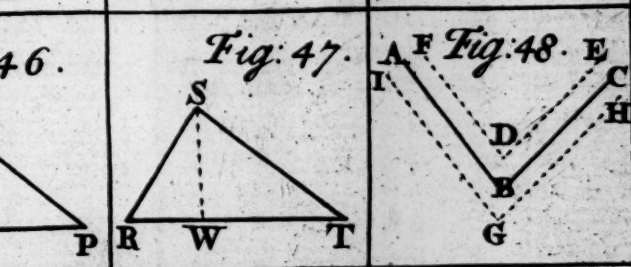
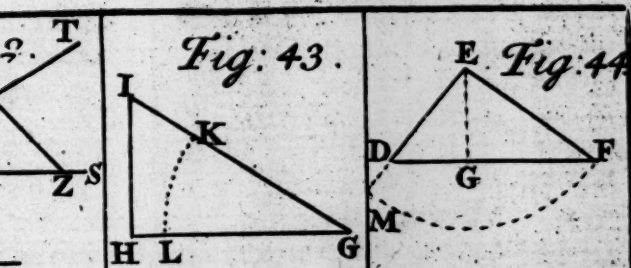
Take a Station in the Field, as L, from whence you can see all the Angles, and setting up your Theodolite there, turn it about till the Needle be just over the Meridian-line, and there screw it fast, and direct the Sights, so as that you may through them see the several Angles, which note down in a Table in your Field-book.

Example. Having fixed the Instrument as above directed, turn the Index till you can, through the Sights, see the Point E, and observe what Angle is cut by the Index, as suppose 25 Deg. 30 Min. then (keeping the Instrument still fixed) turn the Index till you can, through the Sights, see the Angle F, and observe what Degree is cut by the Index, which suppose 87 Deg. 45 Min. which also put down in the Table against its proper Letter F, and so proceed to G, H, I, and K, and place the Degrees cut by the Index, against their respective Letters, and they will appear as in the Table annexed.

Angles at the Station L	
	Deg. Min.
E	25 : 30
F	87 : 45
G	130 : 15
H	209 : 45
I	305 : 00
K	357 : 00

Having





Having thus finished the Angles that every Corner of the field makes with the Meridian at the Station L, set off from our Scale 7 Chains 35 Links, the Length found by measuring from the Station L, to the Corner E, and determine the Point E; then remove your Instrument to the first Station E, and without any Regard to the Meridian-line) place the Index on the Beginning of the Degrees on the Theodolite, and turn the whole Instrument till (having that End of the Index from your eye, that lies upon the Beginning of the Degrees) you can, through the Sights, espy the Angle F, and there screw it fast.

Then turn the Index about till you can, through the Sights, see the Station L, which suppose 39 Deg. 00 Min. put that down against E, as you see in the Table. Then proceed to F, laying the Index again upon 00 Deg. 00 Min. and then with that End of the Index pointing towards G, turn the Instrument about till you can, through the Sights, see the Point G; there fix the Instrument, and turn the Index till you can, through the Sights, see the Station L, and observe the Degree cut by the Index, which suppose to be 55 Deg. 15 Min. set that in the Table against F.

Angles at the several Corners, with the Station L.		
	Deg.	Min.
E	39	: 00
F	55	: 15
G	61	: 00
H	50	: 45
I	28	: 00
K	126	: 00

Remove your Instrument to G, and always placing the Index at 00 Deg. point it forward toward the next Station, which now is H, by turning the whole Instrument about till you can, through the Sights, see the said Point H, and there fix your Instrument fast, and turn the Index about, till you can, through the Sights, see the Station L, and mind the Degree cut by the Index, which is 61 : 00, place that in the Table against G. Proceed in the same Manner with the rest of the Angles, and put them down in the Table against their proper Letters, and they will appear as in the Table annexed; and although the Angle EKL is obtuse, yet the Method of laying down the Angles is the same as if acute: And if you use a Sector that has Chords cut to 60 Degrees, you may set an Angle that is more than 60 off at two or three Extents.

To plot this Field upon Paper by the Help of the Table above.

Having proportioned a Scale to the Size of your Sheet of Paper, or to the Bigness you intend your Draught shall be, assume a Point towards the middle of the Paper to represent your Station L, in the Field, and then draw the Meridian-line NS,

Then have recourse to your first Table, entitled *Angles at the Station L*, and observe what Angles the Station L E, L F, &c. makes with the Line NS, and so lay them down in the Draught: If you do it by the Protractor, you are taught how to perform it in the *Use of the Protractor*. If you do it by the Sector or any Line of Chords, take Radius or the Chord of 60 in your Compasses, and with one Foot in L draw the Circle *abcd*, to set off your Degrees upon.

Then look in the Table against E, you find 25 : 30, set 25 : 30 off the Chords upon the Circle from the Line NS to the Point *a*, and through *a* draw the Line LE at Pleasure; then looking in the Table for F, you find against it 87 : 45, set 87 : 45 off the Chords, from the Line NS to *b* upon the Circle, and through *b* draw the Line LF; also set 130 : 15 off the Chords upon the Circle, from the Line NS to G, and draw LG, and so proceed by setting off 209 : 45 to *c*, and draw LH : Set 305 : 00 to *d*, and through *d* draw LI; and, lastly, set 357 Deg. 00 Min. to K, and draw LK, always minding to begin to set off your Degrees at the Intersection of the Line NS with the Circle.

When this done, set off upon the Line LE, the measured Distance, LE, 7 Chains 35 Links, from L to E, and determine the Point E.

Then look in the Table for the Letter E, and against that you find 39 : 00, that is, the Angle LEF is 39 Deg. 00 Min. Having the Line EL, and Point E given, lay down the Angle LEF 39 : 00 either by the Protractor, or by *Prop. VIII. page 69*, and draw EF, till it cut LF in F.

Look in the Table against F, you find 55 : 15, that is the Angle LFG is 55 : 15; lay down that Angle as before directed, and draw FG to cut LG in G.

Then look in the Table against G, you find 61 : 00, make the Angle LGH 61 : 00, and draw GH to cut LH in H.

You find in the Table against H 50 : 45, viz. the Angle LHI is 50 : 45; make therefore the said Angle at H, and draw HI to cut LI in I.

In the same Manner lay down the Angle LIK 28 : 00, and draw IK to cut LK in K. And lastly, lay down the Angle L

K E,

E, which is found in the Table against K, to be 126.00, and draw K E, which, if your Work is right done, will exactly cut L E in E, and then you have enclosed your Field, ready to be divided into proper Figures in order for casting up,

C H A P. V.

To take the Draught of a whole Manor, Estate, or Farm, consisting of any Number of Enclosures, whether Arable, Pasture, Wood-land, Fenny Ground, &c.

WHEN you are to survey an Estate, Manor, &c. that contains a great Number of Enclosures, your first Work ought to be to take a Guide with you that knows exactly all the Limits of it, and take a true Account of that, and then plot all the Out-boundaries of it, whether Walls, Hedges, Rivers, or Roads, or whatsoever it is bounded by; as it most commonly happens, when a Manor or Estate is bounded on one Side by a River, on another Side by a Road, and on a third Side by only a Hedge between it and another Person's Ground, take care to observe not only the Boundaries of the several Sides, but with what they are bounded; representing every thing as it is, adds very much, not only to the Beauty, but also to the Truth of a Draught, and also to the Reputation of the Workman.

When you are certain of the Boundaries of your whole Work, take what Instrument you think fit to make use of, and plot all the Out-bounds, or form of the whole Manor, as has been taught in the first Section of the last Chapter, and draw all the Out-lines of it.

As you take the Out-lines of it, observe diligently what Bridges or Mills, if upon a River, or what Houses, Wind-mills, &c. you find, as it often happens, especially when bounded by a Road, or whatever other material Thing happens, take care to put it in its proper Place as you go round; for these will be great Helps towards the regulating your Survey, when you go within to lay down the inner Boundaries between Field and Field.

You should, in your first Walk round, observe where is the most convenient Place for you to begin; for though, if exactly per-

performed, the Truth is the same in all, yet there is least Danger of contracting an Error, if you begin at an Angle, where the Boundaries proceeding both ways from it, continue for a great Distance straight; and likewise to begin at an Angle from whence you can see the most considerable part of your Work.

I shall not need to enlarge upon the manner of taking the Out-boundaries, but refer the Learner to the Beginning of the last Chapter, and proceed to dividing it within — only observe——

As you are going round to take the Out-bounds, you may take Notice of the Hedges, Walls, or Ditches, that proceed from the Out-bounds inwards, and divide the Manor into smaller Enclosures, as in the following Example.

Let the Farm ABCDEFGHIK be to be surveyed and plotted, which we suppose to be bounded on the East with Lumley Brook, on the North and West with Ashton Lane, and on the South by the Grounds of Mr. Hopkins and Mr. Fisher. Fig. 61.

Note, It is very proper to represent Lanes that are public Roads, with two pricked Lines, and two Lines without them, as you see in Ashton Lane; and Rivers with two Lines shaded all between them, the Shades tending the same Way as the Banks of the River, or parallel to it: And for better distinguishing the Boundaries of a Farm, represent the Hedges or Walls that are between that and another Man's Ground, by small double Lines; and when there is Occasion to shew where the Ground that butts upon you is divided between Partners, let that be done by a small pricked Line, as in the Draught between Mr. Hopkins and Mr. Fisher; and let a Foot-path be represented by a small double pricked Line, as you see from the House to the Bridge.

Suppose the Manor or Farm-house be at R, and you there beginning your Walk round the Farm, you observe the Lane to be pretty straight to C, and coming at C, you observe it also pretty straight, both back to A, and forward to the Bridge at E; and coming to E, you turn down the Side of the Brook EFGQH, and from thence to OIKA, and so to the House at R; and having fixed upon C for a Station to begin at, proceed to take the Out-bounds, by which you walked about it, by the Rules laid down in the fourth Chapter: Only in your Passage from C to E, observe there is a Hedge butts upon the Lane at D; now, having upon your Plain-table drawn the Lane CE, before you removed from the Station C, observe in

Your Passage, measuring from C to E, how many Chains and Links it is from C to the Hedge at D, which suppose 5 Chains 30 Links, set 5 : 30 on the Line from C to D, and laying the Index upon the Plain-table on the Line C E, turn the Instrument about till you can, through the Sights, see the Point E, and there screw the Instrument fast.

Then laying the Index upon the Point D, turn it about till you can, through the Sights, see the Point S, or so far on one Side of it, as your Eye is on the same Side of the Point D; and then along the Side of the Index draw the Line D S at Pleasure, but do not stay to measure it, but proceed to measure to E; and suppose the whole Line C E, to be 20 Chains 00 Links, set that upon the Line C E, from C to E, then is the Point E the Corner of the Bridge, or Bank of the Brook.

Now the Brook going a little winding from E to G, you must either measure by the Right-line E G, and then take the Offset T P, as is taught in the former Part of this Treatise; or else measure by the River Side from E to P, and from P to G, remembering as you go along to draw the Hedge F M at Pleasure, by the same Method as you did D S, but not regarding to measure it till afterwards.

When you come at G set up the Plain-table again, and with the Needle over the Meridian-line of the Card, fix it fast, and laying the Index upon G, turn it till you can see the Point Q through the Sights, and draw G Q, which suppose 6 Chains 80 Links, which set from G to Q; and before you remove from G, turn the Index parallel to the Hedge G N, and when you can, through the Sights, see the Point N, draw, by the Edge of the Index, the Line G N at pleasure.

In the same Manner proceed to the Stations H, O, and I, observing also when you are at I, to draw the Line or Hedge I N; and then, when you come at K, and have drawn the Line K A, draw also the Line K L, before you remove from that Station, and then proceed to measure the Line K A 11 Chains 20 Links, and then remove to A with your Instrument.

From A to C may be all taken at one Observation, only as you measure from A to C, observe how much it is from A to the Hedge B L, and as you go along lay down the Hedge B L, as you have done the rest, and proceed to C, and then you have closed your Survey.

Note, You ought to be very particular in measuring the Distance from A to the Gate, the Breadth of the Gateway, the Distance from the Gate to the Barn, the Length of the Barn; the Distance from the Barn to the House; the Length of the House;

House; the Distance from the House to the little House at B, which we may call the Brew-house or Dairy, &c. For the Distance AB were truly taken, yet if the above-mentioned Particulars be not also carefully and exactly taken, it will very much impair the Truth and Beauty of the Draught.

We are next to consider what is to be done within; but, indeed, by this expeditious Way of working, the Inside Work is a great Part of it performed; for by drawing GN at pleasure, as directed, from the Station G, and drawing IN from the Station I, till it cut GN in N, the Field GQHON is laid down.

Also by drawing KL of any Length at Pleasure by the foregoing Direction, and drawing BL till it cuts KL in L, the Home-field ABLK is laid down.

Fix your Plain-table at N, and laying the Index upon the Line NI, turn the Instrument about till you can, through the Sights, see the Station I (then will the Needle be directly over the Meridian-line;) there fix your Instrument fast, and laying the Index upon N, turn it about till you can, through the Sights, see the Angle of the Field S, then by the Edges of the Index draw the Line NS, to cut the Line DS in S; then have you closed Brook-meadow, and Bridge-field, and the Side BCD, as also the Side DS of Mill-field is determined. Go into Mill-field and measure the Side BV 6 Chains 30 Links; set 6 : 30 from B to V, and draw the Line or Hedge SV, which separates Mill-field from Broad-close, and closeth them both, and then the Plot is finished.

To perform the same Work by the THEODOLITE.

After all the Examples that can possibly be given, Experience in the Art will best direct where the properest Place is to begin; but in this, after having walked round the Farm, or cross it by the Foot-path which leads from the Gate to the Bridge, it is easily perceived (before it is plotted) that Broad-close lies nearest the Middle or Heart of the Farm, therefore I think it proper to begin with that first; for by having that carefully done, you have done a Part of every Field that lies contiguous to it; and, indeed, in this Case, you have done a Part of every Field in the Farm.

Note, When I speak of beginning near the Middle, it is not meant that the Out-lines, as the Lane, the River, &c. should be drawn first, but reserved to the last, to be determined as the Work

Work proceeds outwards; which I rather chuse to insert, not only as being as true as the other, but for its Variety, that the Learner may not be altogether confined to one Method.

I need not repeat the Method of plotting a single Field, that having been sufficiently treated of before. Therefore having plotted the Close called Broad-close, IKLVSMN, chuse any Angle, as suppose the outward Angle OIN, to begin at, and see what Angle the Hedge or Bounds IO makes with the Hedge IN.

Note, You need not in this Case regard the Meridian-line, but laying the Index of the Theodolite upon the Beginning of the Degrees, turn the Instrument about till you can, from the Angle you are at, see through the Sights the Angle you intend next to go to, and observe the Degrees cut by the Index, it is the Angle required.

Note also, It is best in this Case, when you make your Field-book, to mark the Angles with three Letters, as the Angle N IO, and the Distances with two, as the Distances IO 8 Chains 00 Links, &c.

Example. I begin at the Angle I, and placing the Instrument and the Index upon the Beginning of Degrees, turn the Instrument about, till I can, through the Sights, see the Angle N, and there screw it fast, and move the Index, till I can, through the Sights, see the Point O; observing what Degree is cut by the Index, which suppose 53 Deg. 00 Min. I set that down in the first Column of the Table NIO 53 : 00, and with my Chain measure the Line IO, which we suppose 8 Chains 00 Links; therefore I set in the second Column IO 8 : 00, as you see below.

I then remove the Instrument to N, and laying the Index upon 00 Deg. 00 Min. turn the whole Instrument about, till I can, through the Sights, see the Point S, and there screw the Instrument fast: Then I turn the Index, till I can, through the Sights, see the Point G, observing what Degree the Index cuts, which suppose 120 Deg. 00 Min. And measure also the Distance NG 6 Chains 50 Links; I set the Degrees SNG 120 : 00 in the first Column of the Table, and NG 6 : 50 in the second Column, as you see in the Table.

Note, If when you are in the Angle N of Brook-meadow, you cannot see the Angle S, by reason of the Hedge or Wall MF, you may place your Instrument at N in Horse-pasture, and
set

set down in your Table the Angle ING (instead of SNG) and the Distance NG ; but where the Hedge is all straight the Angle SNG will be the same of MNG , provided you see beforehand that there is no Angle at M in the Hedge NS .

Then remove your Instrument to the Angle M of Bridge-field, and laying the Index as before, upon 00 Deg. 00 Min. turn the Instrument, till you can, through the Sights, see any Mark set up at S in Bridge-field, and there screw the Instrument fast.

Then turn the Index about, till you can, through the Sights, see the Corner F , by the Brook Side, and observe what Angle it makes; and also measure the Distance MF 13 Chains 16 Links, and set down in your Table, as before taught, SMF $94:00$ MF $13:60$.

Then remove your Instrument to the Angle S in Mill-field, and take the Angle DSV $97:00$, and the Sides DS being measured and found 6 Chains 50 Links, put them down as before taught.

The Side VS of Mill-field, is also the Side VS of Broad-clofe; and that you had before in the Survey of Broad-clofe, being 8 Chains 80 Links.

Remove to the Angle V of Mill-field, and find the Angle BVS 110 Deg. 00 Min. and the Side BV 6 Chains 30 Links; place them down as before.

Proceed in the same Manner to find the Angles LBA , also BAK and AKL of Home-field; as also the Sides BA 12 Chains 40 Links, and AK 11 Chains 20 Links (for you have the Side KL before, being the Side of Broad-clofe) and put them down as before.

When you have gone round the whole in this Manner, you will have got the Points G, F, E, D, B, A, K, I , and H for the utmost Limits of your Work.

Note, When the Boundary is not straight between the Points found, you must make the Off-sets as in Mill-field, the Boundary of it not lying in a Right-line between the Angles of it, B and D , go to the Angle B , and while you look directly at the Angle D , let your Assistant be somewhere at X , directed by you by the waving of your Hand one Way or the other, till you find him just between you and the Angle D (and there let him stand) then go to him and take the Off-set XC : Do the same in Brook-meadow, with the Off-set TP , and also in the Horse-pasture, with the Off-set ZQ , or whatever other Off-sets you find in the Work, do the same in all.

By this Work you have got not only the marks before mentioned, but all the rest, viz. H, Q, G, P, F, and E, for the Brook. Also E, D, C, B, and A, for the Lane: And A, K, I, O, and H, for the Partition Fences. By which Marks, if the proper Boundary Lines be drawn, and Care taken to delineate the Brook, with proper Shading to represent Water, and the Lane with double pricked Lines to represent a Road, and each in due proportional Breadth to the Scale you plot by, the Draught will appear as in Fig. 61. Only Note,

If a Lane or Brook be Part of the Boundary of a Farm or Estate, and you plot to so small a Scale, that a Brook of three or four Yards broad, cannot, if represented in a just Proportion, appear broader than a common Line, as would be the Case, if you plot by a Scale of 12 or 16 Chains to an Inch; it is allowable, in such Cases, to take a little Liberty to exceed your Scale, to give Room to represent the Brook, &c. so as it may, without putting a Name to it, be distinguished for what it is.

CHAP. VI.

To plot mountainous, uneven Ground.

IT is to be observed, that in plotting mountainous, uneven Ground, the horizontal Content of it is intended to be taken; or more vulgarly, there is nothing intended to be taken into the Draught, but the Plain upon which the Hills and uneven Ground stand, in Case the Hills themselves were taken away; and indeed it ought to be so for these Reasons.

First, If every Estate was to be measured and plotted by the Ascents and Descents, then an Estate that really contained just a square Mile, would by that Measure, be brought to contain perhaps two or three; and if all the Land in England and Wales were to be so measured and plotted, then, where it is most mountainous, it would be most distorted, and this Kingdom would not only seem to possess a far greater Part of the Surface of the Globe than it really does; but as it is more mountainous in some Places than others, and consequently unequally distorted, the Shape of it would be quite lost.

But another Reason is, that Corn, Grass, &c. always to grow perpendicular to the Horizon; and hence, if they grow out of

L

the

the Side of a Hill, and have their Stems when perpendicular, just as near together as if they grew upon a Plain, the Roots of those that grow below one another must be farther asunder than the Stems, in Proportion as the Hypothenuse of the Hill is more than the Plain of its Base, and consequently, no more Grass can grow upon a Mountain, than can upon the Plain upon which the Mountain stands, though we suppose the Ground equally productive; and hence comes that known and demonstrative Experiment, That the same Number of Pales of the same Breadth, and placed at the same Distance would form a Fence over a Hill of any Dimension (suppose 300 Yards about at the Bottom, and 20 Yards high) that would make a Fence over the Plain on which the Hill stands, if the Hill was taken away.

But yet this is better illustrated in City or Town Surveying, where, if you measure the Length of a Street that lies with a great Descent, you will find it will measure more than the Length of the first Floor of all the Houses in one Side of the Street added together, and that in Proportion to the Steepness of the Street, for the Floors being horizontal, their Sum will bear the same Proportion to the Length of the Street, as the Base of a Triangle bears to the Hypothenuse, when the Angle of the Base is equal to the Angle that the Hill makes with the Horizon.

I therefore think it is proper, first to shew,

S E C T. I.

How to find the Horizontal Line or Plain, upon which a Mountain stands.

LET the Mountain be BAD, supposed to stand upon the Horizontal Line or Plain BCD, and its perpendicular Altitude AC, and the whole Horizontal Line or Plain BD is required.

Fig. 62.

There are several Sorts of Instruments for taking Altitude, which we need not enlarge upon the Use of; but that being known, bring the Instrument to the Bottom of the Mountain or Hill at B, and let your Assistant go to the Top of the Hill at A, and place a Mark that may be seen from B, and elevate your Instrument so that you can, through the Sights, see the Mark at A, and observe the Degree of Altitude, which suppose 37 Deg. 30 Min.

Note,

Note, Your Mark, whether white Paper, Handkerchief, &c. must be just as far above the Top of the Hill, as your Instrument you observe with is above the Ground, or else you will commit an Error in the Observation.

Then measure the Side of the Hill BA as near as possible in the pricked Right-line BA, which suppose 5 Chains 00 Links.

Note, A Wheel is a very improper Instrument to Survey mountainous Ground with, because it passes into the Bottom of all the irregular Cavities, though but of two or three Yards broad, whereas a Chain may be stretched from Side to Side of such, and prevent the Error, or make it less; the Intent, in this Case, being to measure with the strait pricked Line BA (Fig. 52.)

The Angle ABC, found as above, being 37 Deg. 30 Min. the Angle BAC is 52 Deg. 30 Min. (by Theor. V. the Angle BCA being a Right-angle :) Therefore to find the Base BC, by Case the first of Right-angled Plain Triangles,

As Radius	- - - - -	10.00000
To Hypothense BA 5 Chains, or 330 Feet,	- - - - -	2.51851
So Sine of BAC—52 : 30	- - - - -	9.89946
To BC, 262 Feet	- - - - -	2.41797

Then proceed to the other Side of the Mountain or Hill at D, and with your Instrument take the Angle CDA, as before, which suppose 56 Deg. 00 Min. and let the Side DA, when measured, be 270 Feet, from thence find the Base CD, as before.

The Angle D being 56 : 00, the Angle DAC will be 34 : 00 to find CD.

As Rad.	- - - - -	10.00000
To AC—270 Feet	- - - - -	2.43136
So Sine of DAC 34 : 00	- - - - -	9.74756
To CD—151 Feet	- - - - -	2.17892
The Line BC	- - - - -	262 Feet.
Added to CD	- - - - -	151 Feet.
The Sum is the whole Line BD	- - - - -	413 Feet.

This 413 Feet is the whole horizontal Line of the Mountain; whereas, to have gone up from B to A 330 Feet, and down

down from A to D 170 Feet, the Sum 600 Feet, which is more than the horizontal Length that the Mountain possesses, by 187 Feet.

But suppose this to be a Mountain that you can go round about, and let E F G H (*Fig. 63.*) be the horizontal Plain of it; and to make it yet more intelligible, let G and D be the North Point of it, B and E the South Point, let F be the West, and H the East Points of the Plain or Skirt of the Mountain, and let the Point A of the Mountain, when seen at a Distance, which is the Top, be represented by the Point O, or Top of the Mountain in *Fig. 63.* And then, when we observed from B to A in *Fig. 62.* we observe from E to O, in *Fig. 63.* where the Mountain appears orthographically, and from D to A, is from G to O; and when we have found B D, we have found E G.

But if the horizontal Plain of the Mountain be narrower from West to East, as F H, than from North to South, as E G, go to the West Side F, and also to the East Side H, and take the Angles of Altitude and Length of the Slop-sides as you did before, and then you will have found the Line F H, as well as E G, which is B D; and by the Help of the four Points E F G H, it is easy to plot the horizontal Plain of the Mountain, which is all that ought to be plotted in a Survey, only some Shading to shew that it is mountainous, or Trees amongst the Shades, to shew if it is mountainous and woody.

S E C T. II.

How to plot Mountains, hilly or uneven Ground.

WE need not enlarge much upon this, because we have in *Chap. IV.* shewed how to plot a Field by going round it; but that being supposed to be a plain Field, where there are no Ascents or Descents from Angle to Angle, what we have now to do is to shew, how to do it when there are Hills or Valleys between the Angles of Station.

Suppose then the Field to be O P Q R S T V, which suppose to be so woody and mountainous that you cannot plot it by the common Rules of Surveying before laid down, there is nothing to do but follow the Directions of *Chap. IV.* except in Case of Hills

Hills or Mountains intercepting the Sight between the two Stations, which, to perform the Work true, is thus to be accounted for.

Suppose the first Station be V , and you would from thence proceed according to the Directions of *Chap. IV.* to make your Observation to the next Angle or Station O , but when you are at V , it is all rising Ground to w , and then a Descent to O ; so that you cannot from V see the Point O . In this Case take first the Ascent Vw , as you were directed *Sect. I.* to take the Ascent BA , *Fig. 62*, and then go to O , and looking back to w , take the Ascent Ow , as you were directed also *Sect. I.* of this Chapter, and measuring the Distance Vw , and Ow , you will find the true Horizontal Distance VO , as taught in the said *Sect. I.* which is the true Side of the Field VO , to be plotted as in *Chap. IV.*

Then remove your Instrument to O , and suppose from thence you see the Angle P , but that between O and P is a deep Valley, with a great Descent to the Bottom x , where the Brook comes out that runs through the Ground, as x , and an Ascent from the Brook at x to P .

If a Line could be stretched exactly straight from O to P , as the Line OxP , without regarding the Ascent or Descent, it would be the horizontal Line OP required; but that being impracticable, you must from O , take with your Instrument, the Angle of Depression xOw , and measure the Descent Ox , to find Ow also. From x take the Angle of Altitude above the Horizon from x to P , which is equal to the Angle of Depression at P (by *Theor. III.*) and measure the Side of the Ascent xP , then have you given the Angles O and P , and the Sides Ox and xP , to find the Sides Ow and wP , which added together, is the true horizontal Line OP , required, and is to be taken from the Side OP of the Field to be plotted, to keep its true horizontal Measure.

Next, suppose the Sides PQ and QR , are upon the high plain Ground beyond the Brook, and are to be laid down by the Directions of *Chap. IV.* and in descending from the Angle R to the Brook at r , and ascending from thence to the Angle S , the same Rules are to be observed as to going from O to P ; and for the other Sides ST and TV , all the Variety that can be, being either Hill, Valley or Plain, the Plotting of them has been fully explained. As to the two former, *viz.* Hills, and Valleys, in this Section, and where there is no such Difficulty as either occurs, but all plain Ground, the Directions of the fourth Chapter are very full, so that I need not repeat Examples.

C H A P. VII.

To plot a Field, though, by Reason of a River, or some other Obstacle, you cannot come into it nor near it, provided you can but see all the Angles Treas, Houses, and most material Things therein.

IN this Case you must be obliged to have two Stations, some considerable Distance asunder, from both which you can see all the Angles, or what other remarkable Things in the Field you would represent in your Map; and then it is to be performed according to the following Example.

Let the Field be ABCDE, and the Brook or River *rs*,
Fig. 66. by Reason of which you cannot come at the Field.

In this Case chuse two Stations on the Side you are on, as H and O, and measure the Distance H O, which suppose 8 Chains 40 Links; and then——

S E C T. I.

To plot this Field by the Plain-Table.

HAVING fixed your Sheet upon the Plain-table (and indeed by the Way privately considered, which is the longest dimension of the whole Work, as by the Estimation of the bare Eye) it is easily perceived, that the nearest Distance from the remote Angle B, to the Line H O, is a greater Space than the Length of the Line H O; therefore draw a Line parallel to one End of the Plain-table (not to one Side of it, which is the longest Way of it) to represent the Line H O, and in that Line, towards one End of it, assume a Point to represent the Station O, and make the Mark O.

Set from thence the Distance H O, 8 Chains 40 Links, from O to H, and make the Mark H.

Then placing your Instrument at O, lay the Index upon the Line H O upon the Table, and turn the whole Instrument about till you can, through the Sights, see the Station H, which

(if

(if there is no material Mark at it) must be distinguished by a white Paper or Handkerchief put upon a Stick, or held up by an Assistant upon the Spot; and when you have turned the Instrument about, till you can see the said Mark at H, screw the Instrument fast.

Then keeping one Part of the Index upon the Point O, turn the Index about, till you can, through the Sights, see the Corner or Angle of the Field at A, and by the Edge of the Index draw the Line OA, quite through the Paper, or as far as you please.

Then keeping your Instrument fast, turn the Index, keeping one Part upon O, till you can, through the Sights, see the Angle of the Field at B, and by the Side of the Index draw OB, continued at Pleasure.

Turn the Index further (keeping one Part of it still upon O) till you can, through the Sights, see the Angle of the Field at C, and by the Edge of the Index draw the Line OC.

Again, turn the Index back, till you can, from O, through the Sights, see the Angle D, and by the Hedge of the Index draw the Line OD at Pleasure.

N. B. You need not draw the Line OD or OE, so long as the three former, because you see the Distances are not so great.

Then turn the Index further back, till keeping one Part upon O, you can, through the Sights, see the Angle E, and by the Edge of the Index, draw the Line OE continued, and then you have done at the Station O.

This done, remove your Instrument to the Station H, and placing the same Part of the Instrument toward the Field that was so before, lay the Index upon the Line OH, and turn the Instrument about, till you can, through the Sights of the Index, see the former Station O, and there screw the Plain-table fast.

Then laying one Part of the Index upon H, turn the Index, about, till you can, through the Sights, see the Point or Angle of the Field at A, and by the Edge of the Index, draw the Line HA, to cut OA in A.

Again, upon the Point H, turn the Index about, till you can, through the Sights, see the Point B, and draw HB, till it cut OB in B.

Turn the Index upon the Point H, till you can, through the Sights, see the Point C, and draw HC, to cut OC in C.

Then turn the Index yet forward, till you can, through the Sights, see the Point D, and by the Edge of it draw the Line HD, to cut OD in D.

Lastly, turn the Index back upon the Point H, till you can, through the Sights, see the Angle E, and by the Edge of the Index draw the Line H E, to cut O E in E; and your observations are finished.

Then draw Lines between each collateral Intersection of the Lines drawn from H to O one with the other, as the Lines A B, B C, C D, D E, and E A, these shall exactly represent the Field A B C D E, required.

Then to lay down the Brook:

Set up Marks upon the Edge of the Brook, as at r , t , s , and u ; and, as before directed, in the Angles of the Field A B, &c. draw Lines H r , H t , H s , and H u ; also the Lines O r , O t , O s , and O u , to cross the former in r , t , s , and u ; through which Marks draw one Side of the Brook, and if it is narrow, the other Side may be done sufficiently true by Judgment: If it is a broad River, you may yet get the Breadth of it in as many Places as you please, and through Points so found, draw the Banks on each Side: But of this I shall treat further towards the latter End of this Book, under the Head of *Inaccessible Altitudes and Distances*.

Note, In the Practice of thus plotting a Field that is inaccessible with the Brook, which intercepts your coming at it, it is best to draw all the Lines in Black-lead, for they are of no Use but to find the material Lines, and therefore may be rubbed out as soon as done with; such are all that I have made prickable Lines, H A, H B, H r , H s , &c. also O A, O B, O r , O s , &c. And likewise the Stationary Line H O, and the Points H and O, that nothing may appear but the true Form of the Field and Brook in their just Proportion, which is all that is intended in any Plot or Draught.

Note also, If you would lay down the Field in its true Proportion with respect to the Compass, you must bring the Needle over the Meridian-line, as taught, Sect. III. Page 110, and in other Places in this Book: But if it be a Part of an Estate that you have already surveyed the Remainder of, and thus separated by a Brook, &c. take your Station as commodiously as possible in that Ground where you are and have surveyed, and be sure to measure your Stationary Distance by the same Scale you plotted all the rest of the Estate by; and if it extend out of the Sheet, shift your Paper, or put another Sheet, as you are taught, Page 86. And by this Work carefully performed, the inaccessible Field will be plotted as exactly, in Proportion to the rest of the Estate, as if you had been upon the Spot.

If there is any House, Tree, Wind-mill, or any Thing remarkable, worth expressing, as the Wind-mill by the Hedge A B, it is also plotted by the Intersection of the Lines drawn from H and O, as all the Points A B, &c. are; and therefore the Directions need not be repeated.

S E C T. II.

To perform the Work of the last Section by the Theodolite.

ALthough I have distinguished this in a proper Section, yet the Work of it has such a Resemblance with *Plotting a Field from two Stations by the Theodolite*, that I suppose a Repetition of the Directions needless, they being very plain in *Page 126, 127, and 128*, to which I refer the Reader. Fig. 66.

C H A P. VIII.

To find the Content of any Plot or Draught in Acres, Roods, and Perches.

I Have already observed, at the Beginning of this fourth Part, *Page 102*, That, *all Superficies that are to be measured, are either Squares, or Right-angled Parallelograms, or supposed to be reduced to such*, and then given Directions in the first Chapter of this said Part of this Book, how all Right-lined Triangles, also a Rhombus, Rhomboides, Trapeziums, Polygons regular and irregular, and even Circles themselves, may be reduced as above, and measured, with the Demonstration of the Truth of the Method of Reducing, or Equality of the Areas of the Superficies to be reduced, and that to which it is supposed to be afterwards reduced, for Conveniency of finding the Area: As between a Right-angled Triangle and a Parallelogram, one of whose Sides is equal to a Perpendicular, and the other to half the Base of the said Triangle, &c. See *Chap. I. Pag. 103*.

I have also introduced the Treatise with a brief but plain and full Compendium of Arithmetic, not only Vulgar, but Decimal;

mal, for facilitating the Use of Gunter's Chain, as well as several other Instruments which are decimally divided: so that what I have to do now in order to find the Content of any Plot or Draught in Acres, Roods and Perches, is, to apply the above-mentioned Directions to Practice; and, as Decimals are most expeditious, and Gunter's Chain is divided decimally, I shall instance chiefly in that.

Question 1. Suppose a square Field to be measured, whose Length and also its Breadth is 8 Chains 44 Links, How many Acres, Roods, and Perches is in that Field?

Here 8 Chains 44 Links is in Decimals 8.44, which multiplied by itself, according the Directions given in Multiplication of Decimals, the Product is 71.2336, as in the Operation annexed, viz. 71 square Chains and .2336 of a Chain; and as 10 square Chains is an Acre, it will be 7 Acres 1 Square Chain, and $\frac{2336}{10000}$ of a Chain or square Links.

But to reduce this to the common received Measure of Acres, Roods, and Perches, the following Rule is universal, viz.

Place the Product as if it was one whole Number, viz. 712336; from whence cut off five Figures to the Right-hand, and the Remainder, which here is but one Figure, viz. 7, is Acres, and is here 7 Acres. Then multiply the Figures so cut off by 4 (the Roods in an Acre.) And again, cutting off five Figures from the Right-hand of the Product, the Figure remaining is Roods. And lastly, multiply the five Figures last cut off, by 40 (the Perches in a Rood) and from that Product cut off again five Figures towards the Right-hand, the remaining Figure or Figures is Perches, and the Figures cut off are Ten Thousandth Parts of a Perch.

Acres 7|12336

Roods 0|43944

Perches 19|73760

The Work by this Method, will stand as in the Margin, and proves the Field to contain 7 Acres 0 Roods and 19 Perches, and .73760, viz. $\frac{73760}{100000}$ of a Perch, viz. about three Quarters of a Perch.

Note, When the first Product exceeds not five Figures, the Content is less than an Acre; and when the second exceeds not five Figures (as in the Example above) there are no odd Roods; and when it happens so in the third, there are no odd Perches.

Question

Question II. Fig. 2.

Suppose the Parallelogram E F G H be a Field, whose Length is 11 Chains 56 Links, and the Breadth 11 Chains 34 Links, the whole Work, to find the Content in Acres, Roods, and Perches, will stand thus.

1856
1134
7424
5568
1856
1856

Working according to these Directions, the Field is found to contain 21 Acres, 0 Roods, and 7 Perches.

Acres 21 | 04704
Roods 0 | 18816
Perches 7 | 52640

Question III. Fig. 53.

Suppose the Triangle A B C is to be measured, and the Base A C be found to be 16 Chains 58 Links, and the Perpendicular B D 3 Chains 82 Links.

829
382
1658
6632
2487

Multiply half the Base by the whole Perpendicular (which reduces it to the Parallelogram E F G H, as in Page 127.) and the whole Work will stand as in the Margin.

Acres 3 | 16678
Roods 0 | 66712
Perches 26 | 68480

Note, Half the Base is 8 Chains 29 Links, which may be set in one Sum, as if it was a whole Number [829] and proceeding as above, the Content of the Triangle will be found 3 Acres, 0 Roods, 26 Perches.

A Trapezium, or irregular Square, is to be divided into two Triangles, by a Diagonal drawn from any Angle to its opposite, as in Fig. 23. The Trapezium A B C D is divided into two Triangles by the Diagonal A C, which is a Base common to both Triangles, in each of which, if Perpendiculars be let fall from the Angles B and D, to the Base A C, by Prop. II. the Triangles may be measured, as in Question III. above, and their Contents added together, is the Content of the whole Trapezium.

How

How to Measure an irregular Polygon of any Number of Sides.

R U L E.

An irregular Polygon is first to be divided into Triangles, and each Triangle to be measured as in *Question III.* and the Sum of all their Areas added together, is the Content of the whole Polygon.

Example. Question IV. Fig. 67.

Let the Polygon FGHK be to be measured.

This Polygon may be divided into three Triangles, by the Note in *Page 128.* which, for Instance, we will distinguish by the Numbers I. II. and III. and suppose their Dimensions of Base and Perpendicular as below, in Chains and Links.

	Ch.	Links.		Ch.	Lin.
Triangle I. Base K G	10	: 42	Perpend. F m	6	: 48
Triangle II. Base K G	10	: 42	Perpend. I n	4	: 50
Triangle III. Base G I	9	: 38	Perpend. H o	4	: 12

Multiply half of each Base by its respective Perpendicular, and set the Products aside one under another; and when you have worked all the three Triangles, add the Products together, and find the Content of them all together, as in *Question I. Question II.* and *Question III.* and that shall be the Content of the whole Polygon.

Example.

Triangle I.	Triangle II.	Triangle III.
Half Base 521	Half Base 521	Half Base 469
Perpend. 648	Perpend. 450	Perpend. 412
4168	26050	938
2084	2084	469
3126	Product 234450	1876
Product 337608		Product 193228
	Sum of the Products	765286
Product of Triangle I. 337608		Acres 7165286
of Triangle II. 234450		4
of Triangle III. 193228		Roods 2161144
Sum of the Products 765286		40
		Perches 24145760
The Content of the whole Polygon is 7 Acres, 2 Roods, 24 Perches.		

Note,

Note, When you have one Base common to two Triangles, as it often happens, as here the Line K G is the Base to both Triangles K F G; in this Case you may add the Perpendiculars of the said two Triangles together, as F m 648, and I n 450, their Sum 1198 multiplied by half the Base K G, viz. 521, the Product, 572058 is the Sum of the Products of the Triangle I. and II. to which add the Product of the Triangle III. it will make the Sum of the Products 765286, as before; and this may serve either for Expedition, or for Variety, as a Proof of the Work.

There is another Method of finding the Content of a Field when measured, and that is performed by a Table ready calculated; which, with its Use, is as follows:

Having found the Product of your Multiplication, or the Sum of the Products, if more than one, cut off five Figures towards the Right-hand, as before directed, and those remaining towards the Left-hand are Acres, which set down: We will instance in the last Example, Question IV.

The Sum of the Products, is 765286; from whence cut off five Figures towards the Right-hand, there remains a 7, viz. Acres, which set down thus.

Acres. Roods. Perches.

7 00 00

0 02 16

0 00 08

7 02 24

Links.	R.	P.
100000	4	00
90000	3	24
80000	3	08
70000	2	32
60000	2	16
50000	2	00
40000	1	24
30000	1	08
20000	0	32
10000	0	16
8750	0	14
8125	0	13
7500	0	12
6875	0	11
6250	0	10
5625	0	09
5000	0	08
4375	0	07
3750	0	06
3125	0	05
2500	0	04
1875	0	03
1250	0	02
625	0	01

Then because the Figures cut off are 65286, look in the Table for the next less, viz. 60000, and against that you find 4 Rood 16 Perches, which set in its proper Place under the other, as you see above.

Lastly,

Lastly, Look for the Figures yet remaining, viz. 5286; but that not being to be found in the Table, look for the next less, which you find is 5000, against which stands 8 Perches, set that under the other in the Place of Perches, as you see above, and add them together, the Sum 7 Acres 2 Roods 24 Perches is the Content of the whole Field; that 286 that remains being only square Links of 625 to a Perch, and therefore not quite half a Perch.

It is best, if you cast up by this Table, to have it fairly copied and pasted in your Pocket-book, or Field-book, or the like, that you may have recourse to it upon all Occasions, without the Trouble of carrying always your Books of Surveying into the Field with you.

All irregular Polygons, of how many Sides soever, are to be divided into Triangles, and measured as in this Example of *Question IV.* And, as I have already shewed in *Chap. I. Page 103*, how to reduce any Superficies into a Form or Forms, capable of being measured, I conclude further Examples will be needless.

CHAP. IX.

To divided Land in any Proportion required.

THE dividing of Land when surveyed, is one great and necessary Article in Practical Surveying, of which I shall give the Reader all the useful Varieties in the following Problems.

PROB. I.

How to divide a Field that is in Form of a Triangle into two Parts in any Proportion, by a Line drawn from any Angle thereof.

Suppose the Field ABC contain 11 Acres, and at the Angle B there is a Pond, and I am to divide this Field between two Tenants, by a Ditch drawn from the Pond at B, and that each may have the Benefit of the Pond; he that has that Part towards the Angle A, to have 4 Acres and the other to have 7 Acres.

Divide the Line AC at D, in such Proportion, that AD may be 4 Parts, and DC 7 Parts; and from D draw the Line BD, it shall represent the Ditch that divides the Field, as required.

For imagine EF be drawn parallel to AC, and thro' the Point B, then will the Triangles ABD, and DBC, be between

tween the same Parallels, and therefore proportional to their Bases AD and DC (*Theor. XIV.*) But the Base BD is to the whole BC as 4 to 11; therefore the Triangle ABD is to the whole Triangle ABC as 4 to 11, but the whole Triangle ABC is 11 Acres; therefore ABD is 4 Acres, and DBC 7 Acres.

The best Way to perform the above-mentioned Work in the Field upon the Spot, is to measure with your Chain the Length of the Hedge or Ditch AC, the Side of the Field opposite to the Angle where the Pond is, and having found its Length in Links, say,

As 11 (the Parts into which the Ditch is to be divided) to 4 (the Number of Parts in the lesser Segment AD) so the Length of the whole Ditch in Links to the Length of the lesser Segment AD in Links.

Example. Suppose the Length of the Ditch be 16 Chains 17 Links, or 1617 Links; then,

As 11 to 4, so 1617 to 588 or 5 Chains 88 Links; set 5 Chains 88 Links from the Corner of the Field at A to D, and draw B to D, it is the Place where the Ditch is to be made.

$$\begin{array}{r} 1617 \\ 4 \\ \hline 11)6468(588 \\ 96. \\ 88 \end{array}$$

PROB II.

How to divide a Triangular Field in two Parts in any given Proportion, by a Line drawn from any given Point in any Side of it. Fig. 69.

As suppose the Field EFG contain 7 Acres, and there is a Pond in the Hedge or Ditch FG at the Point I, and I am to draw a Hedge from the Point I, to cut the Side EF in K, so that the Triangle IKF may be two Acres, and the Trapezium IKFG may be 5 Acres, one Tenant having the 5 Acres, and the other the 2, but both to have the Benefit of the Pond at I.

Geometrically.

From the given Point I draw the Line IE to the opposite Angle E; then divide the Ditch FG into seven Parts (the Number of Acres in the whole Field :) and from F to H set two Parts (because there is two Acres to be set off towards F) and from H draw HK parallel to IE; and then a Line drawn from K to I, is the Line or Ditch that divides the Field from the given Point I, in the Proportion as 2 to 5, as was required.

Arithmetically.

Divide the whole Ditch FG into 7 equal Parts (the Number of Acres in the Field) and set off two of the same Parts, from

from F to H: As suppose F G be 11 Chains 76 Links, or 1176; that divided by 7, the Quotient is 168, which multiplied by 2 (because we set off two Parts of the seven) is 336, or 3 Chains 36 Links, which measure off from F to H, and measure the Distance from F to I, which suppose to be 7 Chains 00 Links, or 700 Links. Also measure the Distance from F to E, which suppose to be 7 Chains 85 Links, or 785; and then say by the Rule of Three,

As F I 700 to F E 785, so F H 336 to F K $376\frac{460}{1000}$

This $376\frac{460}{1000}$ (the Fraction being the Fraction of a Link, and not to be regarded) only taking one Link, because it is more than half a Link, is 3 Chains 77 Links, which set from F to K; the Line K I being drawn, is the Partition Line required.

This Operation is grounded upon Theor. XV. for by making as F I to F E, so F H to F K, the Lines E I and K H will be parallel, according to the Geometrical Construction.

$$\begin{array}{r}
 785 \\
 336 \\
 \hline
 4710 \\
 2355 \\
 2355 \\
 \hline
 71002637160(376 \\
 53. \\
 47 \\
 5
 \end{array}$$

PROB. III.

A Gentleman has in his Estate a Triangular Field, ABC, containing 3 Acres, and would build a House at or near the Angle A, and would have a Visto or Walk behind his House to contain just one Acre, to be divided from the Field by the Line DE, drawn parallel to A C. Fig. 70.

SOLUTION.

There being two Acres to be left in the Triangle DBE, and but one Acre cut off in the Trapezium DEAC, divide the Line B A in the same Proportion, viz. in three equal Parts (the Number of Acres) of which set two Parts from B to g, and one from g to A: then find a Mean Proportional between Bg and BA, it will be the Line BD: Draw DE parallel to A C, it is the Line or Hedge required.

Note, The Method of finding a Mean Proportion between any two Numbers, shall be in the next Problem.

Note also, When we speak of dividing a Hedge, Wall, &c. in any Proportion, it is best to measure it first with your Chain or Pole, and computing how many Chains, Links, &c. is contained in the Part set of, it is readily done at the next Time of Measuring.

PROB.

PROB. IV.

How to find a geometrical Mean Proportion between two given Lines; as suppose GH and HI.

Fig. 71.

SOLUTION.

Join GH and HI in one Line, as the Line GAHI, and find the Middle of the Line at A, upon which as a Center, draw the Semicircle GKI, then from H, raise the Perpendicular HK, to cut the Semicircle GKI in K, the Line HK shall be a Geometrical Mean Proportion between GH and HI; and it will be

As GH to HK, so HK to HI: Or,
As IH to HK, so HK to HG;

For, drawing the Lines GK, and KI, the Angle GKI is a Right-angle by *Theor.* 10. and the Line KH is perpendicular to GI by Construction; therefore,

As GH to HK, so HK to HI, by *Prob.* 5.

PROB. V.

A Triangular Field, RST, is to be divided equally amongst four Tenants, but in the Side RT there is a Pond at P, which every Tenant is to have an equal Right to, and Benefit of; the Question is, how to set out each Man's Share

Fig. 72.

SOLUTION.

From the Point P, draw the Line PS to the Angle S, and then divide the Side RT of the Triangle into four equal Parts (the number of Tenants) in the Points *a*, *b*, and *c*; from each of which draw Lines parallel to PS, till they cut the Sides RS or ST, and from these Intersections to the Point P draw the Lines Pd, Pe, and Pf, then shall the four Parts RPd, dPe, ePf, and fPT be the Parts required, yet all participating of the Pond at P.

PROB. VI.

To reduce a given Trapezium to a Triangle, by a Line drawn from any Angle of the Trapezium.

SOLUTION.

Let the Trapezium be ABCD, continue the Side BA at pleasure towards E, and draw the Diagonal CA, and parallel to that draw DE from D to E, to cut BA continued in E; and to the Intersection E, draw CE; then is the Triangle EBC equal to the Trapezium ABCD.

DEMONSTRATION.

For as DE is parallel to CA, the Triangles DAE and DCE are equal: Also CDA, and CEA are equal. Therefore, as in the latter Triangle, CDA is equal to CEA (*Theor. 13.*) Take away CFA, common to both, there remains FDC equal to FAE; therefore the Trapezium AFCB added to the Triangle DFC, is equal to the said Trapezium added to FAE (*Euclid. Lib. I. Ax. 2.*) and therefore the whole Trapezium ABCD is equal to the Triangle EBC.

Note, This is of great Use for Expedition in casting up Work; for, whereas, according to the common Operation, the Trapezium must have been divided into two Triangles by a Diagonal, and required two Operations, it is now reduced to one Triangle (EBC) and may be as easily performed at one Operation.

PROB. VII.

To reduce a Trapezium to a Triangle, by a Line drawn from any Point in any of the Sides of it.

SOLUTION.

Let the Trapezium be ABCD, to be reduced to a Triangle, by a Line drawn from the Point E, in the Side DC.

Continue the Side AB (the Side opposite to that in which the given Point is) both Ways at Pleasure to F and G, draw the Line EB from the given Point E to the Angle B; and parallel to that draw the Line CG from the Angle C, to cut AB continued in G; and from that Intersection draw the Line EG.

Also from the given Point E, draw EA to the Angle A, and parallel to that from D, draw DF, to cut BA continued, in F, and

and draw EF; then is FEG the Triangle required, and is equal to the Trapezium ABCD.

The Demonstration is the same as in the last Problem, EB being parallel to CG, and EA to DF.

PROB. VIII.

To reduce an irregular Pentagon, or five-sided Figure, to a Triangle.

SOLUTION.

Let the Figure or Polygon be ABCDE, to be reduced to the Triangle FCG. Fig. 75.

Continue one of the Sides AE at pleasure, both Ways to F and G, and from the Angle C (opposite to the Side continued) draw the Line CA, and CE to the remote Angles A and E; then parallel to CA draw BF from the Angle B, till it cut the Line EA, continued in F, and draw CF.

Draw also DG from D, parallel to CE, to cut AE continued in G; and draw GC; then is FCG, the Triangle required, equal to the Figure ABCDE.

Demonstrated in Prob. 6. BF being parallel to AC, and DG to CE.

I might instance also in reducing an irregular Polygon of 6, 7, or 8, or more Sides, into a Triangle, but it requires so many Lines, that it confuses the Figure more than dividing it into its proper Number of Triangles, as before taught.

PROB. IX.

A Gentleman has a Field in the Form of the Trapezium EFGH, containing 5 Acres, and has let it out to two Persons, so that one is to have 2 Acres, the other 3; but there being but one Gate out of it into the Road, viz. at the Angle E; the Hedge must be drawn from the said Angle E, cross the Field, to give each his Share, and let both have the Benefit of the Gate, the two Acres lying next the Side EH.

SOLUTION.

Continue the Side HG to I, and reduce the Trapezium EFGH, to the Triangle HEI: by Prob. 6. by a Line drawn from the Angle E; then divide the Base, HI, of the Triangle into 5 Parts (the Number of Acres) Fig. 76. and set two of the said Parts from H to B, because the two Acres are to lie next the Side EH; and from E draw the Line EB, then is the Triangle EBH two Acres, and the Trapezium EBFG three Acres.

For the Base HI being divided in B, it is as HB to BI, so HEB to BEI: But HEB and BEI are equal to HEB, and BEFG; therefore BEFG is equal to BEI, equal to three Acres; and HEB is two Acres, as was required.

PROB. X.

Let the aforesaid Field of five Acres be to be divided, as in *Prob. 9.* between two Tenants, one to have two Acres, and the other three, and let the Gate to which they are equally to have free Passage, be in the Side or Hedge EF, viz. at the Point K, and the three Acres to lie towards the Side EH.

SOLUTION.

Continue the Side HG (the Side opposite to the Side in which the Gate or Point K is) at pleasure, both Ways to M, and L, and reduce the Trapezium EFGH to the Triangle MKL, by *Prob. 7.* then divide the Base ML, into five equal Parts, and set three of the said Parts from M to N, and draw the Line KN, it shall divide the Trapezium EFGH into two Trapeziums, viz. EKNH three Acres, and KFGN two Acres from the Point K, as was required.

PROB. XI.

A Gentleman hath a Field in Form of a Trapezium, ABCD, containing 7 Acres, but having his House in the Angle A, he has a Mind to have a Walk or Vista before his House, to contain just one Acre, and to be all the Length of the Side of the Field AD, and to be separated from the rest of the Field by a Hedge parallel to AD.

SOLUTION.

The Partition-line being to be parallel to AD, it will cut the Lines AB and CD, continue the Sides AB and CD (the Sides which the Partition-line was to cut) till they intersect each other in G; and then, by *Prob. 6.* reduce the Trapezium, ABCD to the Triangle FDA, and divide the Base FA of the said Triangle into seven equal Parts (the Number of Acres in the Field) and set off one of the said Parts (because you are to take off one Acre) from A to H; then find a Mean Proportional between GH and GA, viz. between 6 and 7, which will be 6.483. Set 6.483 from G to I, and draw IK parallel to AD, it shall be the Partition-line, and ADKI, the Walk or Vista required.

PROB.

PROB. XII.

An irregular Field containing 11 Acres, is to be divided between two Persons, one to have seven Acres, the other four, and the Partition-line is to proceed from one of the Angles: As suppose the Field be ABCDE, and at the Angle C there is a Pond, Gate, Stack-yard, or some other Conveniency, to which they are equally to have free Access, and the Partition-fence is to be drawn from thence, and to lay the seven Acres towards A, and the four Acres towards E.

SOLUTION.

Continue the Side AE (which is opposite to the proposed Angle C) at pleasure to F and G, and reduce the Polygon ABCDE to the Triangle FCG, as taught in *Prob. 8.* then divide the Base FG of the Triangle *Fig. 75.* into eleven equal Parts (the Number of Acres) and set four of the said Parts from G to O, or seven of them from F to O, and draw the Line CO, it is the Partition-fence required.

PROB. XIII.

A Field, GHIKLM, containing 17 Acres, is to be divided between two Persons, one to have 11 Acres, the other 6, and the Partition-line to go from the Point N, in the Side GH, and to lay the 11 Acres towards G, and the 6 Acres towards H.

SOLUTION.

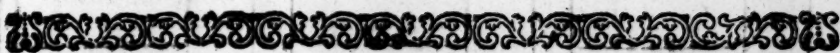
Through the Points M and I draw the Line OP, and by *Prob. 7.* reduce the Trapezium GHIM, to the Triangle ONP; divide OP into 17 equal Parts, and set 6 of them from P to Q. Again, by the same *Prob. 7.* reduce the Trapezium ISLM, to the Triangle RQS, *Fig. 79.* by Lines drawn from the Point Q; and dividing RS into 17 equal Parts, set 6 of them from S to Z, and draw QZ; then will the Lines NQ, and QZ, divide the whole Figure GHIKLM into two Parts; that towards G is 11 Acres, and that towards H is 6 Acres: But as it is to be divided by a Right-line, draw a Line from N to Z, and parallel to that draw QX from Q to X; and draw NX from N to X, it is the Line required; then is NHISX 6 Acres, and XLMGN 11 Acres.

Note, It is best in all such Cases, to draw all the Lines with Black-Lead, except the Out-line, and the Partition-line, that the rest may be rubbed out to avoid Confusion.

It will be needless to give any more Instances of Dividing, for if the Field to be divided hath more Sides and Angles than there are in the above Examples, nothing more is necessary than to reduce it to Triangles, or first to four or five-sided Figures, for an eight-sided Figure may be reduced to two of four Sides each; so that even all that can occur may be reduced to some of the Methods of Practice already taught.

Note, When Land is to be oddly divided, as suppose 17 Acres, 3 Rood, 18 Perches, between two Men; one to have 10 Acres, 1 Rood, 13 Perches; and the other to have 7 Acres, 2 Rood, 5 Perches; here, in the Way of Dividing according to the foregoing Problems, reduce all to Perches, then is the whole 2858 Perches; one Man's Share is 1653, and the other is 1205 Perches: And with these Proportions work as if they were whole Numbers; and if that is not exact enough, reduce lower, viz. to Links of 100000 to an Acre,





CHAP. X.

Containing Tables necessary in Surveying, with their Use, viz.

A Table of Northing, Southing, Easting, and Westing.

A Table of Logarithms from 1 to 1000. And

A Table of Sines and Tangents to every fifth Minute.

The Use of these Three Tables follows immediately after the last of them.



3 Deg.			6 Deg.			9 Deg.		
Distance,	N. S.	E. W.	Distance,	N. S.	E. W.	Distance,	N. S.	E. W.
1	1.0	.1	1	1.0	.1	1	1.0	.2
2	2.0	.1	2	2.0	.2	2	2.0	.3
3	3.0	.1	3	3.0	.3	3	3.0	.5
4	4.0	.2	4	4.0	.4	4	4.0	.6
5	5.0	.2	5	5.0	.5	5	5.0	.8
6	6.0	.3	6	6.0	.6	6	5.9	.9
7	7.0	.4	7	7.0	.7	7	6.9	1.1
8	8.0	.4	8	8.0	.8	8	7.9	1.3
9	9.0	.5	9	8.9	.9	9	8.9	1.4
10	10.0	.5	10	9.9	1.0	10	9.9	1.6
20	20.0	1.0	20	19.9	2.1	20	19.8	3.1
30	30.0	1.6	30	29.8	3.1	30	29.6	4.7
40	40.0	2.1	40	39.8	4.2	40	39.5	6.3
50	50.0	2.6	50	49.7	5.2	50	49.4	7.8
60	59.9	3.1	60	59.7	6.3	60	59.3	9.4
70	69.9	3.7	70	69.6	7.3	70	69.1	10.9
80	79.9	4.2	80	79.6	8.3	80	79.0	12.5
90	89.9	4.7	90	89.5	9.4	90	88.9	14.1
100	99.9	5.2	100	99.5	10.4	100	98.0	15.6
Diff.	E. W.	N. S.	Diff.	E. W.	N. S.	Diff.	E. W.	N. S.
	87 Deg.			84 Deg.			81 Deg.	

12 Deg.			15 Deg.			18 Deg.		
Distance,	N. S.	E. W.	Distance,	N. S.	E. W.	Distance,	N. S.	E. W.
	—	—		—	—		—	—
1	1.0	.2	1	1.0	.3	1	1.0	.3
2	2.0	.4	2	1.9	.5	2	1.9	.6
3	2.9	.6	3	2.9	.8	3	2.8	.9
4	3.9	.8	4	3.9	1.0	4	3.8	1.2
5	4.9	1.0	5	4.8	1.2	5	4.7	1.5
6	5.9	1.2	6	5.8	1.6	6	5.7	1.8
7	6.9	1.5	7	6.8	1.8	7	6.6	2.2
8	7.8	1.7	8	7.7	2.1	8	7.6	2.5
9	8.8	1.9	9	8.7	2.3	9	8.5	2.8
10	9.8	2.1	10	9.7	2.6	10	9.5	3.1
20	19.6	4.2	20	19.3	5.2	20	19.0	6.2
30	29.3	6.2	30	29.0	7.8	30	28.5	9.3
40	39.1	8.3	40	38.6	10.3	40	38.0	12.4
50	48.9	10.4	50	48.3	12.9	50	47.6	15.4
60	58.7	12.5	60	58.0	15.5	60	57.1	18.5
70	68.5	14.6	70	67.3	18.1	70	66.6	21.6
80	78.3	16.6	80	77.3	20.7	80	76.6	24.7
90	88.0	18.7	90	86.9	23.3	90	85.6	27.8
100	97.9	20.8	100	96.6	25.9	100	95.1	30.9
Diff.	E. W.	N. S.	Diff.	E. W.	N. S.	Diff.	E. W.	N. S.
	—	—		—	—		—	—
78 Deg.			75 Deg.			72 Deg.		

Table of Northing, Southing,

21 Deg.			24 Deg.			27 Deg.		
Distance,	N. S.	E. W.	Distance,	N. S.	E. W.	Distance,	N. S.	E. W.
1	.9	.4	1	.9	.4	1	.9	.5
2	1.9	.7	2	1.8	.8	2	1.8	.9
3	2.8	1.1	3	2.7	1.2	3	2.7	1.4
4	3.7	1.4	4	3.6	1.6	4	3.6	1.8
5	4.7	1.8	5	4.6	2.0	5	4.5	2.3
6	5.6	2.1	6	5.5	2.4	6	5.3	2.7
7	6.5	2.5	7	6.4	3.8	7	6.2	3.2
8	7.5	2.9	8	7.3	3.2	8	7.1	3.6
9	8.4	3.2	9	8.2	3.7	9	8.0	4.1
10	9.3	3.6	10	9.1	4.1	10	8.9	4.5
20	18.7	7.2	20	18.3	8.1	20	17.8	9.1
30	28.0	10.7	30	27.4	12.2	30	26.7	13.6
40	37.3	14.3	40	36.5	16.3	40	35.6	18.2
50	46.7	17.9	50	45.7	20.3	50	44.5	22.7
60	56.0	21.5	60	54.8	24.4	60	53.5	27.2
70	65.3	25.1	70	63.9	28.6	70	62.4	31.8
80	74.7	28.7	80	73.1	32.5	80	71.3	36.3
90	84.0	32.3	90	82.2	36.6	90	80.2	40.9
100	93.4	35.8	100	91.3	40.7	100	86.1	46.4
Diff.	E. W.	E. W.	Diff.	E. W.	N. S.	Diff.	E. W.	N. S.
9 Deg.			66 Deg.			63 Deg.		

Fig: 55.

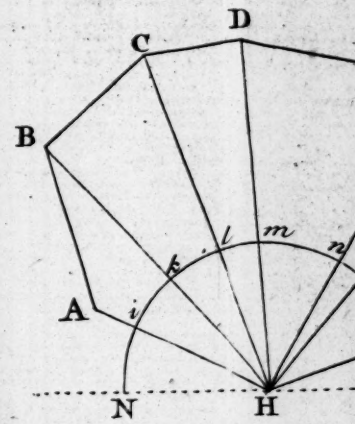


Fig: 57.

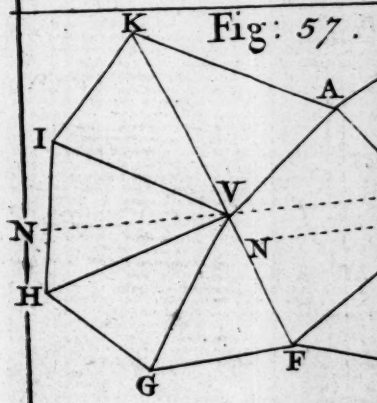
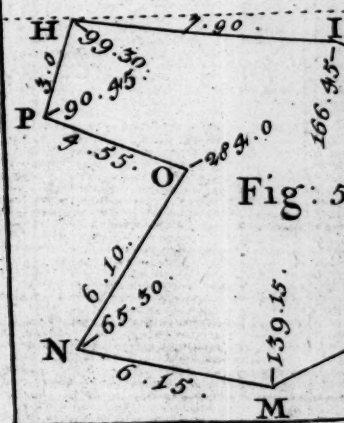
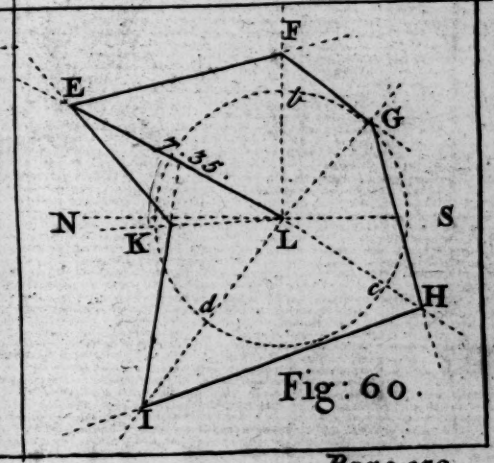
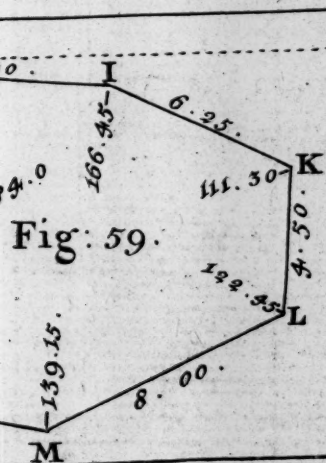
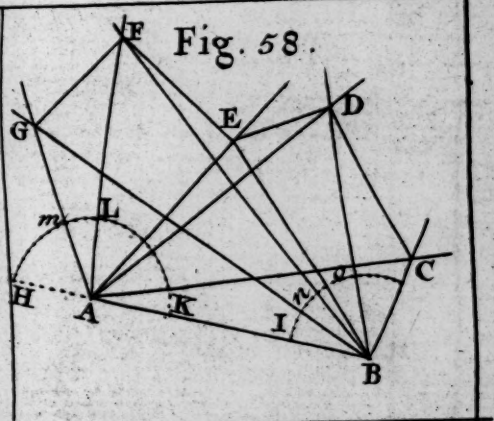
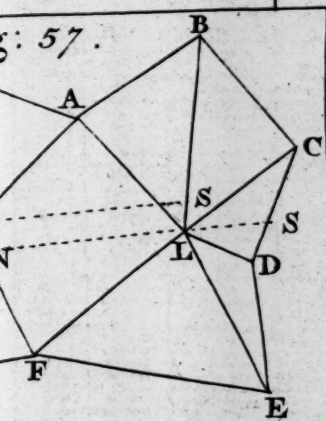
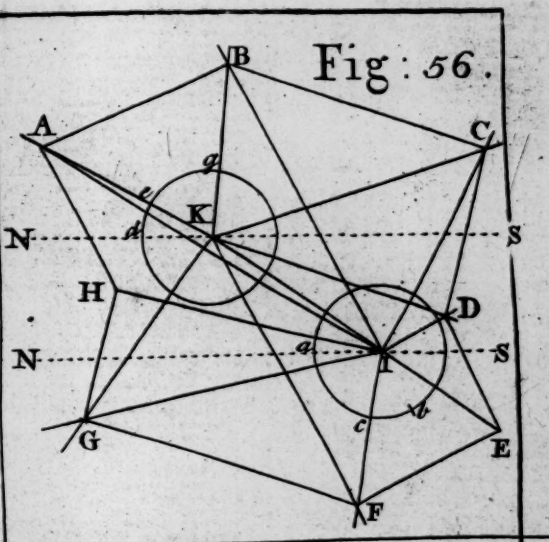
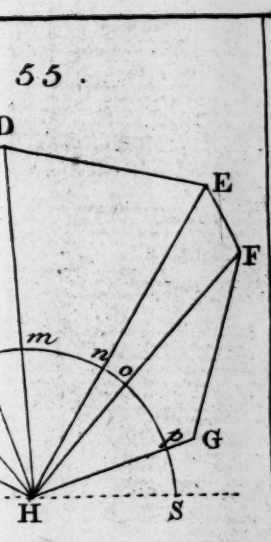


Fig: 5





30 Deg.			33 Deg.			36 Deg.		
Distance,	N. S.	E. W.	Distance,	N. S.	E. W.	Distance,	N. S.	E. W.
1	.9	.5	1	.8	.5	1	.8	.6
2	1.7	1.0	2	1.7	1.1	2	1.6	1.2
3	2.6	1.5	3	2.5	1.6	3	2.4	1.8
4	3.5	2.0	4	3.4	2.2	4	3.2	2.3
5	4.3	2.5	5	4.2	2.7	5	4.0	2.9
6	5.2	3.0	6	5.0	3.3	6	4.8	3.5
7	6.1	3.5	7	5.9	3.8	7	5.6	4.1
8	6.6	4.0	8	6.7	4.4	8	6.4	4.7
9	7.8	4.5	9	7.6	4.9	9	7.2	5.3
10	8.7	5.0	10	8.4	5.4	10	8.1	5.9
20	17.3	10.0	20	16.8	10.9	20	16.2	11.8
30	26.0	15.0	30	25.2	16.3	30	24.3	17.6
40	34.6	20.0	40	33.5	21.8	40	32.4	23.5
50	43.3	25.0	50	41.9	27.2	50	40.4	29.4
60	52.0	30.0	60	50.3	32.7	60	49.5	35.3
70	60.6	35.0	70	58.7	38.1	70	56.6	41.1
80	69.3	40.0	80	67.1	42.6	80	64.7	47.0
90	77.9	45.0	90	75.5	49.0	90	72.8	52.6
100	86.6	50.0	100	83.9	54.5	100	80.9	58.8
Dist.	E. W.	N. S.	Dist.	E. W.	N. S.	Dist.	E. W.	N. S.
60 Deg.			57 Deg.			54 Deg.		

39 Deg.			42 Deg.			45 Deg.		
Distance,	N. S.	E. W.	Distance,	N. S.	E. W.	Distance,	N. S.	E. W.
	—	—		—	—		—	—
1	.8	.6	1	.7	.7	1	.7	.7
2	1.6	1.3	2	1.5	1.3	2	1.4	1.4
3	2.3	1.6	3	2.2	2.0	3	2.1	2.1
4	3.1	2.5	4	3.0	3.7	4	2.8	2.8
5	3.9	3.1	5	3.7	3.3	5	3.5	3.5
6	4.7	3.8	6	4.4	4.0	6	4.2	4.2
7	5.4	4.4	7	5.2	5.7	7	4.9	4.9
8	6.3	5.0	8	5.9	5.3	8	5.6	5.6
9	7.0	5.7	9	6.7	6.0	9	6.8	6.8
10	7.8	6.3	10	7.4	7.7	10	7.1	7.1
20	15.5	12.6	20	14.9	14.4	20	14.1	14.1
30	23.3	18.9	30	22.3	22.1	30	21.2	21.2
40	31.1	25.2	40	29.7	29.8	40	28.3	28.3
50	38.8	31.5	50	37.2	33.5	50	35.3	35.3
60	46.6	37.8	60	44.6	44.1	60	42.4	42.4
70	54.4	44.0	70	52.0	52.8	70	49.5	49.5
80	62.2	50.3	80	59.4	59.5	80	56.6	56.6
90	69.9	56.6	90	66.9	66.2	90	63.6	63.6
100	77.7	62.9	100	74.3	64.9	100	70.7	70.7
Dift.	E. W.	N. S.	Dift.	E. W.	N. S.	Dift.	E. W.	N. S.
	—	—		—	—		—	—
51 Deg.			48 Deg.			45 Deg.		

T A B L E

OF

L O G A R I T H M S,

From 1, to 1000.

Also a T A B L E of S I N E S and
T A N G E N T S, to every fifth Minute
of the Quadrant.

N.	Logarith.	N.	Logarith.	N.	Logarith.	N.	Logarith.
1	0.000000	41	1.612784	81	1.908485	121	2.082785
2	0.301030	42	1.623249	82	1.913814	122	2.086359
3	0.477121	43	1.633468	83	1.919078	123	2.089905
4	0.602060	44	1.643453	84	1.924279	124	2.093422
5	0.698970	45	1.653212	85	1.929419	125	2.096910
6	0.778151	46	1.662758	86	1.934498	126	2.100371
7	0.845098	47	1.672098	87	1.939519	127	2.103804
8	0.903090	48	1.681241	88	1.944482	128	2.107210
9	0.954242	49	1.690196	89	1.949390	129	2.110590
10	1.000000	50	1.698970	90	1.954242	130	2.113943
11	1.041393	51	1.707570	91	1.959041	131	2.117271
12	1.079181	52	1.716003	92	1.963788	132	2.120574
13	1.113943	53	1.724276	93	1.968483	133	2.123852
14	1.146128	54	1.732394	94	1.973128	134	2.127105
15	1.176091	55	1.740363	95	1.977724	135	2.130334
16	1.204120	56	1.748188	96	1.982271	136	2.133539
17	1.230449	57	1.755875	97	1.986772	137	2.136721
18	1.255272	58	1.763428	98	1.991226	138	2.139879
19	1.278754	59	1.770852	99	1.995635	139	2.143015
20	1.301030	60	1.778151	100	2.000000	140	2.146128
21	1.322219	61	1.785330	101	2.004321	141	2.149219
22	1.342423	62	1.792392	102	2.008600	142	2.152288
23	1.361728	63	1.799340	103	2.012837	143	2.155336
24	1.380211	64	1.806180	104	2.017033	144	2.158362
25	1.397940	65	1.812913	105	2.021189	145	2.161368
26	1.414973	66	1.819544	106	2.025306	146	2.164353
27	1.431364	67	1.826075	107	2.029384	147	2.167317
28	1.447158	68	1.832509	108	2.033424	148	2.170262
29	1.462398	69	1.838849	109	2.037426	149	2.173186
30	1.477121	70	1.845098	110	2.041393	150	2.176091
31	1.491362	71	1.851258	111	2.045323	151	2.178977
32	1.505150	72	1.857332	112	2.049218	152	2.181844
33	1.518514	73	1.863323	113	2.053078	153	2.184691
34	1.531479	74	1.869232	114	2.056905	154	2.187521
35	1.544068	75	1.875061	115	2.060698	155	2.190332
36	1.556303	76	1.880814	116	2.064458	156	2.193125
37	1.568202	77	1.886491	117	2.068186	157	2.195900
38	1.579784	78	1.892095	118	2.071882	158	2.198657
39	1.591065	79	1.897627	119	2.075547	159	2.201397
40	1.602060	80	1.903090	120	2.079181	160	2.204120

N.	Logarith.	N.	Logarith.	N.	Logarith.	N.	Logarith.
161	2.206826	201	2.303196	241	2.382017	281	2.448706
162	2.209515	202	2.305351	242	2.383815	282	2.450249
163	2.212187	203	2.307496	243	2.385606	283	2.451786
164	2.214844	204	2.309630	244	2.387390	284	2.453318
165	2.217484	205	2.311754	245	2.389166	285	2.454845
166	2.220108	206	2.313867	246	2.390935	286	2.456366
167	2.222716	207	2.315970	247	2.392697	287	2.457882
168	2.225309	208	2.318063	248	2.394452	288	2.459392
169	2.227887	209	2.320146	249	2.396199	289	2.460898
170	2.230449	210	2.322219	250	2.397940	290	2.462398
171	2.232996	211	2.324282	251	2.399674	291	2.463893
172	2.235528	212	2.326336	252	2.401401	292	2.465383
173	2.238046	213	2.328380	253	2.403121	293	2.466868
174	2.240549	214	2.330414	254	2.404834	294	2.468347
175	2.243038	215	2.332438	255	2.406540	295	2.469822
176	2.245513	216	2.334454	256	2.408240	296	2.471292
177	2.247973	217	2.336460	257	2.409933	297	2.472756
178	2.250420	218	2.338456	258	2.411620	298	2.474216
179	2.252853	219	2.340444	259	2.413300	299	2.475671
180	2.255272	220	2.342423	260	2.414973	300	2.477121
181	2.257679	221	2.344392	261	2.416641	301	2.478566
182	2.260071	222	2.346353	262	2.418301	302	2.480007
183	2.262451	223	2.348305	263	2.419956	303	2.481443
184	2.264818	224	2.350248	264	2.421604	304	2.482874
185	2.267172	225	2.352182	265	2.423246	305	2.484300
186	2.269513	226	2.354108	266	2.424882	306	2.485721
187	2.271842	227	2.356026	267	2.426511	307	2.487138
188	2.274158	228	2.357935	268	2.428135	308	2.488551
189	2.276462	229	2.359835	269	2.429752	309	2.489958
190	2.278754	230	2.361728	270	2.431364	310	2.491362
191	2.281033	231	2.363612	271	2.432969	311	2.492760
192	2.283301	232	2.365488	272	2.434569	312	2.494155
193	2.285557	233	2.367356	273	2.436163	313	2.495544
194	2.287802	234	2.369216	274	2.437751	314	2.496930
195	2.290035	235	2.371068	275	2.439333	315	2.498311
196	2.292256	236	2.372912	276	2.440909	316	2.499687
197	2.294466	237	2.374748	277	2.442480	317	2.501059
198	2.296665	238	2.376577	278	2.444045	318	2.502427
199	2.298853	239	2.378398	279	2.445604	319	2.503791
200	2.301030	240	2.380211	280	2.447158	320	2.505150

N.	Logarith.	N.	Logarith.	N.	Logarith.	N.	Logarith.
321	2.506505	361	2.557507	401	2.603144	441	2.644439
322	2.507856	362	2.558709	402	2.604226	442	2.645422
323	2.509203	363	2.559907	403	2.605305	443	2.646404
324	2.510545	364	2.561101	404	2.606381	444	2.647383
325	2.511883	365	2.562293	405	2.607455	445	2.648360
326	2.513218	366	2.563481	406	2.608526	446	2.649335
327	2.514548	367	2.564666	407	2.609594	447	2.650308
328	2.515874	368	2.565848	408	2.610660	448	2.651278
329	2.517196	369	2.567026	409	2.611723	449	2.652246
330	2.518514	370	2.568202	410	2.612784	450	2.653213
331	2.519828	371	2.569374	411	2.613842	451	2.654177
332	2.521138	372	2.570543	412	2.614897	452	2.655138
333	2.522444	373	2.571089	413	2.615950	453	2.656098
334	2.523746	374	2.572872	414	2.617000	454	2.657058
335	2.525045	375	2.574031	415	2.618048	455	2.658011
336	2.526339	376	2.575188	416	2.619093	456	2.658965
337	2.527630	377	2.576341	417	2.620136	457	2.659916
338	2.528917	378	2.577492	418	2.621176	458	2.660865
339	2.530200	379	2.578639	419	2.622214	459	2.661813
340	2.531479	380	2.579784	420	2.623249	460	2.662757
341	2.532754	381	2.580925	421	2.624282	461	2.663701
342	2.534026	382	2.582069	422	2.625312	462	2.664642
343	2.535294	383	2.583191	423	2.626340	463	2.665581
344	2.536558	384	2.584331	424	2.627366	464	2.666518
345	2.537819	385	2.585461	425	2.628389	465	2.667453
346	2.539076	386	2.586587	426	2.629410	466	2.668388
347	2.540329	387	2.587711	427	2.630428	467	2.669317
348	2.541579	388	2.588832	428	2.631444	468	2.670246
349	2.542825	389	2.589949	429	2.632457	469	2.671173
350	2.544068	390	2.591065	430	2.633468	470	2.672098
351	2.545307	391	2.592177	431	2.634477	471	2.673021
352	2.546543	392	2.593286	432	2.635484	472	2.673942
353	2.547775	393	2.594393	433	2.636488	473	2.674861
354	2.549003	394	2.595496	434	2.637489	474	2.675778
355	2.550228	395	2.596597	435	2.638489	475	2.676694
356	2.551450	396	2.597695	436	2.639486	476	2.677607
357	2.552668	397	2.598790	437	2.640481	477	2.678518
358	2.553883	398	2.599883	438	2.641475	478	2.679428
359	2.555094	399	2.600973	439	2.642465	479	2.680335
360	2.556303	400	2.602059	440	2.643453	480	2.681241

N.	Logarith.	N.	Logarith.	N.	Logarith.	N.	Logarith.
481	2.682145	521	2.716838	561	2.748963	601	2.778874
482	2.683047	522	2.717671	562	2.749736	602	2.779596
483	2.683947	523	2.718502	563	2.750508	603	2.780317
484	2.684845	524	2.719331	564	2.751297	604	2.781037
485	2.685742	525	2.720159	565	2.752048	605	2.781755
486	2.686636	526	2.720986	566	2.752816	606	2.782473
487	2.687529	527	2.721811	567	2.753583	607	2.783189
488	2.688420	528	2.722634	568	2.754348	608	2.783904
489	2.689309	529	2.723456	569	2.755112	609	2.784617
490	2.690196	530	2.724276	570	2.755875	610	2.785229
491	2.691081	531	2.725095	571	2.756636	611	2.786041
492	2.691965	532	2.725912	572	2.757396	612	2.786751
493	2.692847	533	2.726727	573	2.758155	613	2.787460
494	2.693727	534	2.727541	574	2.758918	614	2.788164
495	2.694605	535	2.728354	575	2.759668	615	2.788875
496	2.695482	536	2.729165	576	2.760422	616	2.789581
497	2.696356	537	2.729974	577	2.761176	617	2.790285
498	2.697229	538	2.730782	578	2.761928	618	2.790988
499	2.698100	539	2.731589	579	2.762679	619	2.791691
500	2.698970	540	2.732394	580	2.763428	620	2.792392
501	2.699838	541	2.733197	581	2.764176	621	2.793092
502	2.700704	542	2.733999	582	2.764923	622	2.793790
503	2.701568	543	2.734800	583	2.765669	623	2.794488
504	2.702430	544	2.735599	584	2.766413	624	2.795185
505	2.703291	545	2.736397	585	2.767156	625	2.795880
506	2.704150	546	2.737192	586	2.767898	626	2.796574
507	2.705008	547	2.737987	587	2.768638	627	2.797268
508	2.705864	548	2.738781	588	2.769377	628	2.797960
509	2.706718	549	2.739572	589	2.770115	629	2.798651
510	2.707570	550	2.740363	590	2.770852	630	2.799341
511	2.708421	551	2.741152	591	2.771587	631	2.800029
512	2.709270	552	2.741939	592	2.772322	632	2.800717
513	2.710117	553	2.742725	593	2.773055	633	2.801404
514	2.710963	554	2.743510	594	2.773786	634	2.802089
515	2.711807	555	2.744293	595	2.774517	635	2.802774
516	2.712650	556	2.745075	596	2.775246	636	2.803457
517	2.713491	557	2.745855	597	2.775974	637	2.804139
518	2.714330	558	2.746634	598	2.776701	638	2.804825
519	2.715167	559	2.747412	599	2.777424	639	2.805501
520	2.716003	560	2.748188	600	2.778151	640	2.806180

N.	Logarith.	N.	Logarith.	N.	Logarith.	N.	Logarith.
641	2.806858	681	2.833147	721	2.857935	761	2.881385
642	2.807535	682	2.833784	722	2.858537	762	2.881955
643	2.808211	683	2.834421	723	2.859138	763	2.882525
644	2.808886	684	2.835059	724	2.859739	764	2.883093
645	2.809559	685	2.835691	725	2.860338	765	2.883661
646	2.810233	686	2.836324	726	2.860937	766	2.884229
647	2.810904	687	2.836957	727	2.861534	767	2.884795
648	2.811575	688	2.837588	728	2.862131	768	2.885361
649	2.812245	689	2.838219	729	2.862728	769	2.885926
650	2.812913	690	2.838849	730	2.863323	770	2.886491
651	2.813581	691	2.839478	731	2.863917	771	2.887084
652	2.814248	692	2.840106	732	2.864511	772	2.887617
653	2.814913	693	2.840733	733	2.865104	773	2.888179
654	2.815578	694	2.841359	734	2.865696	774	2.888741
655	2.816241	695	2.841985	735	2.866287	775	2.889302
656	2.816904	696	2.842609	736	2.866878	776	2.889862
657	2.817565	697	2.843233	737	2.867467	777	2.890421
658	2.818226	698	2.843855	738	2.868056	778	2.890979
659	2.818885	699	2.844477	739	2.868644	779	2.891537
660	2.819543	700	2.845098	740	2.869232	780	2.892095
661	2.820201	701	2.845718	741	2.869818	781	2.892651
662	2.820858	702	2.846337	742	2.870404	782	2.893207
663	2.821514	703	2.846955	743	2.870989	783	2.893762
664	2.822168	704	2.847573	744	2.871573	784	2.894316
665	2.822822	705	2.848189	745	2.872156	785	2.894860
666	2.823474	706	2.848805	746	2.872739	786	2.895423
667	2.824126	707	2.849419	747	2.873321	787	2.895975
668	2.824776	708	2.850033	748	2.873902	788	2.896526
669	2.825426	709	2.830646	749	2.874482	789	2.897077
670	2.826075	710	2.851258	750	2.875061	790	2.897627
671	2.826723	711	2.851869	751	2.875639	791	2.898176
672	2.827369	712	2.852480	752	2.876218	792	2.898725
673	2.828015	713	2.853089	753	2.876795	793	2.899273
674	2.828569	714	2.853698	754	2.877371	794	2.899821
675	2.829304	715	2.854306	755	2.877947	795	2.900367
676	2.829947	716	2.854913	756	2.878522	796	2.900913
677	2.830588	717	2.855519	757	2.879096	797	2.901458
678	2.831229	718	2.856124	758	2.879669	798	2.902003
679	2.831869	719	2.856729	759	2.880242	799	2.902547
680	2.832509	720	2.857332	760	2.880814	800	2.903089

A Table of Logarithms.

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N.	Logarith.	N.	Logarith.	N.	Logarith.	N.	Logarith.
801	2.903632	841	2.924796	881	2.944976	921	2.964259
802	2.904174	842	2.925315	882	2.945468	922	2.964731
803	2.904716	843	2.925828	883	2.945961	923	2.965202
804	2.905256	844	2.926342	884	2.946452	924	2.965672
805	2.905796	845	2.926857	885	2.946943	925	2.966142
806	2.906335	846	2.927370	886	2.947434	926	2.966611
807	2.906874	847	2.927893	887	2.947924	927	2.967079
808	2.907411	848	2.928396	888	2.948413	928	2.967548
809	2.907946	849	2.928908	889	2.948902	929	2.968016
810	2.908485	850	2.929419	890	2.949390	930	2.968483
811	2.909021	851	2.929929	891	2.949878	931	2.968949
812	2.909556	852	2.930439	892	2.950365	932	2.969416
813	2.910091	853	2.930949	893	2.950851	933	2.969882
814	2.910624	854	2.931458	894	2.951338	934	2.970345
815	2.911158	855	2.931966	895	2.951823	935	2.970812
816	2.911690	856	2.932474	896	2.952308	936	2.971279
817	2.912222	857	2.932981	897	2.952792	937	2.971739
818	2.912753	858	2.933487	898	2.953276	938	2.972203
819	2.913284	859	2.933993	899	2.953759	939	2.972666
820	2.913814	860	2.934498	900	2.954243	940	2.973128
821	2.914343	861	2.935003	901	2.954725	941	2.973589
822	2.914872	862	2.935507	902	2.955207	942	2.974051
823	2.915399	863	2.936011	903	2.955688	943	2.974512
824	2.915927	864	2.936514	904	2.956168	944	2.974972
825	2.916454	865	2.937016	905	2.956649	945	2.975432
826	2.916980	866	2.937518	906	2.957128	946	2.975891
827	2.917506	867	2.938019	907	2.957607	947	2.976349
828	2.918030	868	2.938519	908	2.958086	948	2.976808
829	2.918555	869	2.939019	909	2.958564	949	2.977266
830	2.919078	870	2.939519	910	2.959041	950	2.977723
831	2.919601	871	2.940018	911	2.959518	951	2.978181
832	2.920123	872	2.940516	912	2.959995	952	2.978637
833	2.920645	873	2.941014	913	2.960471	953	2.979093
834	2.921166	874	2.941511	914	2.960946	954	2.979548
835	2.921686	875	2.942008	915	2.961421	955	2.980003
836	2.922206	876	2.942504	916	2.961895	956	2.980458
837	2.922725	877	2.942999	917	2.962369	957	2.980912
838	2.923244	878	2.943495	918	2.962843	958	2.981366
839	2.923762	879	2.943989	919	2.963315	959	2.981819
840	2.924279	880	2.944483	920	2.963788	960	2.982271

A Table of Logarithms.

<i>N.</i>	<i>Logarith.</i>	<i>N.</i>	<i>Logarith.</i>	<i>N.</i>	<i>Logarith.</i>	<i>N.</i>	<i>Logarith.</i>
961	2.982723	971	2.987219	981	2.991669	991	2.996074
962	2.983175	972	2.987666	982	2.992111	992	2.996512
963	2.983626	973	2.988113	983	2.992554	993	2.996949
964	2.984077	974	2.988559	984	2.992995	994	2.997386
965	2.984527	975	2.989005	985	2.993436	995	2.997823
966	2.984977	976	2.989450	986	2.993877	996	2.998259
967	2.985426	977	2.989895	987	2.994317	997	2.998695
968	2.985875	978	2.990339	988	2.994757	998	2.999130
969	2.986324	979	2.990782	989	2.995196	999	2.999565
970	2.986772	980	2.991226	990	2.995935	1000	3.000000



A

T A B L E

O F

SINES and *TANGENTS*,
to every fifth Minute of the Quadrant.



N 3

A Table of Sines and Tangents.

0.

M.	Sine.	Co-sine.	Tangent.	Co-tangent.	
0	0.000000	10.000000	0.000000	Infinita.	60
5	7.162696	9.999999	7.162696	12.837304	55
10	7.463726	9.999998	7.463727	12.536273	50
15	7.639816	9.999996	7.639820	12.360108	45
20	7.764754	9.999993	7.764761	12.235239	40
25	7.861662	9.999989	7.861674	12.138326	35
30	7.940842	9.999983	7.940858	12.059132	30
35	8.005787	9.999977	8.007809	11.992191	25
40	8.067776	9.999971	8.065906	11.934194	20
45	8.116926	9.999963	8.116963	11.883037	15
50	8.162681	9.999954	8.162726	11.837273	10
55	8.204070	9.999944	8.204426	11.795874	5
60	8.241855	9.999934	8.241921	11.758079	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

89.

I.

M.	Sine.	Co-sine.	Tangents.	Co-tangent.	
0	8.241855	9.999934	8.241921	11.758097	60
5	8.276614	9.999922	8.276691	11.723309	55
10	8.308794	9.999910	8.308883	11.691116	50
15	8.338753	9.999897	8.338856	11.661144	45
20	8.366777	9.999882	8.366895	11.633105	40
25	8.393101	9.999867	8.393234	11.606766	35
30	8.417919	9.999851	8.418068	11.581932	30
35	8.441394	9.999834	8.441560	11.558440	25
40	8.463665	9.999816	8.463849	11.536151	20
45	8.484848	9.999797	8.485050	11.514950	15
50	8.505045	9.999778	8.505267	11.494733	10
55	8.524343	9.999757	8.524586	11.475414	5
60	8.542810	9.999735	8.543084	11.456916	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

88.

2.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	8.542819	9.999735	8.543084	11.456916	60
5	8.560540	9.999713	8.560276	11.439172	55
10	8.577566	9.999689	8.577877	11.422123	50
15	8.593948	9.999665	8.594283	11.405717	45
20	8.609734	9.999640	8.610094	11.389906	40
25	8.624965	9.999614	8.625352	11.374648	35
30	8.639680	9.999586	8.640093	11.359907	30
35	8.653911	9.999558	8.654352	11.345648	25
40	8.667689	9.999529	8.688160	11.331840	20
45	8.681043	9.999500	8.681544	11.318456	15
50	8.693998	9.999469	8.694529	11.305471	10
55	8.706577	9.999437	8.707140	11.292860	5
60	8.718800	9.999404	8.719396	11.280604	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

87.

3.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	8.718800	9.999404	8.719396	11.280604	60
5	8.730688	9.999371	8.731317	11.268683	55
10	8.742258	9.999336	8.742922	11.257078	50
15	8.753528	9.999301	8.754227	11.245733	45
20	8.764511	9.999265	8.763246	11.234754	40
25	8.775223	9.999227	8.775995	11.224005	35
30	8.785675	9.999189	8.786486	11.213514	30
35	8.795881	9.999150	8.796731	11.203969	25
40	8.805852	9.999110	8.806742	11.194325	20
45	8.815599	9.999069	8.816529	11.184711	15
50	8.825130	9.999027	8.826103	11.175389	10
55	8.834456	9.998984	8.835471	11.166452	5
60	8.843585	9.998941	8.844644	11.157356	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

54.

A Table of Sines and Tangents.

4.					
M.	Sine.	Co-sine.	Tangent.	Co-tangent.	
0	8.843585	9.998941	8.844644	11.155356	60
5	8.852525	9.998896	8.853628	11.146372	55
10	8.861283	9.998851	8.862433	11.137567	50
15	8.869868	9.998804	8.871664	11.128936	45
20	8.878285	9.998757	8.879529	11.120471	40
25	8.886542	9.998708	8.887833	11.112167	35
30	8.894643	9.998659	8.895984	11.104016	30
35	8.902596	9.998609	8.903987	11.096013	25
40	8.910404	9.998558	8.911846	11.088154	20
45	8.918073	9.998506	8.919668	11.080432	15
50	8.925009	9.998453	8.927156	11.072844	10
55	8.933015	9.998399	8.944616	11.065384	5
60	8.940296	9.998344	8.941952	11.058048	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

85.

5.					
M.	Sine.	Co-sine.	Tangent.	Co-tangent.	
0	8.940296	9.998344	8.941952	11.058048	60
5	8.947456	9.998289	8.949168	11.050832	55
10	8.954499	9.998232	8.956267	11.043733	50
15	8.961429	9.998174	8.963255	11.036745	45
20	8.968249	9.998116	8.970133	11.029867	40
25	8.974962	9.998056	8.976906	11.023094	35
30	8.981573	9.997996	8.983577	11.016423	30
35	8.988083	9.997935	8.990149	11.009851	25
40	8.994497	9.997872	8.996624	11.003367	20
45	9.000816	9.997809	9.003007	10.996993	15
50	9.007044	9.997745	9.009298	10.990702	10
55	9.013182	9.997680	9.015502	10.984499	5
60	9.019235	9.997614	9.021620	10.978380	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

84.

6.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.019235	9.997614	9.021620	10.978380	60
5	9.025203	9.997547	9.027655	10.972345	55
10	9.031089	9.997480	9.033609	10.966391	50
15	9.036896	9.997411	9.040651	10.960515	45
20	9.042625	9.997341	9.045284	10.954716	40
25	9.048279	9.997271	9.051008	10.948992	35
30	9.053859	9.997199	9.056659	10.943341	30
35	9.059367	9.997127	9.062240	10.937760	25
40	9.064806	9.997053	9.067752	10.932248	20
45	9.070176	9.996979	9.073197	10.926803	15
50	9.075480	9.996904	9.078576	10.921424	10
55	9.080719	9.996828	9.083891	10.916109	5
60	9.085894	9.996751	9.089144	10.910856	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

83.

7.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.085894	9.966751	9.089144	10.910850	60
5	9.091008	9.996673	9.094337	10.905664	55
10	9.096061	9.996594	9.099468	10.900532	50
15	9.101056	9.996514	9.104542	10.895458	45
20	9.105992	9.996433	9.109559	10.890441	40
25	9.110873	9.996351	9.114521	10.885479	35
30	9.115698	9.996269	9.119429	10.880571	30
35	9.120469	9.996185	9.124284	10.875716	25
40	9.125187	9.996100	9.129087	10.870913	20
45	9.129854	9.996015	9.133889	10.866161	15
50	9.134470	9.995928	9.138542	10.861458	10
55	9.139037	9.995841	9.143196	10.856804	5
60	9.143555	9.995753	9.147803	10.852199	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

82.

A Table of Sines and Tangents.

8.

M.	Sine.	Co-sine.	Tangent.	Co-tangent.	
0	9.143555	9.995753	9.147803	10.852197	60
5	9.148026	9.995664	9.152363	10.847637	55
10	9.152451	9.995573	9.156877	10.843123	50
15	9.156830	9.995482	9.061347	10.838653	45
20	9.161164	9.995390	9.165774	10.834226	40
25	9.165454	9.995297	9.170157	10.829843	35
30	9.169702	9.995203	9.174499	10.825501	30
35	9.173908	9.995108	9.178799	10.821201	25
40	9.178072	9.995013	9.183059	10.816941	20
45	9.182196	9.994916	9.187280	10.812720	15
50	9.186280	9.994810	9.191462	10.808538	10
55	9.190325	9.994720	9.195606	10.804394	5
60	9.194332	9.994620	9.199713	10.800287	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

81.

9.

M.	Sine.	Co-sine.	Tangent.	Co-tangent.	
0	9.194332	9.994620	9.199713	10.800287	60
5	9.198302	9.994519	9.203782	10.796218	55
10	9.202234	9.994418	9.207817	10.792183	50
15	9.206131	9.994316	9.211815	10.788185	45
20	9.209992	9.994212	9.215780	10.784220	40
25	9.213818	9.994108	9.219710	10.780290	35
30	9.217609	9.994003	9.223607	10.776393	30
35	9.221367	9.993897	9.227471	10.772529	25
40	9.225092	9.993789	9.231302	10.768698	20
45	9.228784	9.993681	9.235103	10.764897	15
50	9.232444	9.993572	9.238872	10.761128	10
55	9.236073	9.993462	9.242610	10.757390	5
60	9.239670	9.993351	9.246319	10.753681	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

80.

Fig: 61

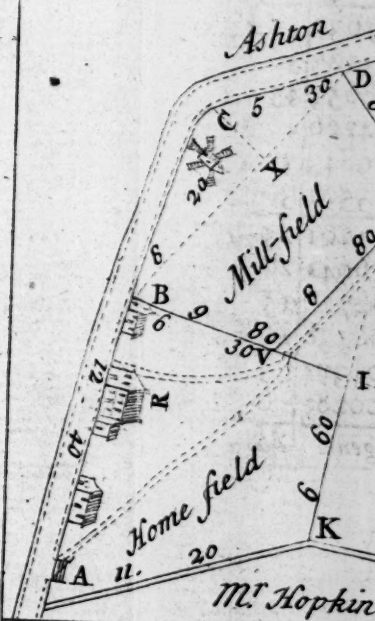


Fig: 62

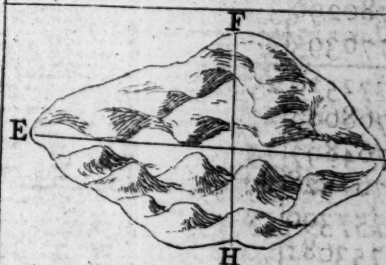
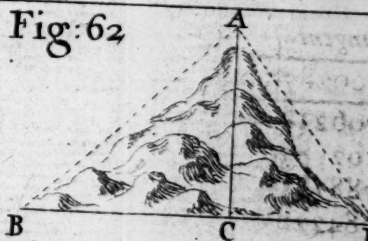
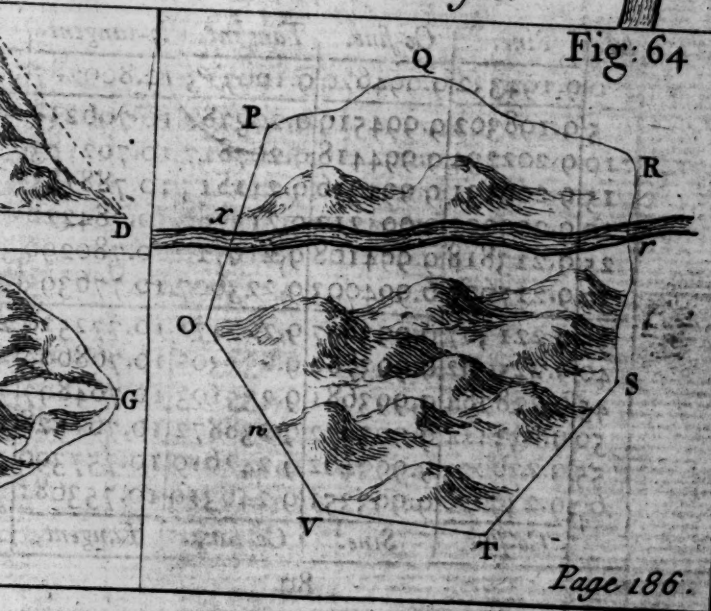
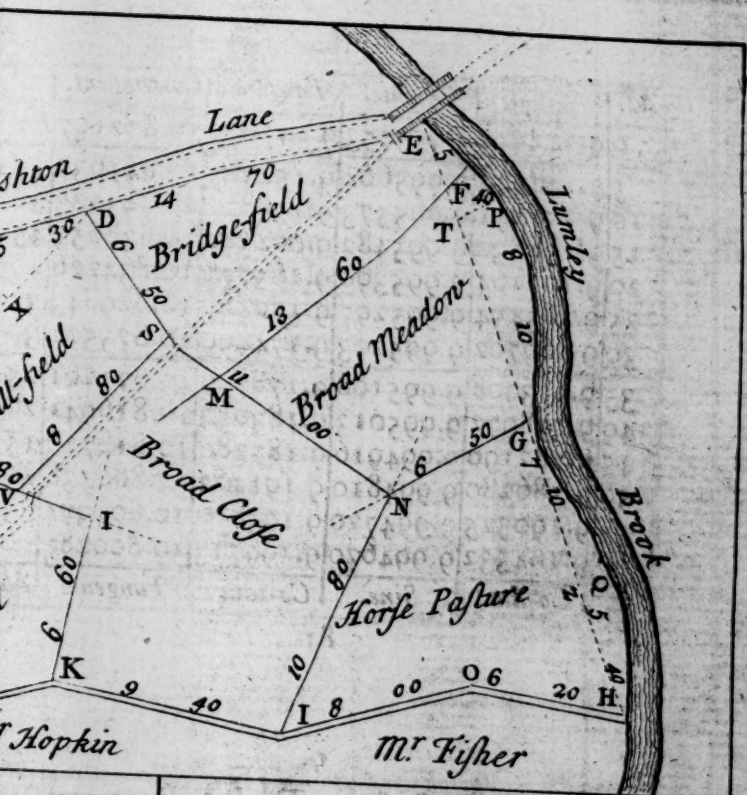


Fig: 63



10.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.239670	9.993351	9.246319	10.753681	60
5	9.243237	9.993240	9.249998	10.750002	55
10	9.246775	9.993127	9.253648	10.746352	50
15	9.250282	9.993013	9.257269	10.742731	45
20	9.253761	9.992898	9.260863	10.739137	40
25	9.257211	9.992783	9.264428	10.735572	35
30	9.260633	9.992666	9.267967	10.732033	30
35	9.264027	9.992549	9.271479	10.728521	25
40	9.267395	9.992430	9.274964	10.725036	20
45	9.270735	9.992311	9.278424	10.721576	15
50	9.274049	9.992190	9.285258	10.718142	10
55	9.277337	9.992069	9.285268	10.714732	5
60	9.280599	9.991947	9.288652	10.711348	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

79.

11.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.280599	9.991946	9.288652	10.711348	60
5	9.283836	9.991822	9.292013	10.707987	55
10	9.287048	9.991699	9.295349	10.704651	50
15	9.290236	9.991574	9.298662	10.701338	45
20	9.293399	9.991448	9.301951	10.698049	40
25	9.296539	9.991321	9.305218	10.694782	35
30	9.299655	9.991113	9.308463	10.691537	30
35	9.302748	9.991004	9.311685	10.688315	25
40	9.305819	9.990934	9.314885	10.685115	20
45	9.308867	9.990803	9.318064	10.681936	15
50	9.311893	9.990671	9.321222	10.678778	10
55	9.314897	9.990538	9.324358	10.675632	5
60	9.317879	9.990404	9.327474	10.672525	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

78.

A Table of Sines and Tangents.

12.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.317879	9.990404	9.327475	10.672525	60
5	9.320840	9.990270	9.330570	10.669430	55
10	9.323780	9.990134	9.333646	10.666317	50
15	9.326700	9.989997	9.336702	10.663298	45
20	9.329599	9.989860	9.339739	10.660261	40
25	9.332478	9.989721	9.342757	10.657243	35
30	9.335337	9.989582	9.345755	10.654245	30
35	9.338176	9.989441	9.348735	10.651265	25
40	9.340996	9.989300	9.351697	10.648303	20
45	9.343797	9.989157	9.354640	10.645360	15
50	9.346579	9.989014	9.357566	10.642434	10
55	9.349343	9.988869	9.360474	10.639526	5
60	9.352088	9.988724	9.363364	10.636636	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

77.

13.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.352088	9.988724	9.363364	10.636636	60
5	9.354815	9.988578	9.366237	10.633763	55
10	9.357524	9.988430	9.369094	10.630906	50
15	9.360215	9.988282	9.371933	10.628067	45
20	9.362889	9.988133	9.374756	10.625244	40
25	9.365546	9.987983	9.377563	10.622437	35
30	9.368185	9.987832	9.380354	10.619646	30
35	9.370808	9.987679	9.383129	10.616871	25
40	9.373414	9.987526	9.385888	10.614112	20
45	9.376003	9.987372	9.388631	10.611369	15
50	9.378577	9.987217	9.391360	10.608640	10
55	9.381134	9.987061	9.394073	10.605927	5
60	9.383675	9.986904	9.396771	10.603229	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	

78.

22.

M.	Sine.	Co-sine.	Tangent.	Co-tangent.	
0	9.383675	9.986904	9.396771	10.603225	60
5	9.386201	9.986746	9.399455	10.600545	55
10	9.388711	9.986587	9.402124	10.597876	50
15	9.391206	9.986427	9.404778	10.595222	45
20	9.393685	9.986266	9.407419	10.592581	40
25	9.396150	9.986104	9.410045	10.589955	35
30	9.398600	9.985942	9.412658	10.587342	30
35	9.401035	9.985778	9.415257	10.584743	25
40	9.403455	9.985613	9.417842	10.582158	20
45	9.405862	9.985447	9.420415	10.579585	15
50	9.408254	9.985280	9.422974	10.577026	10
55	9.410632	9.985113	9.425519	10.574481	5
60	9.412996	9.984944	9.428052	10.571948	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

75.

15.

M.	Sine.	Co-sine.	Tangent.	Co-tangent.	
0	9.412996	9.984944	9.428052	10.571948	60
5	9.415347	9.984774	9.430573	10.569427	55
10	9.417684	9.984603	9.433086	10.566920	50
15	9.420007	9.984432	9.435576	10.564424	45
20	9.422318	9.984259	9.438059	10.561941	40
25	9.424615	9.984085	9.440520	10.559471	35
30	9.426899	9.983911	9.442988	10.557012	30
35	9.429170	9.983735	9.445435	10.554565	25
40	9.431429	9.983558	9.444870	10.552130	20
45	9.433675	9.983381	9.450294	10.549706	15
50	9.435908	9.983202	9.452706	10.547294	10
55	9.438129	9.983022	9.455107	10.544893	5
60	9.440338	9.982842	9.457496	10.542904	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

74.

A Table of Sines and Tangents.

16.

M.	Sine.	Co-sine.	Tangent.	Co-tangent.	
0	9.440338	9.982842	9.457496	10.542504	60
5	9.442535	9.982660	9.459875	10.540125	55
10	9.444720	9.982477	9.462242	10.537758	50
15	9.446893	9.982294	9.464599	10.535401	45
20	9.449054	9.982109	9.466944	10.533055	40
25	9.451204	9.981924	9.469280	10.530720	35
30	9.453342	9.981737	9.471605	10.528393	30
35	9.455469	9.981549	9.473919	10.526081	25
40	9.457584	9.981361	9.476223	10.523777	20
45	9.459688	9.981171	9.478517	10.521483	15
50	9.461782	9.980981	9.480801	10.519199	10
55	9.463864	9.980789	9.483075	10.516925	5
60	9.465935	9.980596	9.485339	10.514661	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

73.

17.

M.	Sine.	Co-sine.	Tangent.	Co-tangent.	
0	9.465935	9.980596	9.485339	10.514661	60
5	9.467996	9.980403	9.487593	10.512407	55
10	9.470046	9.980208	9.489838	10.510162	50
15	9.472086	9.980012	9.492073	10.507927	45
20	9.474115	9.979816	9.494299	10.505701	40
25	9.476133	9.979618	9.496515	10.503485	35
30	9.478142	9.979419	9.498722	10.501278	30
35	9.480140	9.979220	9.500920	10.499080	25
40	9.482128	9.979019	9.503109	10.496891	20
45	9.484107	9.978817	9.505289	10.494711	15
50	9.486075	9.978615	9.507460	10.492540	10
55	9.488034	9.978411	9.509622	10.490378	5
60	9.489982	9.978206	9.511776	10.488224	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

72.

18.

M.	Sine.	Co-sine.	Tangent.	Co-tangent.	
0	9.489982	9.978620	9.511776	10.488224	60
5	9.491922	9.978001	9.513921	10.486079	55
10	9.493951	9.977794	9.516057	10.483943	50
15	9.495772	9.977586	9.518186	10.481814	45
20	9.497682	9.977377	9.520305	10.479695	40
25	9.499584	9.977167	9.522417	10.477583	35
30	9.501476	9.976957	9.524520	10.475480	30
35	9.503360	9.976745	9.526615	10.473385	25
40	9.505234	9.976532	9.528702	10.471298	20
45	9.507099	9.976318	9.530781	10.469219	15
50	9.508956	9.976103	9.532853	10.467147	10
55	9.510803	9.975887	9.534916	10.465084	5
60	9.512642	9.975670	9.536972	10.463028	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

71.

19.

M.	Sine.	Co-sine.	Tangent.	Co-tangent.	
0	9.512642	9.675670	9.536972	10.463028	60
5	9.514472	9.975452	9.539020	10.460980	55
10	9.516294	9.975233	9.541061	10.458939	50
15	9.518107	9.975013	9.543094	10.456906	45
20	9.519911	9.984792	9.545119	10.454881	40
25	9.521707	9.984570	9.547138	10.452862	35
30	9.523495	9.974347	9.549149	10.450851	30
35	9.525275	9.974122	9.551153	10.448847	25
40	9.527046	9.973897	9.553149	10.446851	20
45	9.528810	9.973671	9.555139	10.444861	15
50	9.530565	9.973444	9.557121	10.442879	10
55	9.532312	9.973215	9.559097	10.440903	5
60	9.534052	9.972986	9.561066	10.438934	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

70.

20.

M.	Sine.	Co-sine.	Tangent.	Co-tangent.	
0	9.534052	9.972986	9.561066	10.438934	60
5	9.535783	9.972755	9.563028	10.436972	55
10	9.537507	9.972524	9.564983	10.435017	50
15	9.539223	9.972291	9.566932	10.433068	45
20	9.540931	9.972058	9.568873	10.431127	40
25	9.542632	9.971823	9.570809	10.429191	35
30	9.544325	9.971583	9.572738	10.427262	30
35	9.546011	9.971351	9.574660	10.425340	25
40	9.547689	9.971113	9.576576	10.423424	20
45	9.549360	9.970874	9.578486	10.421514	15
50	9.551024	9.970635	9.580389	10.419611	10
55	9.552680	9.970394	9.582286	10.417714	5
60	9.554329	9.970152	9.584177	10.415823	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

69.

21.

M.	Sine.	Co-sine.	Tangent.	Co-tangent.	
0	9.554329	9.970152	9.584177	10.415823	60
5	9.555971	9.969909	9.586062	10.413938	55
10	9.557606	9.969665	9.587941	10.412059	50
15	9.559234	9.969420	9.589814	10.410186	45
20	9.560855	9.969173	9.591681	10.408319	40
25	9.562468	9.968926	9.593542	10.406458	35
30	9.564075	9.968678	9.595398	10.404692	30
35	9.565676	9.968429	9.597247	10.402753	25
40	9.567269	9.968178	9.599091	10.400909	20
45	9.568856	9.967927	9.600561	10.399071	15
50	9.570435	9.967674	9.602395	10.397239	10
55	9.572009	9.967421	9.604588	10.395412	5
60	9.573575	9.967166	9.606410	10.393590	0
	Co-sine.	Sine.	Co-tang.	Tangent.	M.

68.

22.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.573575	9.967166	9.606410	10.393590	60
5	9.575136	9.966910	9.608225	10.391775	55
10	9.576689	9.966653	9.610036	10.389964	50
15	9.578236	9.966395	9.611841	10.388159	45
20	9.579777	9.966136	9.613641	10.386359	40
25	9.581112	9.965876	9.615435	10.384565	35
30	9.582840	9.965615	9.617224	10.382776	30
35	9.584361	9.965353	9.619008	10.380992	25
40	9.585877	9.965090	9.620787	10.379213	20
45	9.587386	9.964826	9.622561	10.377439	15
50	9.588890	9.964560	9.624330	10.375670	10
55	9.590387	9.964294	9.626093	10.373907	5
60	9.591878	9.964026	9.627852	10.372148	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

67.

23.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.591878	9.964026	9.627852	10.372148	60
5	9.593363	9.963757	9.629606	10.370394	55
10	9.594842	9.963488	9.631355	10.368645	50
15	9.596315	9.963217	9.633098	10.366902	45
20	9.597783	9.962945	9.634838	10.365162	40
25	9.599244	9.962672	9.636572	10.363428	35
30	9.600700	9.962398	9.638302	10.361698	30
35	9.602150	9.962123	9.640027	10.359973	25
40	9.603594	9.961846	9.641747	10.358253	20
45	9.605032	9.961569	9.643463	10.356537	15
50	9.606465	9.961290	9.645174	10.354826	10
55	9.607892	9.961011	9.646881	10.353119	5
60	9.609313	9.960730	9.648583	10.351417	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

66.

A Table of Sines and Tangents.

24.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.609313	9.960730	9.648583	10.351417	60
5	9.610729	9.960448	9.650281	10.349719	55
10	9.612140	9.960165	9.651975	10.348026	50
15	9.613545	9.959882	9.653663	10.346337	45
20	9.614944	9.959596	9.655348	10.344652	40
25	9.616338	9.959310	9.657028	10.342972	35
30	9.617727	9.959023	9.658704	10.341296	30
35	9.619110	9.958734	9.660376	10.339624	25
40	9.620488	9.958445	9.662043	10.337957	20
45	9.621861	9.958154	9.663707	10.336293	15
50	9.623229	9.957863	9.665366	10.334634	10
55	9.624591	9.957570	9.667021	10.332979	5
60	9.625948	9.957276	9.668673	10.331327	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

65.

25.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangents.</i>	<i>Co-tangent.</i>	
0	9.625948	9.957276	9.668673	10.331327	60
5	9.627300	9.956981	9.670320	10.329680	55
10	9.628647	9.956684	9.671963	10.328037	50
15	9.629989	9.956387	9.673602	10.326398	45
20	9.631326	9.956089	9.675237	10.324763	40
25	9.632658	9.955789	9.676869	10.323131	35
30	9.633984	9.955488	9.678496	10.321504	30
35	9.635306	9.955186	9.680120	10.319880	25
40	9.636623	9.954883	9.681740	10.318260	20
45	9.637935	9.954579	9.683356	10.316644	15
50	9.639242	9.954274	9.684968	10.315032	10
55	9.640544	9.953968	9.686577	10.313423	5
60	9.641842	9.953660	9.688182	10.311818	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

64.

26.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.641842	9.953660	9.688182	10.311818	60
5	9.643135	9.953352	9.689783	10.310217	55
10	9.644423	9.953042	9.691381	10.308619	50
15	9.645706	9.952731	9.692975	10.307025	45
20	9.646984	9.952419	9.694566	10.305434	40
25	9.648258	9.952106	9.696153	10.303847	35
30	9.649527	9.951791	9.697736	10.302264	30
35	9.650792	9.951476	9.699316	10.300684	25
40	9.652052	9.951159	9.700893	10.299107	20
45	9.653308	9.950841	9.702496	10.297534	15
50	9.654558	9.950522	9.704036	10.295964	10
55	9.655805	9.950202	9.705603	10.294397	5
60	9.657047	9.949881	9.707166	10.292834	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

63.

27.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.657047	9.949881	9.707166	10.292834	60
5	9.658284	9.949558	9.708726	10.291274	55
10	9.659517	9.949235	9.710282	10.289718	50
15	9.660746	9.948910	9.711836	10.288164	45
20	9.661970	9.948584	9.713386	10.286614	40
25	9.663190	9.948257	9.714933	10.285067	35
30	9.664406	9.947929	9.716477	10.283523	30
35	9.665618	9.947600	9.718017	10.281983	25
40	9.666824	9.947269	9.719555	10.280445	20
45	9.668027	9.946937	9.721089	10.278911	15
50	9.669225	9.946604	9.722621	10.277379	10
55	9.670419	9.946270	9.724149	10.275851	5
60	9.671609	9.945935	9.725674	10.274326	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

62.

28.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.671609	9.645935	9.725674	10.274326	60
5	9.672795	9.945598	9.727197	10.272803	55
10	9.673977	9.945261	9.728716	10.271284	50
15	9.675155	9.944922	9.730233	10.269767	45
20	9.676328	9.944582	9.731746	10.268254	40
25	9.677498	9.944241	9.733257	10.266743	35
30	9.678663	9.943899	9.734764	10.265236	30
35	9.679824	9.943555	9.736269	10.263731	25
40	9.680982	9.943210	9.737771	10.262229	20
45	9.682135	9.942864	9.739271	10.260729	15
50	9.683284	9.942517	9.740767	10.259233	10
55	9.684430	9.942169	9.742261	10.257739	5
60	9.685571	9.941819	9.743752	10.256248	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M</i>

61.

29.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.685571	9.941819	9.743751	10.256248	60
5	9.686709	9.941469	9.745240	10.254760	55
10	9.687843	9.941117	9.746726	10.253274	50
15	9.688972	9.940763	9.748209	10.251791	45
20	9.690098	9.940409	9.749689	10.250311	40
25	9.691220	9.940054	9.751167	10.248833	35
30	9.692339	9.939697	9.752642	10.247358	30
35	9.693453	9.939339	9.754115	10.245885	25
40	9.694564	9.938980	9.755585	10.244415	20
45	9.695671	9.938619	9.757052	10.242948	15
50	9.696775	9.938258	9.758517	10.241483	10
55	9.697874	9.937885	9.759979	10.240021	5
60	9.698970	9.937531	9.761439	10.238560	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M</i>

60.

30.					
<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent</i>	
0	9.698970	9.937531	9.761439	10.238561	60
5	9.700062	9.937165	9.762897	10.237103	55
10	9.701151	9.936799	9.764352	10.235648	50
15	9.702236	9.936431	9.765805	10.234195	45
20	9.703317	9.936062	9.767255	10.232745	40
25	9.704395	9.935692	9.768703	10.231297	35
30	9.705469	9.935320	9.770148	10.229852	30
35	9.706539	9.934948	9.771592	10.228408	25
40	9.707606	9.934574	9.773033	10.226967	20
45	9.708670	9.934199	9.774471	10.225529	15
50	9.709730	9.933822	9.775908	10.224092	10
55	9.710786	9.933445	9.777342	10.222658	5
60	9.711839	9.933066	9.778774	10.221226	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

59.

31.					
<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent</i>	
0	9.711839	9.933066	9.778774	10.221226	60
5	9.712889	9.932685	9.780203	10.219797	55
10	9.713935	9.932304	9.781631	10.218369	50
15	9.714978	9.931921	9.783056	10.216944	45
20	9.716017	9.931537	9.784479	10.215521	40
25	9.717053	9.931152	9.785900	10.214100	35
30	9.718085	9.930766	9.787319	10.212681	30
35	9.719114	9.930378	9.788736	10.211264	25
40	9.720140	9.929989	9.790151	10.209849	20
45	9.721162	9.929599	9.791563	10.208437	15
50	9.722181	9.929207	9.792974	10.207026	10
55	9.723197	9.928815	9.794383	10.205617	5
60	9.724210	9.928420	9.795789	10.204211	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

58.

32.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.724210	9.928420	9.795789	10.204211	60
5	9.725219	9.928025	9.797194	10.202806	55
10	9.726225	9.927629	9.798596	10.201404	50
15	9.727228	9.927231	9.799997	10.200003	45
20	9.729223	9.926831	9.801396	10.198604	40
25	9.729223	9.926431	9.802792	10.197208	35
30	9.730217	9.926029	9.804187	10.195814	30
35	9.731206	9.925626	9.805580	10.194420	25
40	9.732193	9.925222	9.806971	10.193029	20
45	9.733177	9.924816	9.808361	10.191639	15
50	9.734157	9.924409	9.809748	10.190252	10
55	9.735135	9.924001	9.811134	10.188866	5
60	9.736109	9.923591	9.812517	10.187483	0
	<i>Co sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

57.

33.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.736109	9.923591	9.812517	10.187483	60
5	9.737080	9.923181	9.813899	10.186101	55
10	9.738048	9.922769	9.815280	10.184720	50
15	9.739013	9.922355	9.816658	10.183342	45
20	9.739975	9.921940	9.818035	10.181965	40
25	9.740934	9.921524	9.819410	10.180590	34
30	9.741889	9.921107	9.820783	10.179217	30
35	9.742852	9.920688	9.822154	10.177846	25
40	9.743792	9.920268	9.823524	10.176476	23
45	9.744739	9.919846	9.824893	10.175107	15
50	9.745783	9.919424	9.826259	10.173741	10
55	9.746624	9.919000	9.827624	10.172376	5
60	9.747562	9.918574	9.828987	10.171013	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

56.

34.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.747562	9.918574	9.828987	10.171013	60
5	9.748497	9.918147	9.830349	10.169051	55
10	9.749429	9.917719	9.831709	10.168291	50
15	9.750358	9.917290	9.833068	10.166932	45
20	9.751284	9.916859	9.834425	10.165575	40
25	9.752208	9.916427	9.835780	10.164220	35
30	9.753128	9.915994	9.837134	10.162866	30
35	9.754046	9.915559	9.838487	10.161513	25
40	9.754960	9.915123	9.839838	10.160162	20
45	9.755872	9.914685	9.841187	10.158813	15
50	9.756782	9.914246	9.842535	10.157465	10
55	9.757688	9.913806	9.843882	10.156118	5
60	9.758591	9.913365	9.845227	10.154773	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

55.

35.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.758591	9.913365	9.845227	10.154713	60
5	9.759492	9.912922	9.846570	10.153430	55
10	9.760390	9.912477	9.847913	10.152087	50
15	9.761285	9.912031	9.849254	10.150746	45
20	9.762177	9.911584	9.850593	10.149407	40
25	9.763067	9.911136	9.851931	10.148069	35
30	9.763954	9.910686	9.853268	10.146732	30
35	9.764838	9.910235	9.854603	10.145397	25
40	9.765720	9.909782	9.855938	10.144062	20
45	9.766598	9.909328	9.855727	10.142730	15
50	9.767475	9.908873	9.855860	10.141398	10
55	9.768348	9.908416	9.855993	10.140008	5
60	9.769219	9.907958	9.856126	10.138739	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

54.

A Table of Sines and Tangents.

36.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co tangent.</i>	
0	9.769219	9.907957	9.861261	10.138739	60
5	9.770087	9.907498	9.862589	10.137411	55
10	9.770952	9.907037	9.863915	10.136085	50
15	9.771815	9.906575	9.865240	10.134760	45
20	9.772675	9.906111	9.866564	10.133436	40
25	9.773533	9.905645	9.867887	10.132112	35
30	9.774388	9.905179	9.869209	10.130791	30
35	9.775240	9.904711	9.870520	10.129471	25
40	9.776090	9.904241	9.871849	10.128151	20
45	9.776937	9.903770	9.873167	10.126833	15
50	9.777781	9.903298	9.874484	10.125516	10
55	9.778624	9.902824	9.875800	10.124200	5
60	9.779463	9.902349	9.877114	10.122886	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

53.

37.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent</i>	
0	9.779463	9.002349	9.877114	10.122886	60
5	9.780300	9.901872	9.878428	10.121572	55
10	9.781134	9.901394	9.879741	10.120259	50
15	9.781966	9.900984	9.881052	10.118948	45
20	9.782796	9.900433	9.882363	10.117637	40
25	9.783623	9.899951	9.883672	10.116328	35
30	9.784447	9.899467	9.884980	10.115020	30
35	9.784269	9.898981	9.886288	10.113712	25
40	9.786089	9.898494	9.887594	10.112406	20
45	9.786906	9.898006	9.888900	10.111100	15
50	9.787720	9.897516	9.890204	10.109796	10
55	9.788532	9.897025	9.891507	10.108493	5
60	9.789342	9.896539	9.892810	10.107190	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

52.

Fig: 65.

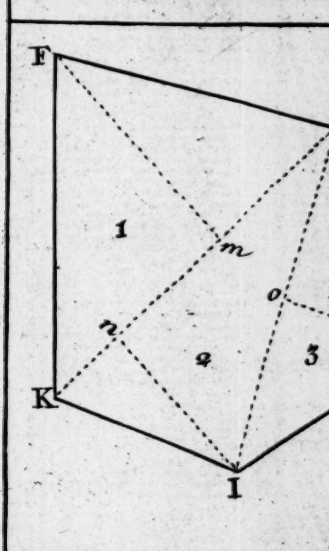
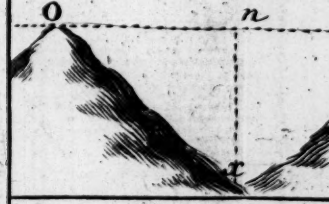


Fig: 72.

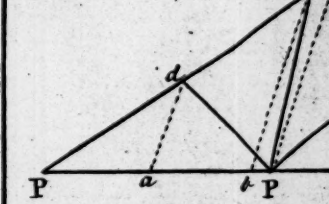


Fig: 74.



Fig: 65.



Fig: 66.

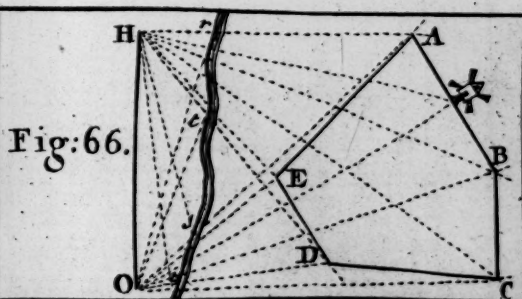


Fig: 67.

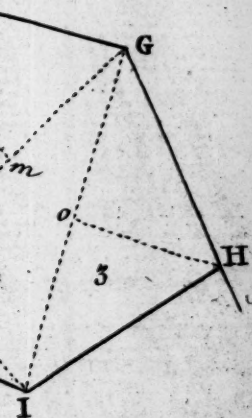


Fig: 68.

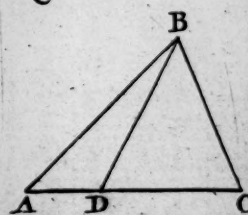


Fig: 69.

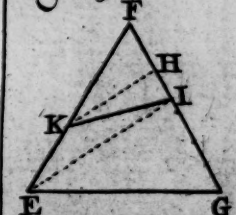


Fig: 70.

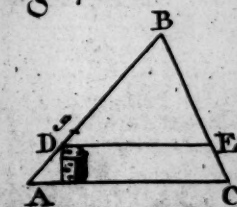


Fig: 71.

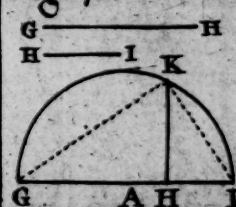


Fig. 73.

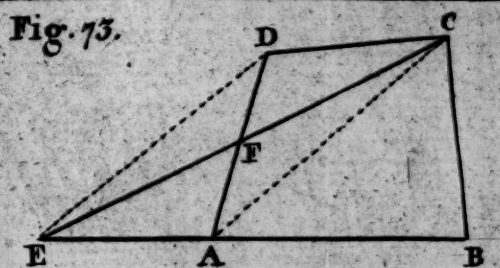
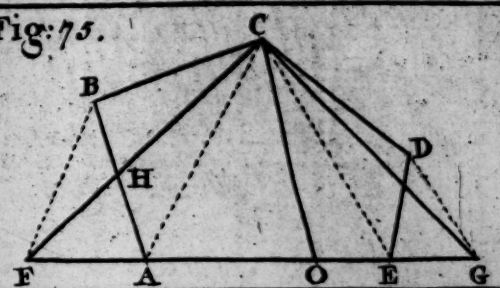


Fig: 75.



38.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent</i>	
0	9.789342	9.896532	9.892810	10.107190	60
5	9.790149	9.896038	9.894111	10.105889	55
10	9.790954	9.895544	9.895412	10.104588	50
15	9.791757	9.895045	9.896712	10.103288	45
20	9.792557	9.894546	9.898010	10.101990	40
25	9.793354	9.894026	9.899308	10.100692	35
30	9.794150	9.893544	9.900605	10.099395	30
35	9.794942	9.893041	9.901901	10.098099	25
40	9.795733	9.892536	9.903197	10.096803	20
45	9.796571	9.892030	9.904491	10.095509	15
50	9.797307	9.891523	9.905785	10.094215	10
55	9.798091	9.891013	9.907077	10.092923	5
60	9.798872	9.890503	9.908369	10.091631	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

51.

39.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent</i>	
0	9.798872	9.890503	9.908369	10.091631	60
5	9.799651	9.889990	9.909660	10.090340	55
10	9.800427	9.889477	9.910951	10.089049	50
15	9.801201	9.888961	9.912240	10.087760	45
20	9.801973	9.888444	9.913529	10.086471	40
25	9.802743	9.887926	9.914817	10.085183	35
30	9.803511	9.887406	9.916104	10.083895	30
35	9.804276	9.886885	9.917391	10.082609	25
40	9.805039	9.886362	9.918677	10.081323	20
45	9.805799	9.885837	9.919962	10.080038	15
50	9.806557	9.885311	9.921247	10.078753	10
55	9.807314	9.884783	9.922530	10.077470	5
60	9.808067	9.884254	9.923814	10.076186	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

50.

A Table of Sines and Tangents.

40.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.808067	9.884254	9.923814	10.076186	60
5	9.808819	9.883723	9.925096	10.070904	55
10	9.809569	9.883191	9.926378	10.073622	50
15	9.810316	9.882657	9.927659	10.072341	45
20	9.811061	9.882121	9.928940	10.071060	40
25	9.811804	9.881584	9.930220	10.069780	35
30	9.812544	9.881046	9.931499	10.068501	30
35	9.813283	9.880505	9.932778	10.067222	25
40	9.814019	9.879963	9.934056	10.065944	20
45	9.814753	9.879420	9.935333	10.064667	15
50	9.815485	9.878875	9.936611	10.063389	10
55	9.816215	9.878328	9.937887	10.062113	0
60	9.816943	9.877780	9.939163	10.060837	5
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

49.

41.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.816943	9.877780	9.939163	10.060837	60
5	9.817668	9.877230	9.940439	10.059561	55
10	9.818392	9.876678	9.941713	10.058287	50
15	9.819113	9.876125	9.942998	10.057012	45
20	9.819832	9.875571	9.944262	10.055738	40
25	9.820550	9.875014	9.945535	10.054465	35
30	9.821265	9.874456	9.946808	10.053192	30
35	9.821977	9.873896	9.948081	10.051919	25
40	9.822688	9.873335	9.949353	10.050647	20
45	9.823397	9.872772	9.950625	10.049375	15
50	9.824104	9.872208	9.951896	10.048104	10
55	9.824808	9.871641	9.953167	10.046833	5
60	9.825511	9.871073	9.954437	10.045563	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

48.

42.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.825511	9.871073	9.954437	10.045503	60
5	9.826211	9.870504	9.955708	10.044292	55
10	9.826910	9.869933	9.956977	10.043023	50
15	9.827606	9.869360	9.958247	10.041753	45
20	9.828301	9.868785	9.959516	10.040484	40
25	9.828993	9.868209	9.960784	10.039216	35
30	9.829683	9.867631	9.962052	10.037948	30
35	9.830372	9.867051	9.963320	10.036680	25
40	9.831358	9.866470	9.964588	10.035412	20
45	9.831742	9.865887	9.965855	10.034145	15
50	9.832425	9.865302	9.967123	10.032877	10
55	9.833105	9.864716	9.968389	10.031611	5
60	9.833783	9.864127	9.969656	10.030344	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

47.

43.

<i>M.</i>	<i>Sine.</i>	<i>Co-sine.</i>	<i>Tangent.</i>	<i>Co-tangent.</i>	
0	9.833783	9.864127	9.969656	10.030344	60
5	9.834460	9.863538	9.970922	10.029078	55
10	9.835134	9.862946	9.972188	10.027812	50
15	9.835807	9.862353	9.973454	10.026546	45
20	9.836477	9.861758	9.974720	10.025280	40
25	9.837146	9.861161	9.975985	10.024015	35
30	9.837812	9.860562	9.977250	10.022750	30
35	9.838477	9.859962	9.978515	10.021485	25
40	9.839140	9.859360	9.979780	10.020220	20
45	9.839800	9.858756	9.981044	10.018956	15
50	9.840459	9.858151	9.982309	10.017691	10
55	9.841116	9.857543	9.983573	10.016427	5
60	9.841772	9.856934	9.984837	10.015163	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>

46.

A Table of Sines and Tangents.

44.					
<i>M.</i>	<i>Sine.</i>	<i>Co sine.</i>	<i>Tangent.</i>	<i>Co tangent.</i>	
0	9.841771	9.856934	9.984837	10.015163	60
5	9.842424	9.856323	9.986101	10.013899	55
10	9.843076	9.855711	9.987365	10.012635	50
15	9.843725	9.855096	9.988629	10.011371	45
20	9.844372	9.854480	9.989893	10.010107	40
25	9.845018	9.853862	9.991156	10.008844	35
30	9.845662	9.853242	9.992420	10.007580	30
35	9.846304	9.852620	9.993683	10.006317	25
40	9.846944	9.851997	9.994947	10.005053	20
45	9.847582	9.851372	9.996210	10.003790	15
50	9.848218	9.850745	9.997473	10.002527	10
55	9.848852	9.850116	9.998737	10.001263	5
60	9.849485	9.849485	10.000000	10.000000	0
	<i>Co-sine.</i>	<i>Sine.</i>	<i>Co-tang.</i>	<i>Tangent.</i>	<i>M.</i>
45.					



S E C T. I.

The Use of the Table of Northing, Southing, Easting and Westing.

THIS Table is of excellent Use in general, but more particularly in large Surveys, or where you cannot see from one Part of the Estate or Field to the opposite Part of it, nor near of it, and indeed is to be accounted as in Navigation, the Mariner can keep account of his Way upon the vast Ocean, and when he cannot see, nor scarce comprehend the Distance that he has to sail in his outward bound Trading, or homeward-bound Voyage; yet by these Sort of Tables (adapted particularly to Navigation) he can account for all the various Turnings and Windings that his Ship has gone thro' and described.

And this Table is more excellent in Surveying, not only because by the Help thereof the Out-lines of vast large Spaces of Ground may be justly plotted and described, but because this Part of the Work may be performed without any Instrument, but only a Chain and Compass; the Compass (as indeed all other Instruments) had need be accurately made and well touched, for if it does not play free and true, the Artist will be subject to contract an Error in the Plotting when the Angles are taken false.

The Method of Surveying and Plotting Land by the Help of these Tables is as follows, and is best illustrated by the succeeding Example.

How to take the Northing, Southing, Easting, or Westing, out of the Whole.

The Column of Distance is Chains, and the Column titled *N. S.* gives the Northing or Southing (not in Chains and Links, but) in Chains and Tenths of a Chain, every one of which contains 10 Links; but the best Way, in any Number of Chains under 10, is to add a Cypher to the Number, and then find it in the Table, in its proper Column; and in the Column of the Degree, find the Northing or Southing, Easting or Westing in its Column, only let the two Figures towards the Right-hand be Links, and the other Chains.

Example. In the following Example, the first Line D E is North 42 Degrees Eastly 5 Chains. Look in the Table for

42 Degrees, and, instead of 5 Chains, look in the lower Part of the Table for 50, and against it you have 37.2 Northing, and 33.5 Easting; but cutting off, or separating the two Figures towards the Right-hand for Links, and leaving the other for Chains, it will be 3.72, viz. 3 Chains 72 Links Northing, and 3.35, or 3 Chains 35 Links Easting; as in the following Table.

Fig. 80. Let the Field to be surveyed and plotted, be in the Form of the Figure DEFGHIKL.

Begin the Work at any Angle of the Field, suppose at D, and observe by your Instrument what Angle the Line or Hedge DE makes with the Meridian, which suppose 42 Degrees, viz. the Line DE lies North 42 Degrees, East; then measure the Distance DE, which suppose to be 5 Chains.

Then in the first Column of the Table set the Letters DE, to represent the Side of the Field, and in the second Column set the Angle that the Line DE makes with the Meridian, whether North-easterly, North-westerly, South-easterly, or South-westerly, and put down the Quantity of the Angle; as here the Line DE lies North-easterly 42 Degrees, therefore set in the second Column N. E. 42; and in the third Column under [Distance] set down the Distance, or Length of the Line DE 5.00, viz. 5 Chains 00 Links.

Then have recourse to the Table of Northing, Southing, &c. and under 42 Degrees, and against 5 Chains, you find Northing 3.72, which place in the Column of Northing, and putting a Dash in your Column of Southing, in your Table place 3.35 in the Column of Easting, which is the Number found in the Column of Easting and Westing, and put a Dash in the Column of Westing, and you have done with the Side DE.

Remove to the Angle E and observe to F, and observe what Angle the Line EF makes with the Meridian, which suppose to be 90 Degrees; then is the Line EF due East from E to F, and measure the Distance 4 Chains 00 Links; this is all Easting, and no Northing, Westing, or Southing; put therefore E F in the first Column, and the Bearing (East) in the second Column, and the Distance 4 Chains in the third (the Column of Distance); and there being no Northing, Southing, nor Westing, dash these three Columns, and place 4 Chains 00 Links in the Column of Easting.

Having put down your Observation of the Line EF in the Table, remove your Instrument to F, and observe what Angle the Hedge FG makes with the Meridian, which suppose to be North 9 Degrees East, and Length FG 4 Chains, look in the Table under 9 Degrees, and against 4 Chains you find North-

ing 3.95, and Easting 0.63, place them in the Table in their respective Columns of Northing and Easting, as before directed.

But Note, *When the Hedge or Side to be measured, consists of Chains and odd Links, as it generally happens, take the Number of Links out of the Table at twice or three times, as shall be explained by the following Example.*

Proceed to G, and observe the Line GH, which let make an Angle with the Meridian of 69 Degrees from the South, viz. South 69 East, and the Distance be 5 Chains 56 Links.

First look in the Table, in the Column of 69 Degrees, and against 5 Chains you find in the first Column 4.7 which is 4.70 Easting (as the Bottom-title of the Column denotes, that being the Title to be regarded, because 69 Degrees is found at the Bottom) and in the next Column 1.8, viz. 180 Southing; put these down, as in the Margin.

Then for 56 Links; look first for 50 in the Table over 69 Degrees, and you have against that in the Table 46.7, and 17.9, which being but Links and Tenths, reject the Tenths and call it 0.18 Southing, and 0.47 Easting; and for the 6 Links find 2 Southing and 5 Easting (not regarding the Tenths of Links;) and these add together make Southing 2.0, and Easting 5.22, which set in their respective Columns in the Table.

S. E.	E.
1.80 :	4.70
0.18 :	0.47
0. 2 :	0. 5
2.00 :	5.22

Then suppose HI be South 36, East 7 Chains, and IK be S. 42 West, 4 Chains, and KL South 75 West, 10 Chains, and LD North 42 East, 7 Chains 42 Links, find all their Northing, Easting, and Westing, and put in the Table as before taught, and it will appear as follows:

<i>Sides of the Field.</i>	<i>Bearing.</i>	<i>Distance.</i>	<i>Northing</i>	<i>Southing</i>	<i>Easting</i>	<i>Westing</i>
	<i>Dist.</i>	<i>Ch. Lin.</i>	<i>Ch. Lin.</i>	<i>Ch. Lin.</i>	<i>Ch. Lin.</i>	<i>Ch. Lin.</i>
DE	NE 42	5 00	3 72	—	3 35	—
EF	East	4 00	—	—	4 00	—
FG	NE 9	4 00	3 95	—	0 63	—
GH	SE 69	5 56	2 00	1 99	5 21	—
HI	SE 36	7 00	—	5 66	4 12	—
IK	SW 42	4 00	—	2 97	—	2 68
KL	SW 75	10 00	—	2 59	—	9 66
LD	NW 42	7 50	5 55	—	—	4 97
			13 22	13 22	17 31	17 31

To prove whether the Work be right.

It is certain, when you are returned to the Angle from whence you set out, you cannot have either Northing, Southing, Easting, or Westing made good; therefore cast up each Column in your Table, and set the Sums under their respective Columns; and if the Easting and Westing be alike, also if the Northing and Southing be alike, it is probable the Work is right; as in this Example, the Northing and Southing are equal, viz. 13 Chains 22 Links: Also the Easting and Westing are alike, namely 17 Chains, 31 Links. From hence we conclude the Work to be right.

The Table of Northing, Southing, &c. to every three Degrees, may be sufficiently exact in all common Cases, but for the sake of those whose Curiosity, or whatsoever other Reason, induces them to be more exact, it is best done by trigonometrical Calculation, taught in the Fourth Chapter of the First Part of this Book, Page 74, and is done by the first Case of right-angled plain Triangles, where *the Hypothenuse, and an acute Angle is given to find the Legs.*

We will instance in the Side K L of this Field, found to lie from K to L South 75 Degrees, West 10 Chains 00 Links.

From L to the Meridian *ff*, let fall the Perpendicular L M, then is L M the Westing, and K M the Southing of the Line K L; that is, the Point L is as much Southward from K as the Line K M, and as much to the Westward of it as the Line L M.

In the Triangle K M L, there is the given Angle K 75 Deg. and L 15 Deg. and L K 10 Chains, or 1000 Links; to find K M the Southing, and M L the Westing, by *Case I.* aforesaid.

As Radius	—	—	—	—	10.00000
To Side K L 1000 Links	—	—	—	—	3.00000
So Sine of the Angle K — 75 Deg.	—	—	—	—	9.98494
To the Westing L M — 966 Links, which is 9 } Chains 66 Links	—	—	—	—	2.98494
As Rad.	—	—	—	—	10.00000
To Side L L 1000 Links	—	—	—	—	3.00000
To Sine of Ang L 15 Deg.	—	—	—	—	9.41299
To the Southing K M, 259 Links, which is 2 } Chains 59 Links, both agreeing with those found by the Tables.	—	—	—	—	2.41299

But in Case, as it often happens, that there be odd Links may take the Chains and Links all in one Sum, as if they were

a whole Number, and work with them, and the Answer will come out in Links.

Example of the Side GH of the Field, which is South 69 Degrees East, 5 Chains 56 Links.

Instead of 5 Chains 56 Links, set down 556, viz. Links, draw or imagine the Perpendicular HN to be let fall from H to the Meridian *ee*, it will fall in the Point N, and then is constituted the Triangle HNG; in which is given the Angle G 69 Deg. and its Complement H, 21 Deg. and the Side GH 556, to find GN the Southing, and NH the Easting.

As Radius	-	-	-	-	-	10.00000
To GH	-	-	-	-	-	2.74507
So Sine of G—69 Deg.	-	-	-	-	-	9.97015
To the Easting HN—520	-	-	-	-	-	2.71522

Again,

As Radius	-	-	-	-	-	10.00000
To GH—556	-	-	-	-	-	2.74507
So Sine of H—21 Deg.	-	-	-	-	-	9.55432
To the Southing GN—200	-	-	-	-	-	2.29939

The Northing, Southing, &c. of all the rest of the Sides, may be found by Calculation, by the same Method; but is needless to repeat them all, for all the Triangles are constructed by letting fall Perpendiculars from the several Angles to their respective Meridians; as PL or DQ is the Easting of the Line LD and PD, or LQ is the Northing of the same; and so in the rest.

How to plot this Field by the Help of the foregoing Table.

Suppose you would begin at D, the Westermost Angle in the Field, draw the Meridian *aa* towards the Left-hand Side of your Paper, and in that assume a Point, as D, to begin at, and proceed to E, &c.

Then look in the Table, you find DE in the first Column, against which you find [*Bearing*] NE 42, [*Distance*] 5 Chains 00 Links. But these first Columns are only of use to construct the Table: The four last Columns, viz. those of *Northing*, *Southing*, *Easting*, and *Westing*, are what we are now to have recourse to.

Against DE you find *Northing* 3.71, viz. 3 Chains 71 Links; therefore (having fixed your Scale to the Size of the Paper that you intend to draw the Plot upon) set 3 Chains 71 Links or

371 Links upon the Meridian *aa*, from D to Z; and from Z erect the Perpendicular ZE.

Then look in the Column of *Eastings*, and against DE you find 3.35. viz. 3 Chains 35 Links: Set 3 Chains 35 Links, or 335 Links off upon the Line or Perpendicular ZE from Z to E, and draw the Line DE.

Then through E draw the Meridian *bb*, parallel to *aa*; and from E raise the Perpendicular EF, and then looking in the Table against EF, you find *East* 4 Chains; and there being therefore no *Northings*, *Southings*, nor *Westings*, set 4 Chains 00 Links, or 400 Links from E to F.

Then through F draw the Meridian *dd*, parallel to *bb*; and, looking in the Table against FG, you find *Northings* 3.95, that is 3 Chains 95 Links, or 395 Links, and *Eastings* 0.63, viz. 0 Chains 63 Links; set the *Northings* 395 upon the Meridian *dd*, from F to *m*, and at *m* raise the Perpendicular *mG*; upon which set off the *Eastings* 63 Links from *m* to G, and draw the Line FG to represent the Side of the Field FG.

Now, having plotted from D to G, the next Line is GH; look therefore in the Table against GH, you find against it *Southings* 1.99, viz. 1 Chain 99 Links, or 199 Links, and *Eastings* 5.21, or 521 Links; therefore through G draw the Meridian *ee* parallel to *dd*, and from G set off the *Southings* 199; which, because it is *Southings*, set it downwards from G to N, and at N erect the Perpendicular NH, upon which, set off the *Eastings* 521, from N to H, and draw GH for the next Side of the Field.

Through H draw the Meridian *ff*, and look in the Table against HI (the Side next to be laid down) against it you find *Southings* 566, and *Eastings* 412; set the *Southings* 566, upon the Meridian downwards from H to *n*, and from *n* erect the Perpendicular *nI*, upon which set off the *Eastings* 412, from *n* to I, and draw HI for the Side or Hedge next to be drawn.

The next being IK, look in the Table for IK, and against it you find *Southings* 297, and *Westings* 268; therefore (having first drawn the Meridian *bb*, through the Point I, parallel to *ff*) set the *Southings* 297, from I to *q*; and from *q* erect the perpendicular *qK*; upon which set off the *Westings* 268, from *q* to K, and draw IK for the sixth Side.

Through K draw the Meridian *gg*, parallel to *bb*; and looking in the Table against KL (the Side next to be laid down) you find *Southings* 259; and *Westings* 966; set the *Southings* 259 upon the Meridian *gg* downwards, from K to M, and at M raise the Perpendicular ML, upon which set the *Westings* 966, from M to L, and draw KL, for the seventh Side.

Lastly,

Lastly, Look in the Table for L.D, and against it you find *North*ing 555, and *West*ing 497; set the *North*ing 550, upwards (because it is *North*ing) from L to Q, upon the Meridian *cc*, first drawn through L (parallel to *bb*) and from Q erect the Perpendicular QD, upon which set the *West*ing 497 from Q to D, and if (as I observed before) your *North*ing was equal to your *South*ing, and your *Easting* and *West*ing also equal: And if you have plotted exactly according to the Table, the Perpendicular QD shall bring you exactly to the Place departed from; and the Line LD being drawn, the Scheme DEFGHIKL shall exactly represent the Field to be plotted, and may afterwards be reduced or divided into Triangles, in order to be measured or divided according to the Directions in the foregoing Part of this Book.

S E C T. II.

Of the Use of the Table of Logarithms.

THIS Table needs not much Explanation, because the Title of the Columns explains what they are for: In each Page are eight Columns, the first, third, fifth and seventh being the Number, and the second, fourth, sixth and eighth the Logarithms of their respective Numbers standing against them.

Example. Suppose I would find the Logarithm of 348, I look in the first, third, fifth or seventh Column under the Title *N*. and I find in the first Column the said Number 348, and find against that 2.541579, the Logarithm of 348 required.

It is to be observed, as a general Rule in Logarithms, that the Index or first Figure of the Logarithm, towards the Left-hand, is always one less than the Number of Figures in the Number proposed; thus in the foregoing Number 348, because it consists of three Figures, or Places, the Index or first Figure is 2, viz. 1 less than the Number 3, &c.

Hence, by having the Index of any Logarithm in the fourth Column of a Question, you may determine how many Figures there are in the Number that it represents; for if the Index be 2, there are 3 Figures in the Number required, viz. if the Logarithm be 2.564666, it is certain there are three Figures in the Number, which, by looking in the Table, you will find is 367.

But suppose in the Answer to a Question, the Logarithm required is 3.523746, and I desire to know the Number belonging to it, I look in the Table, but finding no such Index as 3, I look for 2.523746, and against that I find Number 334; but considering that my Index being 3, I must have four Figures in the Number required, I add a Cypher to 334 on the Right-hand, and then it is 3340, which is the Number belonging to the Logarithm 2.523746.

S E C T. III.

The Use of the Table of Sines and Tangents.

THE Table is disposed in half Pages, so that in the first Page you have 00 Degrees and 1 Degree. In the second 2 Degrees and 3 Degrees. In the third 4 Degrees and 5 Degrees, &c. and these go on to 45 Degrees; and then return backward along the Bottom, from 45 to 90 Degrees; so that in the half Page, where there is 5 Degrees at the Top, there is 84 at the Bottom; and in all other Cases, whatsoever Degree is at the Top, the Degree at the Bottom added to it makes 89 Degrees, which, with the Minutes on each Side added together, making one Degree, makes always in all 90 Degrees; and hence the Sine of any Degree is the Sine Complement of that Degree subtracted from 90; and therefore the Use of the Table is,

If it is required to find the Sine or Tangent of any Degree or Minute under 45 Degrees, find the Degree at the Top of the half Page, and the Minute on the Left-hand, and against that in the common Angle under the respective Title, whether Sine or Tangent, you have the figures required.

Example. I desire to find the Sine of 7 Deg. 40 Min. Look for 7 Deg. at the Top of the half Page (because it is less than 45° and under the Title [*Sine*] and against 40 Min. you find 9.125187, which is the artificial or logarithmal Sine of 7 Deg. 40 Min required.

For the Tangent of 7 Deg. 40 Min. look at the Top of the half Page for 7, as aforesaid, and on the Left-hand for 40 Min. and under the Title [*Tangent*] you find 9.129087, the Tangent of 7 Deg. 40 Min. required.

But if it is required to find the Sine or Tangent of any Degree and Minute above 45 Degrees, look at the Bottom of the half

half Pages for the Degrees, and upon the Right-hand for the Minutes, and in the common Angle you have the Sine or Tangent required, observing always to take the Title [*Sine* or *Tangent*] at the Bottom of the half Page, and over it, against the Minute proposed (found in the Right-hand Column of the Table, as above directed) you have the Sine or Tangent required.

Example. I desire to know the Sine and Tangent of 51 Deg. 30 Min. Look for 51 Deg. at the Bottom of the half Pages, and over it, and against 30 Min. on the Right-hand, find 9.893544, and over the Title Tangent, 10.099395, the Sine and Tangent required.

But sometimes the Sine Complement, or Tangent Complement of an Angle is required, in which this general Rule may be observed; that the Sine Complement of any Angle is the Sine of the Complement of that Angle to 90; thus the Sine Complement of 30 is the Sine of 60; and the Sine Complement of 55 is the Sine of 35, &c. Observe the same in the Tangents and Tangent Complements.

This I think a sufficient Direction for the Use of the Table of Sines and Tangents; but lest any Thing should be wanting, I shall only add a Method, how any Figure, or Row of Figures, in any Sine or Tangent, if defaced or obliterated, may be recovered; which is thus:

Suppose a Tangent is misprinted or obliterated, and you can see in the Table the Sine and Sine Complement, the Rule is,

Subtract the Sine Complement from the Sine added to Radius, the Remainder is the Tangent of the same Degree and Minute.

Example. Suppose the Tangent of 54.20 were by some Accident obliterated, but the Sine and Sine Complement are visible,

From the Sine of 54.20 added to Radius	-	19.909782
Subtract the Sine Comp. of 54.20	-	9.765720
The Remainder is	-	<u>10.144062</u>

This 10.144062 is the Tangent of 54.20 required.

2. Suppose a Sine is misprinted or defaced, and the Sine Complement and Tangent appear legible, the Rule is,

Add the Tangent and Sine Complement together, and from their Sum cut off 1 or Unit, towards the Left-hand, and the remaining Figures shall be the Sine required.

Example. I desire to know the Sine of 37.50, having the Sine Complement and Tangent given.

To the Tangent 37.50	-	-	-	9.890204
Add Sine Complement 37.50	.	-	-	9.897516
				19.787720

The Sum is 19.787720; from which cut off the 1 towards the Left-hand, the Remainder is 9.787720, the Sum of 37.50 required.

Note, What is said of the Sine and Tangent, or of finding them, may be said of the Sine and Tangent Complement, the Method being the same, the Sine Complement of any Degree being only the Sine of its Complement to 90, as above observed.

3. Suppose the Tangent, as also both the Columns of Sines (*viz.* Sine and Sine Complement,) were defaced or not legible, yet you may, by the Help of the Tangent only, recover the Tangent Complement, and *vice versa*; and the Rule is,

Subtract the Tangent of any Degree and Minute from twice Radius, the Remainder is the Tangent Complement of the same Degree and Minute; and consequently subtract the Tangent Complement of any Degree and Minute from twice Radius, the Remainder is the Tangent of the same.

Example. I desire to know the Tangent of 39.45.

From twice Radius	-	-	-	20.000000
Subtract the Tangent Com.	-	39	45	- 10.080038
				9.919962

Which 9.919962 is the Tangent of 39.45 required.

But, 4. The Secants may also be deduced from the Sines and Tangents; and this may be of great Use, when a Table of Sines and Tangents only can be had: But a Secant is upon some particular Occasion required, and that may be done from either of these two Proportions.

1. As Sine Comp. to Radius, so Radius to Secant.
2. As Sine to Radius, so Tangent to Secant.

Hence in the first, the Rule is,

Subtract the Sine Complement of any Degree from twice Radius, the Remainder is the Secant of that Degree.

Example.

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Example. I desire to know the Secant of 35.40.

From twice Radius	-	-	-	20.000000
Subtract Sine Comp. 35.40	-	-	-	9.909782
				10.090218
Remains	-	-	-	10.090218

Which 10.090218 is the Secant of 35.40 required.

The second Proportion is,
As Sine to Radius, so Tangent to Secant.

Hence the Rule is,

Subtract the Sine of any Degree from the Tangent of the same, with Radius added to it, the Remainder is the Secant of the same.

Example. In the last Instance for the Secant of 35.40.

From the Tangent of 35.40 with	}	-	19.855937
Radius added to it, viz.	}	-	9.765720
Subtract the Sine of 35.40	-	-	10.090217
There remains	-	-	10.090217

Which 10.090217 is the Secant of 35.40, differing only an Unit in the last Place from the former, which is inconsiderable, and may be sufficiently accounted for.

For a further Demonstration of these Proportions, see my Supplement to the last Impression of *Barrow's Euclid*, to be had at Mr. Rivington's in St. Paul's Church-Yard.

C H A P. IX.

S E C T. I.

To reduce any given Plot of any Survey of a County, Estate, or Farm, from any Scale to a Lesser.

SUPPOSE I would reduce the Inclosure, ABCDEX, to a Scale of half the Size, and that the Plan should possess just half the Space it now possesses in Length, and also half in Breadth, which would reduce it to one Fourth of the Superficies. Fig. 81.

P 4

First,

First, Inclose it with the Square $F G H I$, and divide $G H$ into any number of equal Parts, as in the Points $a b c$, &c. and do the same by the Side $F I$, and draw the Lines $a a$, $b b$, &c.

Then set the same Distance $F a$, $a b$, &c. which are equal, from F to g , h , i , &c. and also from I upon the Line $I H$, for the Points g , h , i , and draw the Lines $g g$, $h h$, $i i$, &c. to cross the Lines $a a$, $b b$, &c. at right Angles; then shall the whole Square $F G H I$ be all divided into little Squares.

This done, draw another Square $K L M N$, whose Sides $K L$ and $M N$ let be just half as long as the Sides $F G$ and $H I$ *Fig. 82.* of *Fig. 81*, and the Sides $K N$ and $L M$, just half as long as the Sides $F I$ and $G H$ of the said Figure; Then divide $K N$ and also $L M$ into just the same Number of equal Parts as $F I$ and $G H$, (*viz.* 7 in this Example;) then will the Divisions be just half the Length of the Divisions in *Fig. 81*. Mark these also with $a b c$, &c. Set also the same Divisions from K towards L , upon the Line $K L$, and from N towards M , upon the Line $N M$, and mark them $g h i$, &c. And by the Help of these Divisions, make the lesser Square $K L M N$, *Fig. 82.* into just as many small Squares, by drawing the Lines $a a$, $b b$, $c c$, &c. to cross the Lines $g g$, $h h$, &c. at right Angles: then have you just as many small Squares in the lesser of the great Squares, as in the bigger.

Then (suppose you begin at the Angle A) observe where, in the Original or larger Draught, the Angle A falls, which you find towards the Left-hand Side of the Bottom of the Square, marked at the Top $k l$; therefore in the same Square [$k l$] in the lesser Draught, and in the same Proportion towards the Left-hand Side of it, and towards the Bottom, assume the Point for A ; and from thence observing, that the Line $A F$ crosses the Bottom Line of that Square, two-fifths from the Left-hand Bottom Corner, towards the Right-hand, cross it so in your new Draught—Again, it crosses the Right-hand Bottom Corner of the second Square below $k l$, therefore describe it so in the new Draught; and so carry it on to cross the Lines $m m$ and $n n$, in the same Proportion above the Line $c c$, as in the Original, and then determine the Point B in the third Square, below $n o$, and in the same Proportion towards the Right-hand and the bottom of it, in the same Draught, as you see in the Original.

Proceed in the small Form with the Sides $B C$, $C D$, $D E$, $E X$, and $X A$; and also with the Partitions $H I$, $F G$, $G K$, and $L M$; then have you the Plot laid down to a small Scale, yet bearing all the Proportions in itself exactly as the great one: And

And as the Divisions in the latter were but just as long as the Divisions in the former, the Scale of one Chain, ten Chains, one Mile, &c. is just half as long also in the latter as in the former.

Note, The Divisions in the Sides of one Square, being made just half as long as the Divisions in the other, is, because in this Example the Plot was proposed to be reduced to half the Scale; but by the same Rule any Plot may be reduced to any Proportion: As for Instance.

Example. Let it be required to reduce any large Map of, suppose four Sheets of an uncommon Size and unknown Dimensions, to one Sheet of Paper much less, or handsomely to answer the Size of any Book proposed that is to be printed in, or Place that it is intended to be fixed exactly to.

First, draw the Out-lines of the Paper you are to reduce your Map to, or to draw the new one upon, in a handsome Form, and in an exact Square or Parallelogram, as your Paper will admit, and the Shape of the Draught require.

Secondly, Inclose all the Map that is to be reduced in a Square (if the Place you are to reduce it to be a Square) or a Parallelogram; but if it be a Parallelogram, in which you enclose the large original Draught, bear the same Proportion to the Sides of it, as the Ends of the lesser bear to its Sides, always taking Care to include the longest Dimensions of the Map to be reduced, and, if its Form is not proportionable to the Paper you are to reduce it to, leave the rest of that Part of the small draught for the Title or other Imbellishments, or to express what contiguous County, Forest, or Estate, that blank Space is Part of.

Having thus drawn the Out-lines of both, divide the Top and Bottom Out-lines of the large Map, into any competent Number of equal Parts (the more, the more exact :) and keeping your Compasses carefully at that Extent, begin at the Top of each Side, and set as many equal Parts as each Side will admit (It is no matter whether it ends at the Bottom in an equal Part;) then draw Lines from each Division at the Top to its respective Division at the Bottom, and cross these Lines at Right-angles, by Lines drawn cross the Map, from each Division on one Side, to its respective Division on the other Side; then will your Map be all full of little Squares.

Then divide the Top and Bottom Lines of the small Draught into exactly as many equal Parts as the said Lines in the Original (and then the Divisions will be less than the Original, accord-

according to the intended Difference in the Bigness of the Maps;) and keeping the Compasses exactly at that Extent, set the same Divisions down each Side, and by the Help of them rule it also into Squares; and then transferring every Thing remarkable, whether Roads, Rivers, Hedges, Churches, or whatever else is worth Notice, from the Original to the small Draught, placing every thing in, or tracing it through the respective Squares, from one to the other, and in a due Proportion, and you will have exactly the greater Map in the lesser, but to a smaller Scale, in Proportion to the Different Lengths and Breadths of the two Draughts; and the Method of tracing Roads, Rivers, &c. is the same as above expressed, in the Use of Fig. 81 and 82.

Note, A Map or Draught may be enlarged by the same Method inverted, viz. ruling the white Paper with Divisions and Squares bigger than the Original, in Proportion to the Difference of the Scales; but it is not always adviseable, because there is often Room in a great Map to express some remarkable Things that cannot be contained in a small one.

S E C T. II.

How to reduce a Survey of an Estate, Farm, &c. from any Scale to a lesser, according to any given Proportion, only by the Help of a Sector, and a Pair of Compasses.

Fig. 81 I Shall instance in the foregoing Field, ABCDEX, to be reduced to a Scale of half the Quantity, or as it is said,

As 2 to 1, or as 10 to 5, &c.

Having first determined which is North, South, East, and West, of your original Draught, and (as Conveniency will permit) put the North Part towards the Top of the Book; draw a Line at pleasure, either through the Plot, or at some small Distance from one Side of it; we shall here chuse the

latter, and draw the Line FI at some Distance from the Side of it, and let it be a North and South Line, because then it will serve for one of the Boundary Lines, it being on the West Side of the Draught, and in it assume two Points at pleasure, at a competent Distance asunder, as the Points F and I.

On the West, or Left-hand Side of your small Draught, draw also the Line FI; and then taking the Extent FI in the Original, in your Compasses, open the Sector, till the Compasses (kept at the same Extent) will reach *Fig. 82.* from 10 on one Leg, to 10 on the other, on the Line of Lines, or equal Parts on the Sector; then, keeping the Sector at the same Opening, set one Foot of the Compasses in 5 of the said Line upon one Leg of the Sector, and bring the other Point into 5, on the other Leg of the Sector, and with that Extent, and one Foot near the End of the Line FI of the lesser Map, extend the other Foot towards the other End of the Line, and with the two Points of the Compasses make two Marks, of which let the uppermost be F, and the lowest I, as in the Draught.

Note, The Reason of placing the Compasses upon 10, and then upon 5, is because the Map is to be reduced in Proportion, as 10 to 5; and although as 2 to 1 is the same Proportion, yet the small Divisions are not so readily and exactly found in Numbers near the Center of the Sector, as in those more remote; nor will the Sector take such a large Extent (if required) in the former, as in the latter.

Then extend the Compasses from the Point I in the Original, to the Point E; and keeping the Compasses at that Extent, open the Sector, till the two Points fall in 10 : 10, of the Line of Lines; and, keeping the Sector so, set one Foot of the Compasses in 5 on one Leg, and bring the other Point into 5 on the other Leg; and, with that Extent, and one Foot in I of the lesser Draught, sweeping the Arch E.

Set one Foot of the Compasses in F, in the Original, and extend the other to E, and, with that Extent, open the Sector, till the Compasses will stand in 10 : 10, of the Line of Lines, with the Sector at that Opening, and set one Foot of the Compasses in 5, and contract the Compasses till the other Foot will fall in 5 on the other Leg of the Sector; and, with that Extent, and one Foot in F of the new Draught, cross the Arch E, in E.

Next

Next, extend from I to the Angle D in the Original, and, as before, setting that Extent from 10 to 10, keep the Sector so, and contract the Compasses, till they only extend from 5 to 5 on the Sector; and, with that Extent, and one Foot in I of the small Draught, sweep the Arch D: Likewise extend from F to D, and, with that Extent open the Sector, till the Points fall in 10 : 10, of the Sector, and, keeping it so, extend from 5 to 5 on the Sector, and with that Extent, and one Foot in F, cross the Arch D in D.

Note, That if the Extent I D, or any other, be so short, that though the Sector be quite shut, yet the two 10's at the Ends of it cannot be brought near enough together, you may instead of 10 : 10, 5 : 5, use 6 : 6, and 3 : 3 or 2 : 2, and 1 : 1, or any other two Numbers that bear the same Proportion as 10 to 5.

Proceed in the same Manner to find all the rest of the Angles or Points required, as I, C, K, B, F, A, X, L, as also HG and M. And when there are only Right-lines, draw them between their respective Points, as AX, XE, ED, DC, and CB; also LH, GH, FG, and GK, they are the Lines required.

But where there is a Curve, as AFB, either find the several Points in it, *r, s, t*, by the foregoing Directions, and carry the Curve through them from A to B, or else draw the Right-line AB, and from Z (the Point in the Right-line, where the Curve is farthest from it) to (the Point in the Curve farthest from the Line AB) and make the Off-set ZF in the new Draught, half the Length of FZ in the Original, and F is the Point through which to carry the Curve, and is sufficiently exact in most Cases; but the more Points are found where a Curve is irregular, the more exact the Work is likely to be: And, indeed, this Method of reducing is so expeditious, that the whole Work may be performed in less Time than these Directions can be written, or the two Maps ruled into Squares; but in Land-maps, or other Plans that are very full of Work, it is better to use Squares; though this Method is much more expeditious for Surveys, or any Plan that consists mostly of Right-lines.

But there is another Way to reduce a Plot when only the Out-lines are required, as it often happens in a Field, Wood, or more particularly in a Chase, Common, or Forest, where

only

only the Out-lines, to represent the Shape of the Whole, are required; and the Method is,

Draw Lines from any Angle to all the rest, and divide one of the Sides next to the said Angle, into two Parts, in the same Proportion as the Scale is to be reduced, and from that Point draw Lines parallel to each Side, till they touch the next Line drawn from the first Angle, &c. These Lines shall inclose the Shape of the Field, Forest, &c. in its just Form, and reduced to the proposed Scale; but this Way of reducing requires all the Sides or Boundaries to be Right-lines.

Example. Let the Field or Plot ABCDE be to be reduced to a Scale in Proportion to itself, as 3 to 5. or to three Fifths of the Scale it is now in. Fig. 83.

From any Angle, as A, draw Lines to all the rest, as AC and AD (for AB and AE are Out-lines, and drawn before) then, because the reduced Draught is to be a Scale three Fifths as large as the Original, divide either of the Sides adjacent to the Angle A (from whence the Lines were drawn) into five equal Parts, and set three of the said Parts from A to F.

Draw FG parallel to BC, and continue it till it cut AC in G. Then from G draw GH parallel to CD, to cut AD in H: And lastly, from H draw HI parallel to DE to cut AE in I; then is AFGHI, the true Shape of the Field ABCDE, and reduced to a Scale in Proportion to the said Field ABCDE, as 3 to 5.

The Proof of this is reduced from *Theor.* 15. for it is there proved, that as AF to AB, so AG to AC; and, as AG to AC, so AH to AD. Also, as AH to AD, so AI to AE, because FG is parallel to BC, GH to CD, and HI to DE; and therefore each Side FG, GH, HI, and IA, are in Proportion to their respective Sides of the original Plot, to which they are severally parallel, as 3 to 5; as was proposed.

C H A P. XII.

How to reduce a Plan of an Estate, Garden, &c. to a Prospect, as it will appear from any Point conveniently situated so sufficiently near the Estate, and of such a competent Altitude above the Horizon, that from thence all the Plan or Estate may be seen, whether it be from the Top of a House or Steeple, or from any imaginary Point assigned.

IT is very evident, that the farther distant any Object is, the lesser Angle it makes at the Eye; and hence in any Plan with parallel Sides, whether Square, Parallelogram, &c. although the End farthest remote is of the same Dimension with that End next the Eye, yet it shews less, and the containing Sides seem to incline, as they recede from the Eye, and that in a kind of reverse Proportion to that of a projected natural Tangent Line, whose Radius is the Perpendicular Height of the Eye above the horizontal Line of the Plane.

For, whereas the Tangents are projected by transferring the equal Divisions of the Quadrant, or Quarter of a Circle, to the Tangent Line, where they become unequal, and increase as they proceed upward: This is done transferring the equal Divisions of the Tangent Line (by Lines drawn to the Center) to the Circle, where they become unequal.

Example. Draw the Quadrant BC upon the Center A, and set, suppose, 80 Degrees of the Chords from B to *d*, and draw the Tangent Line BD, perpendicular to AB, and through *d* draw Ad to cut BD in D; then is BD the Tangent of 80 Degreee to Radius AB.

Then (instead of dividing the Arch Bd into four equal Parts, which would project the unequal Divisions on the Tangent Line BD to every 20 Deg.) divide the Line BD into any Number of equal Parts, as suppose 4, in the Points *g*, *i*, *l*, and draw the Lines Ag, Ai and Al, and they divide the Quadrant unequally in the Points *o*, *q*, and *s*, and consequently, make unequal Angles at the Center A, decreasing according to their Remoteness from the Point B; so that though the Space Dl be equal to the Space Bg, yet by reason of the Remoteness

of the Space Dl from the Center A , compared with the Distance of the Space Bg from the same, the latter possesseth the Arch Bt of the Quadrant, or appeareth to make at the Center A the Angle $B A f$, about 35 Degrees, whereas the Space Dm possesseth but the Space en the same Quadrant, or makes at A only the Angle $D A m$, viz. not 2 Degrees.

From hence it is easy to conceive, in what Proportion Objects of equal Magnitude are increased or diminished, according to their Distance from the Eye, and their Position with respect to it. And this I think sufficient to introduce so much of Perspective, as is necessary for turning a Plan into a Prospect, by the Help of a general Notion of Trigonometry.

Example. Let the Parallelogram $AB C D E F$, consisting of Gardens, Orchards, and other Divisions, as are expressed, be to be reduced to a Prospect from the Tower or Steeple at A , supposing the Height of the Tower *Fig. 85.* to be 96 Feet, the Length of the whole Parallelogram AD (equal to BC , or EF) to be 576 Feet, and the Breadth BF , equal to CE , 180 Feet.

Here is given,

First, the Base AD , the Length of the Parallelogram, 576 Feet.

Secondly, The Perpendicular, the Height of the Tower 96 Feet.

To find the Hypothenuse or Distance from the Eye at the Top of the Tower to the Point D , the remote End of the middle Line of the Parallelogram.

As the Line AD	-	-	576	-	-	2.76042
To Radius	-	-	-	-	-	10.00000
So the Height of the Tower	-	96	-	-	-	1.98227
To the Tangent of the Angle which the Tower will make at the Eye at D	}		9.28	-	-	9.22185

Then,

As the Sine of the Angle D	-	9.28	-	-	9.21609
To the Height of the Tower	-	96	-	-	1.98227
So Radius	-	-	-	-	10.00000
To the Distance from the Eye	}	584	-	-	2.76618
to the Point D					

Again,

Again,			
As the Distance of the Eye	} 584	-	2.76618
from the Point D			
To Radius	-	-	10.00000
So half the Breadth of the Pa-	} 90	-	1.95424
rallelogram DE			

To the Tangent of the Angle	} 8.46	-	9.18806
that the Line DE makes at			
the Eye at the Top of the			
Tower			

And,			
As the Height of the Tower	- 96	-	1.98227
To Radius	-	-	10.00000
So AF equal to DE	- 90	-	1.95424

To the Tangent of the Angle	} 43.9	-	9.97197
that AF makes at the Eye			
at the Top of the Tower			

Lastly,			
As Tangent of	- 43.9	-	9.97195
To Tangent of	- 8.46	-	9.18812
So half the Breadth of the Pa-	} 90	-	1.95424
rallelogram AF			

To the Appearance of DE in	} 14.8	-	1.17043
Perspective			

This 14.8, viz. $14\frac{3}{10}$ is the Line DE, as it appears in Perspective from the Eye at the Top of the House or Tower at A; as appears thus,

The Line AF is seen but at the Distance 96, the Height of the Tower; but the Side DE is seen at the Distance of the Point D, from the Top of the Tower 584—Therefore,

Draw the Line AD 584, and upon that set-off OA 96, from O to A. Through A draw BF; and through D draw CE, perpendicular to OAD, and set off 90, (half the Breadth of the Parallelogram) from A, both Ways, to B and F; and from D, both Ways to C and E; it is evident that although the Space DE be just equal to AF, yet to the Eye, placed at O, it possesses no more Space,

nor

nor appears any bigger than the Space A G, and is to be represented so contracted in its perspective Appearance, in Proportion to its Distance from the Eye.

The equal Divisions upon the Line A D upon the Plan (Fig. 85.) are also to be diminished, as their Distance from the Eye increases, as I have proved before (from Fig. 84.) that equal Divisions upon a Tangent-line make unequal Angles at the Eye, in Proportion to their unequal Distances from it; and if the equal Divisions of the Line A D (Fig. 85.) be reduced to unequal ones, according to the different Angles that they make at the Eye, (as in Fig. 84.) the equal Parts of the Line A D will diminish as they appear farther from the Eye, as the Arches B s, s q, q o, and o d (Fig. 84.) decrease as they proceed from B towards these unequal Arches, being subtended by the equal Divisions of the Tangent-line B D, as divided in g, i, and D, as is represented in Fig. 84.

Having thus reduced the right Squares, Fig. 85, to the Oblique ones, Fig. 86, transfer the Hedges or Divisions, also Houses, Wind-mills, and whatsoever is remarkable, from the several Squares in the Plan, to their respective Squares in the Prospect, and the last Draught will appear as the real Plan would appear, if the Eye was fixed at the Place proposed.

Note, It is common in Perspective, to suppose the Eye elevated to a Point of an inaccessible Altitude above the Horizon, when the Magnitude or Beauty of the Prospect requires it; but in that Case the Rules are the same, although there is not an Opportunity for an ocular Proof of the Work.

There might be Tables adapted to this Method of performing Perspective, whose Construction would be (as in Fig. 84.) by (instead of dividing the Quadrant equally, and the Tangent Line unequally) dividing the Tangent-line equally, and the Arch unequally; and then, as in the Tangents, that of 90 Degrees is infinite, so here the Divisions upon the Arch decrease infinitely; so that no less than an infinite Number of Divisions can reach the Arch of 90 Degrees.

Example. Divide the natural Tangent of 80 Degrees (which is 56712816) into (suppose) 8 equal Parts, one of these Parts is 7089102, and two of them is 14178204, &c. as in the Table below, and the first of these Numbers is the Natural Tangent of 35 Deg. 20 Min. and the second is the Natural Tangent of 54 Deg. 48 Min. &c. as it is also expressed in the Table, where the eighth Number is the natural Tangent of 80 Degrees: And

Q

if

if we still continually add the first Number 7089102 to the last, then the next will be the Tangent of 81 Deg. 5 Min. &c. And thus, although the first equal Division advance to 35 Deg. 20 Min. yet the next Addition above 80 Degrees comes but to 81 Deg. 6 Min. and so decreasing infinitely, but never reaching to 90 Degrees.

<i>P.</i>	<i>Tangent.</i>	<i>Deg.</i>	<i>Min.</i>	<i>Total Min.</i>
1	7089102	35	— 20	2120
2	14178204	54	— 48	3288
3	21267306	64	— 49	3889
4	28356408	70	— 35	4235
5	35445510	74	— 15	4455
6	42534612	76	— 46	4606
7	49623714	78	— 36	4716
8	56712816	80	— 00	4800
9	63801918	81	— 6	4866

The first Column of this Table is the Number of equal Parts, into which the Tangent-line is supposed to be divided, the Tangent of 80 Degrees being the eighth equal Division.

The second Column is the equal Divisions of the Natural Tangent-line.

The third is the Degrees and Minutes corresponding to its respective Natural Tangent, against which it stands.

The fourth is the same Degrees and Minutes reduced all into Minutes, and are the Number of Minutes comprehended between the Beginning of the Quadrant at B, and the Line drawn from A to its respective Division on the Tangent-line.

The Use of the Table is this.

Suppose BD was a Horizontal Plan to be reduced to a Prospect, and BA was a Tower, from the Top of which at A, you are to take the Prospect of the Plan BD.

Let the Length of the Plan BD be in such Proportion to the Height of the Tower BA, that the perpendicular Tower may make with the Visual Line AD, an Angle of 80 Degrees.

Suppose

Suppose the Length of the Plan B D be 120 Yards, or 360 Feet, divide that into 4 equal Parts in *g*, *i*, and *l*; then will each Part be 90 Feet, and will measure equal upon the Spot, but will diminish in Perspective when seen from the Point A; and I would know by Calculation in what Proportion they decrease as they proceed towards D; there is this

General R U L E.

As the Number of Minutes contained in the Degrees subtended by the Tangent-line (the Height of the Eye being Radius)

To the real Length of the Base, or Tangent-line which here is B D 360)

So the Number of Minutes contained in that Quantity of the Arch, which is cut by the Division proposed (which here is the Arch B *s*, determined by the Line A *g*, which is drawn from the Center A, to the Point *g*, which is one Fourth of the Tangent-line) that is, 3288,

To the Number of Parts that will express B *g* in Perspective, of such Parts as the whole Plan B D contains 360 viz. Feet.

Hence in this Case,

		Co. Ar.
As the Minutes contained in 80 Degrees	— — — — — }	4800 - 6.31876
To the Length of the Plan B D	—	360 - 2.55630
So the Minutes in $\frac{1}{4}$ of the Tangent of 80 Degrees	— — — — — }	3288 - 3.51693
To	— — — — —	247 - 2.39199

And,

		Co. Ar.
As	— — — — —	4800 - 6.31876
To the Length of the Plan B D	—	360 - 2.55630
So the Minutes in $\frac{1}{2}$ the Tangent of 80	— — — — —	4235 - 3.62685
To	— — — — —	318 - 2.50191

Q 2

Again,

Again,

			Co. Ar.
As	—	—	4800 - 6.31876
To the Length of the Plan BD	—	360	- 2.55630
So the Min. in $\frac{1}{4}$ of the Tangent of 80		4606	- 3.66332
To	—	—	345 - 2.53838

The Height of the Eye above the Horizon may be thus determined.

As Radius	—	—	10.00000
To BD	—	360	2.55630
So Tangent-angle D	1.00	—	9.24632

To the Height of the Eye BA 63.5 — 1.80262

The Height of the Eye is 63.5 Feet, or 63 Feet and a Half; and the Line BD is 360 Feet.

Fig. 84. Draw the Line BD, and upon that set 360 from B to D, and at B erect the Perpendicular BA 63.5.

Then, according to the first Operation, set 247 from B to *g*; and, according to the second Operation, set 318 from B to *i*; and also, by the third, set 345 from B to *l*; then are the equal Divisions B *g*, *gi*, *il*, and *lD*, *Fig. 84.* truly represented in Perspective, by the unequal Divisions B *g*, *gi*, *il*, and *lD*, *Fig. 84.* and all the Parts will diminish in the same Proportion as the Divisions of that Line do, according to their Distance from the Eye; and doubtless this Method might be improved; but this I think to be a sufficient Hint, as to the Nature of Perspective in general, and reducing a Plan to a Prospect in particular.

I think it may not be improper to observe, that there is a Property something remarkable in the Tangents, expressed in the second Column of the Table, *viz.*

The two Figures towards the Right-hand in each Tangent, taken together and numbered downward, advance successively by two, *viz.* 02, 04, 06, 08, 10, &c.

The third Figures from the Right-hand advance one in each Tangent successively, beginning at the Top, and continuing downwards, as 1, 2, 3, &c.

The

The fourth from the Right-hand in each is just the Reverse of the third, beginning at the Bottom, and encreasing by 1 successively upwards.

The fifth begins at the Bottom at 0 and increases to 8, each Excess being 1 successively; and

The sixth is again the Reverse of the fifth, beginning at the Top at 0, and advancing 1 each Time, counts downwards to 8.

This Remark may be an Amusement, but of what further Use it may be, or what Improvements it is capable of, is beside my present Purpose.

C H A P. XIII.

Of County Surveying, in which is included Surveying Roads, Rivers, &c.

First. **A**S a Foundation to the Whole, if an Opportunity offers, chuse two Eminences, as Churches, Castles, Gentlemens Seats, or Houses, &c. from each of which you can see the other, and as many other Churches, or other Eminences as possible, the more the better, you can see from both, and also the farther asunder your Stations are, it is the better; then finding first the horizontal Distance between your two Stations, find the several Distances from each Station to every one of the other Objects, &c. as taught in *Chap. III. Page 125.* so have you the horizontal Distance between each two of the said Eminences; which though it commonly falls short of the Distance found by measuring, yet it is the truest, and will assist in correcting the rest, provided the true horizontal Distance between the two Stations be found; in order to which, great Care must be taken to reduce all the Ascents, Descents, and Curvity of the Road to a true horizontal Line, as is taught in *Chap. VI. Page 87, and 145.*

Secondly, If some Part of the County is bounded by the Sea, as the Case is in many Counties of the Island, as in *Yorkshire, Suffolk, Cornwall, &c.* take care, first, to be very exact in surveying all the Sea-coasts, with the entrance of the Rivers, &c. (for the Rivers themselves may more properly be called In-land Surveying) and make a fair Draught thereof, inserting every thing

thing remarkable that occurs, as Towns, Houses, Churches, Wind-mills, Trees, &c. in their Places; and by the Help of a Boat, or otherwise, set all Rocks and Sands with your Compass by observing therefrom any two Objects in your Draught, that by means of their Bearings from those two Objects, you may truly lay them down, which in the Draught will demonstrate the best Way for Shipping to come in, and where to place your Buoys; and also the Draught will direct you where to place Sea-marks to bring your ships in best, &c. and be of considerable Use otherwise: However, take this Example; where suppose this Figure to be some Part of the Sea-coast to be surveyed; then make as fair a Draught as you can, drawing the River's Mouth as far up as Occasion shall require, and to note down every Thing remarkable; but the Rocks and Sands which you thus take in a Boat, traverse up and down till you find the Sands; as suppose A is the first you light on, you being at *a* set the Red-house and Tree with your Compass, that is find their Bearing from you: Accordingly let the Red-house lie off of you S. W. 86 Degrees, and the Tree S. E. 6 Degrees; then to plot the same you must consider, that as the Red-house did lie S. W. 86 Degrees, that you at that Time was N. E. of the House as many Degrees, and by a like Reasoning you was N. W. 6 Degrees from the Tree; so that from the Red-house in your Draught draw a Line that is N. E. 86 Degrees, and from the Tree draw a Line that shall be N. W. 6 Degrees; for where these two Lines meet each other, will be the very Place *a* of the Sand A where you made your Observation. Now, as we will suppose *a* to be the Beginning of the Sand A; row along the same till you find a considerable Bending (which you may do, founding as you go) where make two Observations, also, repeating your Observations, as the Matter requires, till you are round the Sand: Then lay them down as you did the first Observation; whereby you will have pointed out your several Places of Observation; through which Points draw a pricked Line to inclose the Sand. Thus may you proceed from

Fig. 87. Sand to Sand, till you have described them all; and as it is obvious that you may effect the Thing, as well by observing other Objects, as the Red house and Tree.

Having described the Sands in your Draught, observe that there are two Passages for the Ships to come in by, *viz.* one between the Sands B and C, and the other between the Sands A and C, and as drawing a Line through the Middle of the same, then the Passage does cut both the Church and Wind-mill:

You

You must sail in a Right-line with those two Objects, till you are directly against the River's Mouth, where you may turn off and sail up it; but the better to direct your turning off, lest you touch upon the Sands B and A, cause the Beacon G to be put up in a Right-line with the Middle of the River and the Red-house; then sail in a Right-line with the Church and Wind-mill, till you be also in a Right-line with the Red-house, and the Beacon G, which is the very Place for turning off; whence you are to sail in a Right line with the House and Beacon up the River: But as to the Northern Passage between the Sands A and C, a Line being drawn through the Middle of it, happening only to cut the Tree that stands alone, cause the Beacon H to be put up, whereby you will have always the Tree and the Beacon H to direct you in, and the former two Objects, the Red-house and Beacon G, to direct your turning off.

Now, as to laying Buoys, you must sound the Channels all up and down, and put the Depths down, which may be best done at low Water, especially when it is a Spring-tide.

A third Thing to be performed in County Surveying, is an exact Survey and Description of the Roads, both principal and cross By-roads, which is to be done as follows.

Having a Field-book, with wide Margins on the Right and Left, to enter Remarks in, the Column in the Middle being subdivided into three other, and titled M, F, and P, to insert the Miles, Furlongs, and Poles in, of your stationary Distances.

Place your Instrument at some remarkable Object, in some Town of Note, and insert the Name of the Place at the Top of your Book, putting down 1, to denote it is the first Station; then send a Person with a white Handkerchief in his Hand along the Road, as far as you can see, and by waving your Handkerchief here, let him stand; then observe him with your Instrument, setting down the Degrees at the Bearing of the first stationary Distance.

Now, let any Person drive the Surveying-wheel from you, in order to measure the stationary Line, with this Caution, to stop at every Object of Note as he goes along, till you take its Bearing, and he to note down the several Distances for you, and whether they be on the Right or Left; to what Places all Roads, principal or bye, lead to, and how they incline, whether at Right-angles or not, with the Road you are measuring, making

all useful Observations : But the better to insert the Nature of these Roads in your Survey-book, let *a* denote a Road in general, which if on the Right-hand, put it so on the right Side of the Station Column, but if on the Left, do the contrary.

In like Manner, let *b* signify a Road that inclines forwards, and *c* when it inclines backwards ; but when at Right-angles with your Road, denote it with *d* ; and when another crosses yours, by *e* ; you must be sure at least, to observe the Distances of all Roads and Lanes that come into the Road you are measuring, in respect of your Stations.

You must take notice of all Bridges you go over, and insert your Distances from the last Station in your Book, with the Name of its Water, &c. So you must enter Fords and the like, and also the Distance when you come at a Hill, when at the Top, and when over it : and when you come at any Town, &c. and when at the End of it ; not failing to enter its Name, and what may be judged necessary to describe it.

Even in measuring along the Road, it will not be amiss to take and enter the Bearing of every remarkable Object you can see, as Churches, Buildings of Note, Wind-mills, Towns, whether great or small, and the Distances of several Places of Observation from your last Station ; because, in measuring on a great Way, you may only by taking the Bearings of the same Objects, lay them down, which is a very facile way of coming at them.

Be as full with Perspicuity in your Observations and Entries as possible, observing these Directions ; and having done your Day's Work, it will be proper to protract the same, before you survey any farther, whilst you have a perfect Idea of the Thing in your Mind.

However, to protract the same, fill your Paper (the rough Draught is to be on) with Meridians, pretty near one another ; that is, draw very many Lines parallel-wise : Then chuse a convenient Place in one of these Lines for your first Station, and make an Angle with that Meridian equal to the first Angle of Observation, whereby you may have the true Position of the first stationary Line : Then take its several Distances (out of your Book) and lay thereon, whereby you will have the true Places of the Objects of Note, and where you took Remarks : Draw these Objects and the Bearings of the rest observed.

As suppose a Church, a Mile or more from the Road, did bear from your N. W. 76 Degrees, you must apply the Center of the Protractor to the Place of Observation, and at the De-
gree

gree of Bearing make a Mark, by which, and the Point of Observation, draw a Line long enough; then do the same at the other Place where you observed the said Church; and where these two Lines meet or intersect, will be the very Place of the Church in your Draught.

In the same Manner all Buildings of Note, Wind-mills, &c. are to be protracted; and if you thus lay down a Village in your Draught that is some Distance off, you must distinguish it by writing against it: but Market towns may be distinguished therefrom by a different Character, or the like. And as to the protracting Villages on the Road, where Houses stand scattering on both Sides, you must be governed by your Book, for those entered on the left Margin, must be put on the left Side, and the like.

Roads must be described with black parallel Lines, where there are Hedges; but in open Places, without any Hedges, by pricked Lines: Also you may put down the Quality of the Land, whether Arable, Common, Moorish, &c. And if the Road goes through a Wood, draw a Parcel of Trees about it: But if it be a Road over a Hill, at the Foot of the Hill shadow it deep, and so lighter and lighter till you come at the Top; and if the Hill makes any Thing of Angle, and be pretty high, you must protract its horizontal Line, or else you run into an Error.

All Rills you may signify by drawing single Lines cross the Road; Brooks by double ones; and Rivers by more Lines; and to all Rivers put their Names.

I might have instanced, that when a Road passes over a Ford, to draw the River quite cross the Road, to shew there is no Bridge; though when there is a Bridge, I advise to draw the River up to the parallel Lines of the Road, and not cross it, and to write the Name of the Bridge: You may write also the Name of the River, &c.

If a Road goes through a Park, &c. mark out its Beginning and Ending, and place Trees about that Park in the Park, and among those Trees write the Park's Name.

If there be a Village situated on the Side of a Hill, it must be shadowed that the Houses may be seen; and when an Object stands on a Hill, draw something like a Hill and the Object at the Top.

You may, as to Roads, take notice of the Way, whether stony, craggy, or clayey, or what will still add to the Credit of the Surveyor, though Practice will in a great measure direct them in these things.

A fourth Part of the Work is an exact Survey of all the Rivers, Brooks, and Rills that are remarkable: And although you may have crossed them, or several of them, in the Survey of the Roads, yet it may be necessary to trace them up to their Original (if in the County proposed to be surveyed) or down to the Sea, if the River, &c. in the County extend so far.

Note, Small Brooks, or Rills, that frequently have many Turnings and Windings, cannot have all their small Turnings expressed in the Map, if done to a small Scale; but yet all Brooks, and whatsoever Branches of Rivers, ought to be expressed; and particularly where they run into a main River.

The Method of surveying a River is not much unlike that of surveying a Road, and yet the following particular Directions may be necessary.

As in the Road, let a Person place himself at the next considerable Bending of the River, the farther off the better, and you being at its Mouth observe his Position, which will be as a Point in the River's Side; which note down in your Book as Bearing: Then with a Wheel (for the Readiness of Measuring) measure his Distance from you, and direct him to go to the next considerable Winding, or Bounding, or Bending, where, from the first Place he stood at, you are again to observe him, setting down the Bearing and Distance in your Book, as before; for thus are you to proceed along the River-side till you have Points enough to draw a Line by for its Description.

As to the River's Breadth, it may be determined by observing a Beacon or Mark on the opposite Side from two Stations on this, as has been before observed in Road-surveys, and otherwise.

Now, if the River should not be equally broad throughout, you have an Opportunity, from the Beacon on the opposite Side, to survey that Side as you did this; otherwise you must suppose the Sides parallel, as the Hedges in Road-surveys.

As to the protracting these Observations, it is almost needless to mention, because it is the same as hath been taught all along; however, chuse a Place on your Paper, as in some Meridian, to represent your first Station, and project your first Observation according to Angle and Length; then at your second Station do the like: Thus proceeding from Station to Station, till you have protracted the Whole.

When there is a Ferry over the River, the River more especially then must have its true Breadth; and you may draw a
pricked

pricked Line cross the same, to show its Passage over ; and it is not amiss to write the Name of the Ferry in your Draught.

As in Road-surveys, observe and enter all Remarkables that occur in this Survey, such as Bridges, Mills, &c. with their real Distances from the River's Mouth and last Station. And you may in your Draught distinguish the Nature of the Bridge, whether Wood or Stone, or such-like Things.

Take also notice of all Sluices, Flood-gates, Locks, &c. as you proceed, and to enter their Names : And if there be any Rivulets, Brooks, and cut Rivers, from that you are surveying, note their Places, as you do Bye-roads and Lanes ; and observe what Places are fordable, and also to enter the Distance of the same from your last Station, and Mouth of the River.

You are also, as in Road-surveys, to take notice and enter all Towns the River passes through, and every Thing worth Observation, with their Distances from your last Station, and the Mouth of the River, your first Station.

And, as in Road-surveys, you must not neglect taking the Bearing of all Churches and Objects of Note from two Places or Stations, and properly to enter the same, with their Names, &c.

Having measured the principal Rivers, proceed to the Rivulets, or small ones branching therefrom, which also survey with equal Exactness.

And observe, as to all Rivers whatever that are navigable, to take every Bending exactly, not regarding small Creeks ; but as for small Brooks, which perhaps bend at every two or three Poles, or less, thus survey :

Take a Sight as far as you can to the next considerable Bending, and account for the little Turnings or Windings by Off-sets from that Line, as in Field-surveying ; for thus you may proceed in all Cases of like Nature, only observing, that if a Mile or two be an Inch, that you cannot describe these small Bendings in your Draught from so small a Scale.

As to the protracting this Observation, it differs from that of Road-surveying, only in laying off the Off-sets, which is also a Matter of no Difficulty.

C H A P. XIV.

Of computing the Content of a Survey by the Pen only, without any Draught, geometrically considered and demonstrated.

SUPPOSE the Tra^ct of Land ABCDEFGHA (see Fig. 88.) to be surveyed, and that by my Instruments I find the Lengths and Positions of its Sides respectively, as follows:

<i>Sides of the Land.</i>	<i>Their Positions.</i>	<i>Their Lengths in Chains.</i>
AB —	NE - 42	5 : 00
BC —	East. -	4 : 00
CD —	NE - 9	4 : 00
DE —	SE - 69	5 : 56
EF —	SE - 36	7 : 00
FG —	SW - 42	4 : 00
GH —	SW - 75	10 : 00
HA —	NW - 39	7 : 50

Then I first of all cast up the same as a Traverse in Sailing, whereby I know how much B is more Northerly and Easterly than A, and so on from Point to Point; which if you do not think to take out of the Table, you may thus calculate:

As Radius is to the Length of any Side,
So is the Sine of the Angle of Position or Bearing,
To the Easting or Westing required.

And as Radius is to the Length of any Side,
So is the Sine Complement of the Bearing,
To the Northing or Southing desired.

And you may observe, that as you began your Survey at A, and left off at the same Place, that the Total of your Northings must be equal to that of your Southings: And also, that the Sum of all your Eastings must be equal to that of your Westings; for if a Person was to travel and traverse the World
never

never so long, and at last was at the same Place, undeniably he must have travelled as much Southerly as ever he did Northerly, and as much Westerly as ever he did Easterly, or else it would be impossible ever to be at the same Place again. However, the Traverse cast up, as mentioned, is as follows :

Sides of the Land.	Bearing.	Distance	Northing	Southing	Easting	Westing
	Deg.	Chains.	Chains.	Chains.	Chains.	Chains.
AB	NE 42	5—	3.71	—	3.35	—
BC	East	4—	—	—	4—	—
CD	EE 9	4—	3.95	—	0.63	—
DE	SE 69	5.56	—	1.99	5.21	—
EF	SE 36	7—	—	5.66	4.12	—
FG	SW 42	4—	—	2.97	—	2.68
GH	SW 75	10—	—	2.59	—	9.66
HA	NW 39	7.5	5.55	—	—	4.97
Totals - -			13.21	13.21	17.31	17.31

As AB is N. E. 42° there's Northing and Easting ; which may be actually done according to the Canons before specified thus logarithmically :

As Radius	—	—	—	10.000000
To 5 Chains	—	—	—	0.698970
To the Sine 42°	—	—	—	9.825511
				—
To Easting 3.35	—	—	—	0.524481

And,

As Radius	—	—	—	10.000000
To 5 Chains	—	—	—	0.698970
To Co-sine 42	—	—	—	0.978017
				—
To the Northing 3.71	—	—	—	9.871073

And thus may the Northings in the four Left-hand Columns be always calculated : But as to those that incline to use a transverse Table, let them observe, that Northing or Southing is Difference of Latitude there, and that Easting or Westing is Departure there ; for so it will not be difficult to use the same.

And

And to proceed, let us consider that every Tract of Land has an extreme southerly Point, as H; and let us reckon so much as any other Point in the Figure is distant from the East and West Line IK, that passes through H, to be its Latitude North, as AI to be the Latitude or Breadth North of the Point A; BL the Latitude of B; CM the Latitude of C; and so every-where else. Then I assert, that if from the Content of the Figure I A B C D E F K I, be subtracted the Figure F K I A H G F, the Remainder will be the Content of the Tract of Land, as is manifest from the Scheme.

But to find the Content of the multangular Figure I A B C D E F K I, you are to observe, that the same is composed of all these Trapeziums, *viz.* I A B L I, L B C M L, M C D N M, N D E O N, and O E F K O; and that finding the Content or Area of any of these Trapezia, is only to multiply the middle Latitude by the particular Easting of the two Points of the Land that belongs to the same: As for Instance;

Suppose the Trapezium N D E O N, the middle Latitude will be $\frac{1}{2}$ the Sum of DN and EO, which may be imagined as a Line drawn from the Middle of NO to the Middle of DE; and it is plain, that the same multiplied by NO, which is so much as E is to the Eastward of the Point D, will produce the Content of the said Trapezium; and so for the rest.

The like we may argue as to the computing the Quantity of the Figure F K I A H G F, where the middle Latitude between the Points F and G, multiplied into P K, so much as G is to the Westward of F, will produce the Content of the Trapezium P G F K P, and that the mean Latitude between G and H, which is $\frac{1}{2}$ GP, multiplied by O H, the Quantity H is to the Westward of G, will produce the Area of H G P H, &c. whence I infer the following General Rule.

R U L E.

Multiply the mean Latitude of every easterly 2 Points, by the real Easting of the latter Point from the former, and collect all into one Sum.

Then do so as to the Points that are West, by multiplying their several mean Latitudes respectively into their proper Westings,

ings, and subtract the latter Sum from the former, for the Content of your Survey.

Now, before you can actually compute the Quantity of Land from this Rule, you must calculate the several Latitudes, which you may do from the last Table thus, as the Scheme demonstrates :

When at B you had northed 3.71, and when at D you had northed more 3.95, so that at D you had northed in all 7.66.

But, when at E you having southed 1.99, you were not so much North as at D by that Quantity ; so taking away 1.99 from 7.66 shews, that at E you had northed from A but 5.67.

Having at F southed more 5.66, taking 5.66 from 5.67, shews that at F you was to the North of A but 0.01.

And having at G southed more 2.97, it is plain at G you was entirely to the South of A 2.96.

But coming to H, you southed more 2.59 ; therefore adding 2.96 and 2.59 together, shews that H is South of A 5.55.

But 5.55 being the greatest Southing from A ; H is the extreme southern Point of the Land.

And you may also observe, that 7.66 being the greatest Northing from A, that D must be the extreme North Point of the Land.

However, as H is South of A 5.55, A must, of course, be as much North of H, whence the Latitude of A is 5.55, viz. A I is equal to 5.55.

Now, having thus reasoned out the Latitude of A, it will not be difficult from the Table (in like manner by Addition, and subtracting the Northings and Southings) to obtain the Latitudes of the rest of the Points, thus :

For, if to Latitude of A 5.55 be added 3.71, the Northing of B, there will result 9.26 for the Latitude of B ; so that B L is 9.26 ; and, I suppose, I need not mention the Latitude of C is the same, because C is neither Northing nor Southing, but only Easting.

2dly, If to the Latitude of C be added 3.95, there will result 13.21 for the Latitude of D, that is, D N equal to 13.21.

Again ; If from the Latitude of D, (because it is Southing) you subtract 1.99, there will result 11.22 for the Latitude of E, equal to E O.

4thly, If from the Latitude of E you subtract 5 66, you have the Latitude of F 5.56 equal to FK.

Lastly, If from the Latitude of F you subtract 2.97, the Remainder will be 2.59 for the Latitude of G, equal to GP.

Thus having calculated the Latitudes of all the Points, they thus stand.

The Latitude of A	is	5.55
B	is	9.26
C	is	9.26
D	is	13.21
E	is	11.22
F	is	5.56
G	is	2.59
H	is	0.00
And so round to A's	is	5.55

And from the Scheme it is also manifest, that if you add the Latitude of A to that of B, that you have twice the Latitude between those Points: The like is to be understood as to the adding the Latitudes of B and C together, and so on; whereby the Doubles of the mean Latitudes are as follow, according to Order :

14	:	81
18	:	52
22	:	47
24	:	43
66	:	78
8	:	15
2	:	59
5	:	55

Now, though the Rules mention operating by the mean Latitudes, I propose here to work by their Doubles, because they will not justly halve, and so take half the Content, thus found, for the true Content of the Land.

Agreeable to which I place the above Northing in a Column to the Right of that of Westing, and add two other Columns

Columns for the Easting and Westing Products, as immediately follows ;

<i>Easting Chains.</i>	<i>Westing Chains.</i>	<i>Double Lat. Chains.</i>	<i>East Products.</i>	<i>West Products.</i>
3.35	—	14.81	49.6135	—
4.—	—	18.52	74.08	—
0.63	—	22.47	14.1561	—
5.21	—	24.43	127.2803	—
4.12	—	16.78	69.1336	—
—	2.68	8.15	—	21.842
—	9.66	2.59	—	25.0194
—	4.97	5.55	—	27.5835
Totals - -		{	334.2635	74.4449
			74.4449	
Difference -			259.8186	
Semi-difference			129.9093*	
* The Content in Square Chains.				

Note, I came by 49.6135 by multiplying 14.81 by 3.35, and 21.842 by multiplying 8.15 and 2.68 together. The like is to be understood as to the other Products, only this Caution, that if one of the Factors be East, it will be an East Product, and vice versa.

Now, as ten Chains in Length and one over, is a Statute Acre, I divide the last Northing by ten Square Chains, bring it into Acres decimally, &c. thus :

Acres	—	—	12	99093
				4
Roods	—	—	3	96372
				40
Perches	—	—	38	54880

So that the Content of the Survey is very near thirteen Acres.

R

Now,

Now, whereas in this Survey from the extreme Point West to that East, was entirely Easting, and no Westing withal; and that, from the extreme East Point to that West, intirely Westing; I shall give you an Instance where it happens otherwise; and accordingly demonstrate its Solution, on Purpose to confirm this general Method of Computation; agreeable to which, let us suppose by the Method of Perambulation, or Circulation, that the Field QRSTUVQ (see *Figure 89.*) is surveyed, and that to stand in the Field-book.

<i>Sides.</i>	<i>Bearings.</i>	<i>Distances.</i>	} Chains.
QR	N E - 25	9 . 00	
RS	S E - 66	9 . 12	
ST	S W - 46	5 . 42	
TU	S E - 83	8 . 98	
UV	S W - 71	15 . 24	
VQ	NW - 27	6 . 03	

Then, as before, I cast the same up as a Traverse, either by the Table formerly mentioned, or Trigonometrically, by the Canons of Operation used in the last Example; however the same cast up as follows;

<i>Sides.</i>	<i>Bearings.</i>	<i>Distance</i> <i>Chains.</i>	<i>Northing</i> <i>Chains.</i>	<i>Southing</i> <i>Chains.</i>	<i>Easting</i> <i>Chains.</i>	<i>Westing</i> <i>Chains.</i>
QR	NE 25	9 . 00	8 . 157	—	3 . 803	—
RS	SE 66	9 . 12	—	3 . 709	8 . 332	—
ST	SW 46	5 . 42	—	3 . 765	—	3 . 899
TU	SE 83	8 . 98	—	1 . 094	8 . 913	—
UV	SW 71	15 . 24	—	4 . 962	—	14 . 41
VQ	NW 27	6 . 03	5 . 373	—	—	2 . 739
Totals - -			13 . 53	13 . 53	21 . 048	2 . 048

Next I compute the Latitude of Q, at I did that of A in the last Example, and so of the rest of the Points, as follows:

R is more Northing than Q by 8.157; and S being to the Southward of R 3.709, I subtract 3.709 from 8.157, and the Remainder 4.448, is what S is to the Northward of Q.

And

And, as *T* is southerly of *S* 3.765, I subtract the same from 4.448, and the Remainder .683, is what *T* is to the Northward of *Q*.

Again, as *U* is more southerly of *T* than *T* is northerly of *Q*, I subtract .683 from 1.094, and the Remainder .411, is evidently what *U* is southerly of *Q*.

And, as *V* is southerly of *U* by 4.962, I add .411 (which is what the last Point *U* is southerly of *Q*) to it, and the Sum is 5.373, what *V* is southerly of *Q*; consequently *Q* is as much to the North of *V*.

Having obtained the Southings and Northings of each Point in respect of *Q*, it is not amiss to set them down thus,

In respect of <i>Q</i>	{	<i>R</i> is	8.157	}	North.
		<i>S</i> is	4.448		
		<i>T</i> is	0.683		
	{	<i>U</i> is	0.411	}	South.
		<i>V</i> is	5.373		

And now you may observe, that *V* is the extreme southern Point of the Land, and consequently the Latitude of *Q* is 5.373.

You may, if you please, also observe, that as 8.157 is the greatest Number North; that the Letter *R* is the extreme north Point of the Land.

However, having the main Hinge of the Matter, viz. the Latitude of *Q*, therefrom, and by the Help of the Columns of Northing and Southing, I shall now calculate the Latitudes of the rest of the Points, thus:

As *R* is to the Northward of *Q* (by the Column of Northing) 8.157, I add the same to the Latitude of *Q*, and there results 13.53 for the Latitude *R*.

Secondly, as *S* is southerly of *R* 3.709; I subtract the same from the Latitude of *R*; and the Remainder 9.821, is the Latitude of *S*.

Again, as *T* is southerly of *S* 3.765, I take the same from the Latitude of *S*, and there results 6.056 for the Latitude of *T*.

Lastly, As *U* is southerly of *T* 1.094, I deduct it out of the Latitude of *T*, and 4.962 left, is the Latitude of *U*.

Thus having obtained the Latitude of all the Points, I rank them according to the following Order.

The Latitude of Q is 5.373
 R is 13.53
 S is 9.821
 T is 6.056
 U is 4.962
 V is 0.000
 And so round to Q is 5.373

Fig. 89.

Further, it is evident, that if I add the Latitudes of any two adjacent Points together, that I have twice the mean Latitude between the same two Points; therefore I compose the Doubles of the mean Latitudes which follow, by adding the first and second of the above Numbers together, and by adding the second and third, and so on.

Chains.

18 . 903
 23 . 351
 15 . 877
 11 . 018
 4 . 962
 5 . 373

These Numbers are to be placed on the Right-side of the Column of Westing, and the other Columns there must be still for the East and West Products; and to remember, that as these Numbers are double what they should be, to halve the Content so found for the real Content of the Field: but this I shall now wave, till I have demonstrated, that the same Method of determining the Content will also succeed in this Survey; because you wested from S to T, before you came to the extreme Point V.

DEMONSTRATION.

The Easting Product WQRYW, and the Eastern Product YRSaY, is equal to WQR SaY; from which, if the West Product ZTSaZ be subtracted, now as all West Products at least are to be by the Rule, the Remainder will be

W Q R S T Z W ; to which, if you add the East Product Z T U X Z, the Sum will be W Q R S T U X W.

From which, if the West Products V U X V and V W Q V be subtracted, the Remainder manifestly will be Q R S T U V Q the Quantity, or the Field itself.

Now, as it is not to be doubted, if three Numbers are to be subtracted, that subtracting one first, and then the Sum of the other two, that the Remainder is the same as if I had omitted Subtraction to the last, and then subtracted the Sum of all three : Therefore, &c. as was to be demonstrated.

Now, seeing the Westing of T (which occasions the superfluous Triangle T S b) has no Effect on the Method of Procedure ; I shall go on with tabulating the said Latitudes, &c. and exactly proceed as in the Conclusion of the last Example.

Eastings Chains.	Westings Chains.	Double Lat. Chains.	East Products.	West Products.
3.803		18.903	71.888109	
8.332		23.351	194.560532	
	3.899	15.877		61.904423
8.913		11.018	98.203434	
	14.41	4.962		71.50242
	2.739	5.373		14.716647
Totals - - {			364.652075	148.123490
			148.12349	
Difference -			216.528585	
Semi-difference			108.2642925*	
* The Content in Square Chains.				
Acres - - 10			82642	
			4	
Roods - - 3			30568	
			40	
Perches - - 12			22720	
			A. R. P.	
So that the Content of the Field is			10 : 3 : 12	

By this Time I have clearly demonstrated and sufficiently exemplified this Method of Computation by the Pen, which is not only a Matter of Exactness, but of Use, as the ingenious Artift may find; for Practice will make a thing very ready and familiar to a Person, though at first View it appears to be a Difficulty.

However, before I quite conclude this Section, it will not be amifs to add this excellent Theorem, for determining the Area of a plain Triangle, when your two Sides and the Angle included by them are given.

THEOREM.

As Radius is to the Sine of the Angle; so is $\frac{1}{2}$ the Product of those Sides to the Triangle's Area.

DEMONSTRATION.

Let ABC be the Triangle, (see *Figures 90 and 91*) B the given Angle, and BA and BC the given Sides; then on BC describe the Parallelogram BEGC B of the same Altitude with the said Triangle, and make BD equal to BA, completing also the Parallelogram BDFCB: Now considering BA or BD as Radius, BE, according to the Definition of a Sine, is the right Sine of the Angle B.

But $BD : BE :: BDFCB : BEGC B$.

That is, as Radius to the Sine of the Angle B:
So is the Parallelogram, whose Sides are AB and BC,
To the Parallelogram BEGC B, which is twice the Triangle's Area.

Therefore $BD : BE :: \frac{1}{2} BDFCB : \frac{1}{2} BEGC B$.

Q. E. D.

Now as a Specimen of the Use of the said Theorem, let us suppose a four-sided Field, as ABCDA; (see *Fig. 92.*) to stand in the Field-book as follows.

Sides.	Bearings.	Chains.
AB —	NE - 32° —	27
BC —	SE - 51 —	33
CD —	SW - 47 —	29
DA —	NW - 46 : 34' —	25 : 38

Then

Then imagine or draw the pricked Diagonal BD, dividing the same into two Triangles; and consider, that as A is N. W. $46^{\circ} : 34'$ of D, then D is South-easterly as much of A; that is, AD is S. E. $46^{\circ} : 34'$.

Draw also the North and South, or Meridian Line EF, and the Angle EAB (the middle Letter denoting the angular Point) will be equal to the Bearing of AB; and the Angle FAD will be equal to the Bearing of AD; whence, if you subtract the Sum of these two Bearings, from 180, as follows, you will have the Angle BAD.

Add	$\begin{array}{r} 32^{\circ} \\ 46 : 34' \\ \hline \end{array}$	
Take	$\begin{array}{r} 78 : 34 \\ 180 \\ \hline \end{array}$	Sum
From		
Rem.	$\begin{array}{r} 101 : 26 \\ \hline \end{array}$	For the Angle BAD.

In like Manner we compute the Angle C thus;

Add	$\begin{array}{r} 51^{\circ} \\ 47 \\ \hline \end{array}$	
Take	$\begin{array}{r} 98 \\ 180 \\ \hline \end{array}$	Sum
From		
Rem.	$\begin{array}{r} 82 \\ \hline \end{array}$	

Remains 82 for the Angle C.

Having thus obtained the opposite Angles A and C, observe in computing the Content of the Triangles ABD, and BAD by the said Theorem; that if the whole Product instead of $\frac{1}{2}$ be used, that the Content so found must be halved: And also observe that the Sine of any Angle is also the Sine of what that Angle wants of 180° ; agreeable to which, the Content of the Triangle ABD is thus computed by Logarithms.

			<i>Logarithms.</i>
As Radius	- - - - -		10.000000
Is to the Sine of 101 . 26, or Angle A			9.991295
So 27 multiplied by 25 . 38	{ 27 - - -	1.431364	
	{ 25 . 38 -	1.404492	
To twice the Content of the Tri-			
angle ABD	- - - - - }	2.827151	From
	2 —————	0.301030	Take
335.8 Chains	—————	2.526121	

So that the Content of the Triangle ABD is 335.8 square Chains. And after this manner, as follows, I also compute the Content of the Triangle BDC.

	<i>Logarithms.</i>
As Radius - - - - -	10.000000
To the Sine of the Angle C 82° - -	9.995753
So BC multiplied by DC { 33 - -	1.518514
{ 29 - -	1.462398
 To twice the Content of the Tri- } angle BCD - - - - - }	- - 2.976665 From
	2 ———— 0.301030 Take
The Content of BCD 473.9 - - -	2.675635

$$\text{Add } \left\{ \begin{array}{r} 335.8 \\ 473.9 \end{array} \right.$$

Square Chains 809.7 The Area or Content ABCDA.

Acres	-	-	-	-	-	80		97
								4
Roods						3		88
								40
Perches	-	-	-	-	-	35		20

Whence the Content of the Field is 80 Acres, 3 Roods,
35 Perches.

I shall

I shall forbear to multiply Instances of this Kind, as judging this sufficient to shew the Usefulness of this excellent Theorem, as to its Facility and Conciseness of Operation; which is a thing really worthy of Observation.

C H A P. XV.

How to correct the Work in a Survey, when though performed with the utmost Accuracy, and by the Help of the best Instruments, yet Errors may be contracted.

OBERVE in the Course of your Survey, whether of Roads, Rivers, &c. when you see the Sun shine right along any Part of the Road or Lane, go directly up or down any particular Reach of the River to be surveyed, or the like; and being provided with a good Watch, or universal Dial, see the Time of the Day when the Sun shines up or down any Part of a Lane or River, as above, and compute, by Astronomical Calculation, the Sun's Azimuth; which by the Latitude of the Place, the Declination of the Sun, and the Hour of the Day, is obtained by the Rules of Astronomy; and may be of Use, not only in surveying Roads and Rivers, but also in correcting the Survey of the whole County, when a Town, Castle, or any other remarkable Object can be seen, although at never so great a Distance; and is also of excellent Use in correcting and adjusting the Ichnography or Ground plot of any City, Town, or Village, that is required to be drawn: And although this Book is not intended as a Treatise of Spherical Trigonometry, yet it may not be amiss to give the following Directions for finding the Sun's Azimuth, or Point of the Compass that he bears upon; which may be performed, either by the Latitude of the Place, the Hour of the Day, and the Sun's Declination given; or by the Latitude of the Place, the Sun's Altitude and Declination given; I shall instance in both, as supposing Surveyors are, or ought to be, furnished with Instruments that may be applied to the finding the Sun's Altitude, as well as the Hour of the Day.

And

And first,—The Latitude of the Place, the Sun's Declination, and the Hour of the Day being given, to find the Sun's Azimuth, or Point of the Compass that he bears upon; which is thus performed,

As Tangent of Declination

To Radius,

So Sine Complement of the Hour (accounting 15 Degrees of the Equator for an Hour of Time)

To the Tangent of a fourth Arch, from which take the Complement of Latitude, and observe the Remainder.

Then,

As Sine of that fourth Arch

To the Tangent of the Hour from Noon (allowing as above)

So Sine of the Remainder above-mentioned

To Tangent of the Sun's Azimuth from the Meridian, whether North or South.

One Example will explain the whole.

Suppose in the Survey of an Estate or Manor, as mentioned Page 139, and 140, and described *Fig. 61*. I am going along *Ashton-lane* from the Gate at A, to the Wind-mill near the Corner C, and this is on the twelfth of *August*, 1759, I see the Sun shine right along that Lane, from A towards the Wind-mill; and then observe by a Watch, or rather an universal Dial, that it is just two o'Clock, and the Latitude of the Place is 52 Degrees, 28 Minutes.

Find the Sun's Declination, which on the twelfth of *August* is 15 Degrees 2 Minutes North.

Then you have given

The Latitude of the Place, 52.28,

The Sun's Declination, 15 Deg. 2 Min. North.

The Hour of the Day two o'Clock (which, at 15 Degrees to an Hour, is 30 Degrees, the Angle that the Meridian the Sun is in makes with the Meridian of the Place)

To find the Sun's Azimuth or Point of Bearing.

			<i>Co. Ar.</i>
As Tang. of Declination	- - 15.2	- -	9.42906
To Radius	- - - -	- -	10.00000
So Sine-Comp. of the Hour	- 30.0	-	9.93753
To Tang. of a fourth Arch	- 72.46	-	10.50847

From which take the Complement of Latitude
37.32, the Remainder is 35.14.

			<i>Co. Ar.</i>
As Sine of the Remainder	- 35.14	-	0.23889
To the Tang. of the Hour	- 30.00	-	9.76144
So Sine of the fourth Arch	- 74.46	-	9.98005
To Tangent of the Sun's Azi-	} 43.42		9.98038
muth from the South			

Which is South-west 1 Deg. 18 Min. Southerly, and North-east 1 Deg. 18 Min. Northerly, that Part of the proposed Lane, Street, &c. lies

The Proportion to find the Azimuth, by having the Altitude, Declination, and Hour given, is

As Sine-Comp. of Altitude
To Sine of the Hour from Noon,
So Sine Comp. of Declination
To Sine of the Azimuth.

Or, if by having the Latitude of the Place, the Sun's Altitude and Declination given, it be required to find the Azimuth, it is three Sides of an oblique spherical Triangle, given to find an Angle, which, as I suppose, those that understand Surveying, are, or ought to be acquainted with, I shall not enlarge upon it.

C H A P. XVI.

Of Artificer's Work, and how to measure and cast up the same, the Dimensions being taken in Feet and Inches.

GLASIERS and gauged Brick-work is computed by the Foot Square, but Plaistering, Paving, Wainscotting, and Painting, by the square Yard, containing 9 square Feet.

Tiling and Flooring by the Square that contains 100 square Feet

Brick-work is measured by the square Rod of $227 \frac{1}{4}$ square Feet; that is, a Rod of Brick-work is a Square, whose Length or Breadth is $5 \frac{1}{2}$ Yards, when the Wall is a Brick and a half thick; and the thicker the Wall is, the more Rods there are in the same Measure, which is easy to be accounted for.

As here we are to multiply Feet and Inches by Feet and Inches, without reducing them to a like intire Denomination: Observe, as a general Rule, that Feet, multiplied by Feet, produce Feet; and that Feet and Inches, multiplied together, produce Inches; but Inches, multiplied by Inches, give Twelfths of Inches.

I suppose I need not mention, that always any plain Superficies to be measured may be imagined to be made up of Oblongs or Triangles, which is their Halves; and so the Dimensions of those Oblongs taken and computed, by multiplying their Length by their Breadths, as Examples will better evidence.

Example 1. Quære, how many Feet of Glas there are in a Window that is 7 Feet 5 Inches high, and 4 Feet 8 Inches over.

$$\begin{array}{r}
 7 : 5 \\
 4 : 8 \\
 \hline
 4 : 11 : 4 \text{ Twelfths.} \\
 29 : 8 : \\
 \hline
 \text{Facit } 34 : 7 : 4 \text{ Or, } 34 : 7 \frac{1}{3}
 \end{array}$$

Feet. Inches.

The Operation explained.

Eight Times 5 is 40 Twelfths; set down 4, and go 3 Inches; 8 Time 7 is 56, and 3 is 59 Inches, which set down 4 Feet 11 Inches: Now having multiplied the Multiplicand throughout by 8, I begin to multiply by 4, saying 4 Times 5 is 20 Inches, set down 8 Inches, and go one Foot; then 4 Times 7 is 28, and that 1 is 29 Feet, which I also set down, and add the two several Products together for the Product of the whole.

Example 2. Suppose a Glasier has done the following Work, let it be required to cast the same up.

- 1 Window 8 Feet 9 Inches high, and 5 Feet 7 Inches broad.
- 3 Windows, each 7 Feet 5 Inches high, and 4 Feet 10 Inches broad.
- 1 Window 5 Feet 7 Inches high, and 3 Feet 8 Inches over.

Feet.	Inches.	Feet.	Inches.	Feet.	Inches.
8	: 9	7	: 5	5	: 7
5	: 7	4	: 10	3	: 8
5	: 1 : 3	6	: 2 : 2	3	: 8 : 8 12ths.
43	: 9	29	: 8	16	: 9 :
48	: 10 : 3	35	: 10 : 3	20	: 5 : 8
		107	: 6 : 6		

	Feet.	Inches.	12ths.
First Window - - -	48	: 10	: 3
The three Windows -	107	: 6	: 6
The last Window -	20	: 5	: 8
	176	: 10	: 5 Answer.

Example

Example 3. A Carpenter has made a Table of an oblong Form, whole Length is 17 Feet 8 Inches, and Breadth 5 Feet 7 Inches, which I would have cast up, because I am to pay him for it by the Foot.

Feet. Inches.

17 : 8

5 : 7

10 : 3 : 8

88 : 4

Facit 98 : 7 : 8

Example 4. I have done a Piece of Wainscoting that is 21 Feet 7 Inches long, and 12 Feet 8 Inches broad : Now, how many Yards are there in this Work ? Because I am to be paid by the Yard.

Feet. Inches.

21 : 7

12 : 8

14 : 4 : 8

259 : 0

9)273 : 4 : 8

Facit 30 :

Feet. Inch. Twelfths.

Which is 30 Yards, and 3 : 4 : 8

Note As in 21 there is one 12, and 9 besides; you may count it 1 : 9 in multiplying by 8, saying, 8 times 9 is 72, and the 4 I go is 76; set down 4, and go 6; and 8 times 1 is 8, and 6 is 14.

The like might have been considered in the Work of the Example immediately preceding, viz. in multiplying 17 by 7; which Observation is of Use, as it eases the Work.

Example 5. A Wall $1\frac{1}{2}$ Brick thick measured is found to be 67 Feet 8 Inches long, and 10 Feet 7 Inches high; *Quære*, How many Rods there are in the same.

Feet. Inches.

67 : 8

10 : 7

39 : 5 : 8

676 : 8

Here I counted 67 : 5 : 7 as the above Note specifies, which saves the Work, and is equally easy.

Feet. Inch.) 716 : 1 : 8

272 : 3) 544 : 6

Remain 171 : 7 : 8

Facit 2 Rods, and 171 Feet, 7 Inches 8 Twelfths, which is better than $2\frac{1}{2}$ Rods.

Example 6. A Wall two Bricks thick, is 89 Feet 7 Inches long, and 11 Feet 8 Inches high; *Quære*, How many Rods there are in the same?

Feet. Inches.

89 : 7

11 : 8

59 : 8 : 8 Twelfths.

985 : 5

1045 : 1 : 8

Now these are the Feet, &c. where the Wall is $1\frac{1}{2}$ Brick thick : But, as it is 2 Bricks thick, increases the Number of Feet by a Third; thus,

3)1045 : 1 : 8

348 : 4 : 6

Feet. Inch.) 1323 : 6 : 2 | Rods.

272 : 3) 1291 : 3

Remains - 32 : 3 : 2

Facit 5 Rods, 32 Feet, 3 Inches, 2 Twelfths.

Had the Wall been $2\frac{1}{2}$ Brick thick, the Increase must have been $\frac{2}{3}$; and if 3 thick, just as much again : All which is very obvious and plain, with a little Consideration.

Example:

Example 7. The Foundation of a Wall 97 Feet 7 Inches long, is 4 Feet 8 Inches deep, and $3\frac{1}{2}$ Bricks thick: From thence to the Water-table is 7 Feet 5 Inches, and three Bricks thick; and from the Water-table to the Top of the Wall is 9 Feet 10 Inches, and but $2\frac{1}{2}$ Bricks thick; *Quære*, the Rods in the said Wall?

	97 : 7		97 : 7
	7 : 5		4 : 8
	40 : 7 : 11		65 : 0 : 8
	683 : 1 :		390 : 4 :
$1\frac{1}{2}$ Bric.	723 : 8 : 11	$1\frac{1}{2}$ Bric.	455 : 4 : 8
Water-tab.	2		2
	1447 : 5 : 10	$3\frac{1}{2}$ Bric.	910 : 9 : 4
		$\frac{1}{2}$ Bric.	151 : 9 : 9 $\frac{1}{2}$
		Found.	1062 : 6 : 10 $\frac{1}{2}$

Feet. Inches.

97 : 7
9 : 10

81 : 3 : 10
878 : 3 :

$\frac{1}{2}$ Brick. 959 : 6 : 10
 $\frac{1}{2}$ Brick. 319 : 10 : 3
 $\frac{1}{2}$ Brick. 319 : 10 : 3

Upper Part 1599 : 3 : 4

Feet. Inches. T.

Foundation 1062 : 6 : 10

Water-tab. 1447 : 5 : 10

Upper Part 1599 : 3 : 4

Feet. Inch.) 4109 : 4 (10
272 : 3) 2722 : 6 (5

1386 : 10 : 15 Rods.
1361 : 3

Remainder 25 : 7

Facit 15 Rods, and 25 Feet 7 Inches.

Example

Example 8. The Roof of a House is 57 Feet 10 Inches long, and the Length of the Rafter is 15 Feet 9 Inches; how many Square of Tiling is there?

Feet. Inches.

57 : 10

15 : 9

43 : 4 : 6

297 : 6

57

910 : 10 : 6 $\frac{1}{2}$ the Roof
2

100)1821 : 9

Facit 18 Squares, 21 Feet, 9 Inches.

Example 9. How many Yards of Painting is there in a Room that is 17 Feet 7 Inches long, and wide 12 Feet 5 Inches, that is, 19 Feet 2 Inches high.

Feet. Inches

17 : 7

12 : 5

33 :

2

60 The Circuit of the Room.

Feet. Inches.

19 : 2

60

9)1150 (7

Facit 127 Yards, and 7 Feet.

Example 10 How many Yards is there in a Piece of Painting 95 Feet 3 Inches long, and 17 Feet 7 Inches broad?

Feet.	Inches.
95	: 3
17	: 7
55	: 6 : 9
669	: 3
95	
9)1674	: 9 : 9

Facit 186 Yards, 9 Inches, 9 Twelfths.

As to measuring Timber, it is customary to girth the Tree in several Places, and to take a Quarter of the mean Girth for the Side of a Square; which Square, multiplied by its Length, gives its Solidity, which though not mathematically true, yet Custom has prevailed.

Example 11. I girth a Tree that is 35 Feet 7 Inches long, in three Places, which Girths are 5 Feet 8 Inches, 6 Feet 10 Inches, and 7 Feet 3 Inches; *Quære*, How many Feet of Timber there is in it?

Feet.	Inches.
5	: 8
6	: 10
7	: 3
3)19	: 9
4)6	: 7 Mean-girth.
Multiply these Numbers	{ 1 : 8
	{ 1 : 8
Square	- - - 2 : 9
	35 : 7
Multiplication by 5	- 1 : 7 : 5 : 4
	97 : 2 : 8
	9 : 10 : 1

Facit 97 Feet, 10 Inches, and 3 Twelfths of an Inch.

12ths of 12th.
for multiplying
by 5 and by 7
thus, does the
Work by 35.

These

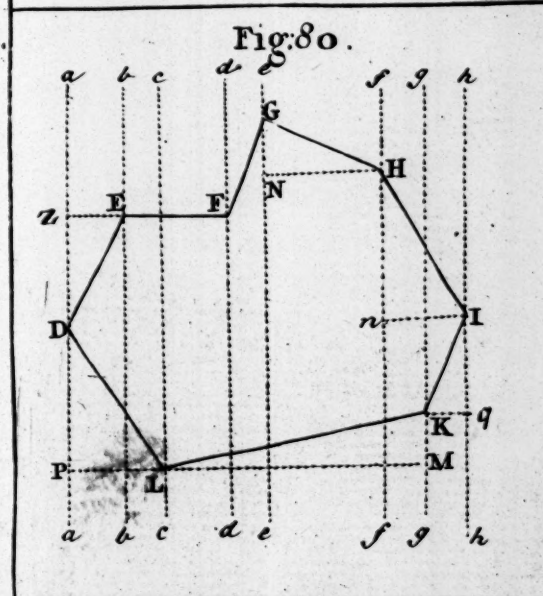
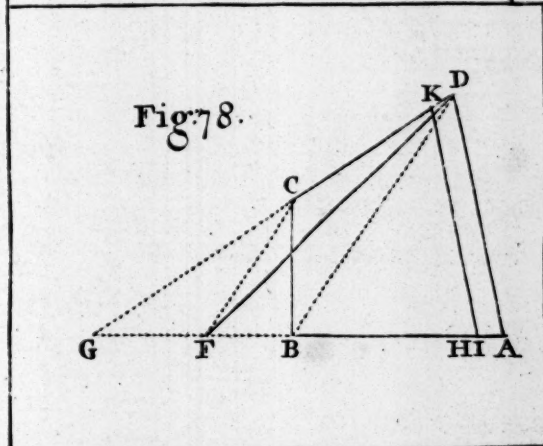
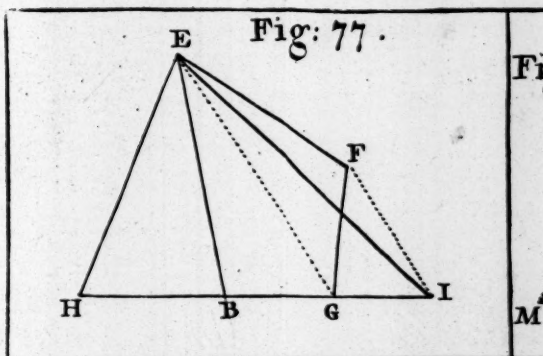


Fig: 77.

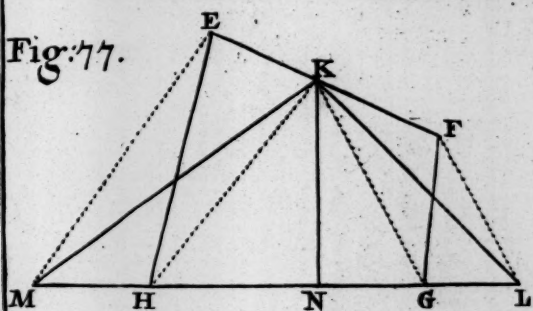


Fig: 79

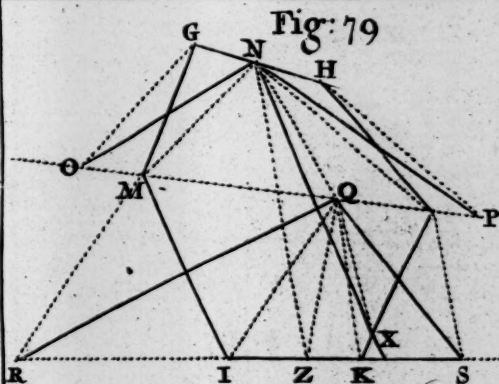
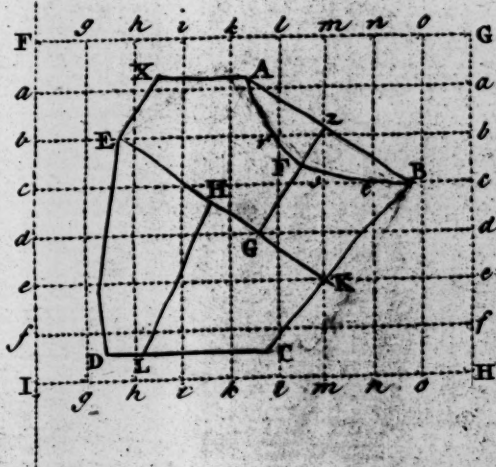


Fig: 81.



These Operations by Twelfths is properly what is called Duodecimal Arithmetic: for Inches may be reckoned Primes; the next Sub-division Seconds, and so on: and a Series of these Sub-divisions may be conveniently noted down thus; Feet 17 : 2' 11" 10''' ; which is 17 Feet 2 Inches, 11 Twelfths, and 10 Twelfths of a Twelfth, and are thus multiplied.

$$\begin{array}{r}
 \text{Feet.} \\
 \begin{array}{r}
 5 : 2' : 3'' \\
 4 : 7 : 5 \\
 \hline
 \end{array} \\
 \begin{array}{r}
 2' : 1'' : 11''' : 3'''' \\
 3 : 0 : 3 : 9 \\
 20 : 9 : 0 \\
 \hline
 \end{array} \\
 \text{Product, Feet } 23 : 11' : 5'' : 8''' : 3''''
 \end{array}$$

To understand this you must know that Seconds multiplied by Seconds give Fourths, and that Seconds by Primes give Thirds, and the like, according to the Addition of the Indices. But to explain the Operation a little more, I say 5 times 3 is 15''', set down 3'''' and go 1; then 5 times 2 is 10'', and the 1 is 11'', and so on.

You may observe, according to this Method of Notation, that Feet, &c. may be thus very properly set down
5° : 2' 3''.

Note, It is a Rule among Masons in measuring Chimnies to multiply its Height from the Foundation to the Hearth of the next Chimney by its Girth, taking the Thickness of the Jambs for its Thickness, proceeding thus till they come at the Shafts; which they girth likewise, and multiply by their Heights, and augment this last Product by a Third.

Example 12. Suppose a Chimney, from the Foundation to the Hearth of the next, to be 11 Feet 7 Inches, let its Girth be
S 2 29 Feet

29 Feet 5 Inches, and the Thickness of the Jamb be $2\frac{1}{2}$ Bricks, then how many Rods in it?

	<i>Feet.</i>	<i>Inches.</i>	
	29	: 5	
	11	: 7	
	17°	: 1' : 11"	
	323	: 7	
1 $\frac{1}{2}$ Brick.	340	: 8 : 11	
$\frac{1}{2}$ Brick.	113	: 6 : 11	
$\frac{1}{2}$ Brick.	113	: 6 : 11	
<i>Feet. Inch.</i>			
272 : 3	567	: 10 : 9	2 Rods.
	544	: 6	
Remains -	23	: 4 : 9	

That is 2 Rods, and $23^{\circ} : 4' : 9''$

After this Manner are you to proceed : for by measuring the Chimnies between every Floor, and computing the Shafts, at last, you will have the whole Stack.

Observe in measuring Returns in Brick-work, not to measure the Thickness of the Wall twice: As, suppose a square Place walled about, and that you measure the same on the Out-side; then in two Sides you measure the full Length, and in either of the other two abate twice the Wall's Thickness, or rather measure them so much short; but Practice will inform you in Things of this Nature, better than many Words: And it will not be improper to form to yourself some Method of Book-keeping, to enter such Surveys in, as perhaps it will be more convenient to cast them up afterwards.

C H A P. XVII.

How to take an inaccessible Altitude.

I Desire to know the Height of the Tower B C ; but the horizontal Line A B being unpassable, I have no other Means than to take two Stations of different Altitude in the Tower A D. Fig. 93.

The Eye being at A, I observe the Top of the Tower C, and find by my Instrument, the Angle C A B to be 52 Degrees : Then from the Top of the little Tower A D, which is 32 Feet high, at D, I again observe C, and find the Angle A D C to be 124 Degrees.

Having finished my Operation, it is plain, in the Triangle A D C, there is known the three Angles, besides the Side A D ; for 90 Degrees, lessened by 52 Degrees, is the Angle A ; and the Angle A, and the Sum of the Angles A and D, taken from 180 Degrees, is the Angle C of it.

From 90°	A d { 124	From 180°
Take 52	38	Take 162
38 the Angle A.	Sum 162	18 the Ang. C.

Now it is only necessary to find A C in this Triangle, which is as follows.

As Sine of C 18°	—	—	—	9.489982
To A D, 32 Feet	—	—	—	1.505150
So is the Sine of D 124	—	—	—	9.918574
				11.423724
To A C—86	—	—	—	1.933742

Now, in the right-angled Triangle A B C, I have the Logarithm of A C, besides the Angle A : Therefore,

As Sine of B, or Radius	—	—	—	10.000000
To A C	—	—	—	1.933742
So Sine of A 52°	—	—	—	9.896532
To B C 67.65	—	—	—	1.830274

S 3

Which

Which 67.65 added to your own Height, or Height of your Eye at A, when you first observed the Top C of the Tower, is the Height required.

But if you can have Access from the Station A, to the Bottom of the Tower at B, measure the Space AB, and by the foregoing Directions find the Angle $\angle CAB$; and then the Proportion will be

As Radius
To AB,
So Tangent of the Angle $\angle CAB$,
To Height of the Tower BC.

Or, it may be done by a Staff or Rod set perpendicular (if the Ground be plain, no matter whether it is level or not.)

Let the Staff be set up on that Side of the Tower from the Sun, so that the Shadow of the Top of the Tower may just fall upon the Top of the Staff, and then will the Shadow of the Top of the Staff and of the Tower fall in one Point, and each will be in Proportion to the Length of its Shadow.

Example. Let the Tower be BC, and the Shadow of the Top of the Tower fall upon the horizontal Line AB; at A set up the Staff between B and A, so that the Shadow of the Top of the Staff, or Pole, may likewise fall at A, then it will be,

As AE (the Distance from the Point A, where the Shadow of the Top of the Tower falls to the Point where the Staff is erected)
To EF, the Height of the Staff or Pole;
So AB, the Distance from A to the Foot of the Tower,
To BC the Height of the Tower required.

C H A P. XVIII.

Of Colours proper to be used in washing or shading Maps and Plots, and how to prepare them and lay them on.

BEFORE I speak of the Colours themselves, it will be necessary to give an Account of the proper Liquids to mix them with, so as that they may be so transparent as not to obliterate or deface the Lines, Trees, Houses, or whatsoever is necessary or ornamental to the Map, and yet that they may be durable, so as not to fade and decay; and likewise so strong, as not to sink so much into the Paper, as to destroy the original Beauty and Strength of the Paper: And these Waters are made as follows:

To make ALUM-WATER.

Take a Pound of Alum and beat it to Powder; then put it into a Gallon of Water and boil it, till the Alum be melted; and when cold put it in a Vessel for your Use.

Note, If you wet your Paper with it before you lay the Colours on, it will not only keep them from sinking into it, but also add a Lustre and Beauty to them when laid on.

To make GUM-WATER.

Take a Quantity of the whitest and clearest Gum-Arabic, and bruise it into small Pieces; then put them into a fine Linnen Rag, and hang the same in clean Water till it be dissolved; then put your Fingers into the Water, and try its Stiffness, for if it feel too glutinous, add Water; but otherwise add more of the Gum at your Discretion.

Note, You temper most of your Colours with this Water.

The Names of COLOURS proper to wash and adorn Maps, &c.

GREENS.

Verdigrase, Sap-green, Verditer, Bice.

YELLOWS.

Saffron, Yellow-berries, Arsnick, Masticot, Gamboge.

BLUES.

Ultramarine is the best Blue, Indico, Logwood, Bice, Verditer, Litmose.

BLACKS.

Lamp-black, Printers-black, *Indian Ink*, burnt Shavings of Ivory or Harts-horn, &c.

WHITES.

White-lead in Flakes.

BROWNS.

Soot of Wood, Rinds of Walnuts, *Spanish Brown*, and Umber.

REDS.

Carmine the best, *Indian Cakes*, Vermilion, Red-lead, Roset, Turnsole, Brazil, Scarlet Flocks, Lake, &c.

Now, as these Colours (according to their Nature) are to be ground, washed, steeped, or dissolved, boiled, or perhaps burnt before they are ground: Here follow the various Methods of ordering them.

To grind a Colour.

Put the Colour on the Grinding-stone and break it pretty small with your Muller, if it be in Lumps, and pour a little Water at a time to it, as Occasion requires; and so grind it with your Muller till it be very fine, and then put it into any thing, and when dry reserve it for your Use.

Note, *The Colours that want grinding are, Vermilion, Lake, Indico, White-lead, and Masticot.*

To

To wash Colours.

Put the Colour and some clean Water into a Bason, often stirring it together ; then let the Colour settle, and pour the Water off, and put in some more clean, doing as before ; so repeating the Method till the Water you pour off has nothing of Filth swimming at the Top : Now, before you take the Colour out of the Bason daub it thin about its Sides, and as it dries, some will drop down, tho' what at last sticks to the Sides will be the better Colour ; all which things put on a Sheet of Paper, the better still to dry it, and it is done.

Note, The Colours to be washed are Red-lead, Arsnick, Rice, Rosset, Verditer, and Spanish Brown.

To steep Colours,

Is to dissolve them in Liquor, either hot or cold ; and the Colours to be dissolved in cold Liquors are Litmose, Saffron, Yellow-berries, Sap-green, *Indian Cakes*, and Gamboge ; but the Colours to be steeped or boiled are Rinds of Walnuts, *French Verdigrease*, Turnsole, Brazil, Logwood, and Wood-foot.

Note, Colours steeped in hot Liquors are to be kept close in Glasses.

To burn Colours.

Put your Colour and Water into a Crucible, and cover the Crucible with Clay, setting it into a hot Place of the Fire till it be red ; then take it out of the Fire, and the Colour out of the Crucible when it is cold, and it is fit to grind.

Note, Colours requiring burning are Lamp-black, Hartshorn, Spanish Brown, Ivory, Printers-black, Umber, and other gross ones.

To temper Colours.

Put your Colour into a deep Shell and some Gum-water to it, which will soon mollify it, and with your Finger do the Colour against the Shell, till all the knots be dissolved ; then with your Pencil stroke the Colour from the Side of the Shell to the Bottom, and it is done ; only observing if it be too thick, to add more of the Gum-water to it.

Note, Washed Colours are thus to be tempered, but as to steeped ones the Liquor of them is only to be used.

The

The ordering each Colour, without Commixture, to the best Advantage.

REDS.

Vermilion, if ground and tempered with weak Gum-water gives a deep Red or Scarlet Colour.

Roset, washed and tempered with Gum-water, is at first almost like Lake, tho' subject to fading; but tempered with Brazil-water, it will be of a deep Colour.

Carmin, tempered with Gum-water not stiff, gives the best of Reds.

Lake, ground or tempered with Gum-water, is a deep Pink or bloom Colour.

Red-lead washed is a Colour between a Red and an Orange.

Turnsole, put into Vinegar with some Gum-water over a Chafing-dish, and well squeezed in the Liquor, will give a good Colour for shadowing any Yellow with.

Indian Cakes, thus managed, is a good transparent Red.

Brazil, the Filings or Raspings in Vinegar and small Beer in an Earthen Vessel, with Powder of Allum boiled to heighten the Colour, gives a light violet Colour.

Note, *After you have strained it off, it is proper to add a little Gum Arabic to it.*

Scarlet-flocks boiled gently in Water about five Hours, putting in a Spoonfull of Soap-lees, gives a transparent Red or Scarlet Colour.

GREENS.

Verditer, washed and tempered with Gum-water, makes no transparent Green: But half a Pound of French Verdigrease, with an Ounce of Argol boiled in a Quart of Water, makes a good transparent Green, inclining to a Blue.

Note, *This Colour requires about five Hours gentle boiling, and when it is done, it must stand a considerable time a-settling, that you may pour the clear from it.*

Bice, washed and tempered with Gum-water, makes a good Green, tho' not transparent.

Sap-green steeped in Water, with a little Alum powdered, is a Green to shadow withal.

YEL-

Y E L L O W S.

Saffron, steeped at Night in Water, makes an excellent clear gold Colour.

Yellow-berries, steeped in Alum-water, is a transparent Yellow.

Arsnick, washed and tempered with Gum-water, is a Gold-colour; though there are several Kinds of it, as more Red, or more Yellow.

Masticot, ground and tempered with Gum-water, is a good, tho' no transparent Yellow

Gamboge, with Water only, does at last make the best and most transparent Yellow.

B L U E S.

Ultramarine, tempered with weak Gum-water, is the best Blue.

Indico, tempered and ground with Gum-water, is a deep Blue, and fit to shadow Blues withal.

Logwood, managed as *Brazil* (which see among the Reds) is a very good Purple.

Bice, washed and tempered with Gum-water, is a good Blue, tho' not transparent, of which there are several Sorts, lighter or darker.

Verditer so managed is a good Blue, but not transparent.

Litmoſe cut into small Slices, and steeped in weak Water (made of Gum Black) a Day or more, is a transparent Blue.

B L A C K S.

Lamp-black, *Printers-black*, *Ivory*, and *Hart's-horn* Shavings burnt, ground, and tempered with Gum-water, are all good Blacks.

Indian Ink, ground and tempered with a weak Gum-water, is the best Black to shadow deeper Blacks withal.

W H I T E S.

White-lead, ground and tempered with Gum-water, is the best White.

B R O W N S.

Wood-foot, or Rinds of Walnuts boiled in Water and strained with some Gum-water to it, either of them is a good Colour to distinguish Roads, &c. by.

Umber burnt, ground, and tempered with Gum-water, is a good Haw colour, and looks well on Gold.

Spanish Brown so ordered makes a good Liver Colour.

Now,

Now, as these Colours are in all respects sufficient in themselves to beautify any Draught: I shall only instance how they may be made fadder or lighter, by mixing those of different Kinds together, as Greens with Yellows, &c. For tho' the Colours mentioned be sufficient, yet it may be requisite it should be paler or fadder. Therefore as to a Green, Verdigrease-water, and Yellow-berry-water, according as they are mixed, make a lighter or darker Green, and, as being transparent, is a Colour better for this Use.

And you may observe also, that a different Colour may be produced by mixing different Colours together; as a deep Green may be made of Litmose and Yellow-berry-water mixed together, viz. a Blue and a Yellow, &c.

Knowing how to make a Colour lighter or fadder, you are able to make Colours proper to shadow with, as light Colours are always shadowed with fadder of the same Nature, viz. lighter Greens with deeper, &c. as for instance, observe;

Verdigrease may be shadowed with Indigo and Yellow-berry-water mixed together.

Gamboge and Yellow-berries may be shadowed with Umber mixed with Red-lead or Vermilion.

Masticot may be shadowed with Red Arsnick.

Verditer and Bice with Indico.

Spanish Brown may be shadowed with burnt Umber and Brazil-water mixed together.

Wood foot and Walnuts with Umber, and Umber with burnt Umber, &c.

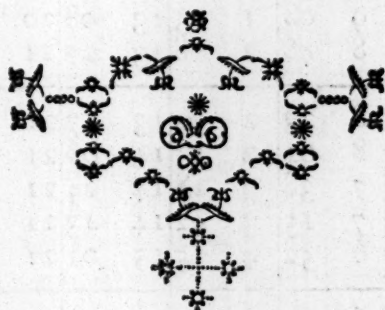
Which is obvious to any that can rightly distinguish Colours singly of themselves, and have a Notion of producing others by co-mixing those of different Kinds, &c. as I have already instanced.

Having your Colours ready for shadowing, and all in order to lay them on your Draught, it is presupposed you are stocked with Pencils, so as to have Pencils of different Sizes for each Colour, to prevent the Trouble of frequently cleansing them (in Water with a Linen Rag) as you have Occasion to use different Colours for one Use or another.

Chuse Pencils by their Fulness of Hair next the Quill, which ought *gradatim* to lessen to the End in a Point, as you may try by drawing it through your Lips.

Now, as to the actual laying on the Colour: Suppose you had the Plan of a Survey only in its Lines, and would lay the Colour about a Field in it; dip your Pencil into the Colour, and draw it along of an equal Breadth about the Field, in the Inside thereof, broader or narrower, as the Field is more or less
in

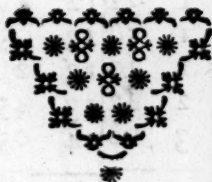
in Quantity ; then just wet your Pencil in Water, and pencil off by degrees what you before did, towards the Middle of the Field ; only, *Note*, You are to pencil off only from the Inside of the coloured Breadth ; for by this means the Colour from the Outside of the Field will seem by Degrees to vanish, and so look the more beautiful to the Eye. And when Landscapes are divided into several Tenures, or great Estates so divided, it is proper in one intire Colour to strike out each Parcel, and a different Colour to each Parcel is needful : And it is adviseable that every adjacent Field should be bounded with a Colour somewhat different, which must be refered to the Judgment of the Artist, as the colouring of Maps is not always for Ornament only.



Days.	Jan.		Feb.		March.		April.		May.		June.	
	South.		South.		South.		North.		North.		North.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.
1	21	41	13	44	3	22	8	37	18	06	23	10
2	21	31	13	24	2	59	8	59	18	21	23	14
3	21	21	13	04	2	35	9	21	18	35	23	17
4	21	10	12	43		11	9	42	18	50	23	20
5	20	59	12	22	1	48	10	03	19	04	23	23
6	20	47	12	01	1	24	10	25	19	18	23	25
7	20	35	11	40	1	00	10	46	19	31	23	26
8	20	22	11	19	0	37	11	07	19	44	23	28
9	20	10	10	58	0	13	11	27	19	57	23	29
10	19	56	10	36	Nor.	10	11	48	20	10	23	29
11	19	43	10	14	0	34	12	08	20	22	23	29
12	19	29	9	52	0	58	12	28	20	33	23	29
13	19	15	9	32	1	21	12	48	20	45	23	28
14	19	00	9	08	1	45	13	08	20	56	23	27
15	18	45	8	46	2	08	13	27	21	07	23	25
16	18	30	8	23	2	32	13	47	21	17	23	23
17	18	24	8	01	2	55	14	06	21	27	23	21
18	17	58	7	38	3	18	14	24	21	36	23	18
19	17	42	7	15	3	42	14	43	21	45	23	14
20	17	25	6	52	4	05	15	01	21	54	23	11
21	17	08	6	29	4	29	15	15	22	33	23	07
22	16	51	6	06	4	52	15	37	22	11	23	02
23	16	34	5	43	5	15	15	55	22	19	22	57
24	16	16	5	20	5	38	16	12	22	26	22	52
25	15	58	4	57	6	01	16	19	22	33	22	46
26	15	39	4	33	6	23	16	46	22	40	22	40
27	15	21	4	09	6	46	17	02	22	46	22	34
28	15	02	3	46	7	08	17	19	22	51	22	27
29	14	43			7	31	17	35	22	57	22	20
30	14	23			7	53	17	50	23	02	22	12
31	14	04			8	15			23	06		

Days.	July.		August.		Sept.		Octob.		Nov.		Dec.	
	North.		North.		North.		South.		South.		South.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.
1	22	04	15	06	4	18	7	20	17	42	23	07
2	21	54	14	48	3	55	7	42	17	58	23	11
3	21	46	14	30	3	32	8	05	18	14	23	15
4	21	37	14	11	3	08	8	27	18	29	23	19
5	21	28	13	52	2	45	8	50	18	45	23	22
6	21	18	13	33	2	22	9	12	18	59	23	24
7	21	08	13	14	1	59	9	34	19	14	23	26
8	20	57	12	54	1	35	9	56	19	28	23	28
9	20	46	12	35	1	12	10	17	19	42	23	28
10	20	35	12	15	0	48	10	40	19	56	23	29
11	20	23	11	55	0	29	11	00	20	09	23	29
12	20	11	11	35	0	02	11	22	20	22	23	29
13	19	59	11	14	Sou. 22	11	43	20	34	23	28	
14	19	46	10	53	0	45	12	03	20	46	23	27
15	19	33	10	33	1	09	12	24	20	58	23	25
16	19	20	10	12	1	32	12	45	21	09	23	22
17	19	06	9	51	1	56	13	05	21	20	23	19
18	18	52	9	29	2	19	13	25	21	30	23	16
19	18	38	9	08	2	42	13	45	21	40	23	12
20	18	23	8	46	3	06	14	05	21	50	23	09
21	18	08	8	25	3	29	14	24	21	59	23	04
22	17	53	8	03	3	53	14	44	22	08	22	59
23	17	38	7	41	4	16	15	03	22	16	22	53
24	17	22	7	19	4	39	15	21	22	24	22	47
25	17	06	6	56	5	02	15	40	22	32	22	41
26	16	50	6	34	5	25	15	58	22	39	22	34
27	16	33	6	12	5	48	16	16	22	45	22	27
28	16	16	5	49	6	11	16	34	22	52	22	19
29	15	59	5	26	6	34	16	51	22	57	22	11
30	15	42	5	04	6	57	17	08	23	02	22	02
31	15	24	4	41			17	25			21	53

IT is necessary to insert this Table of the Sun's Declination, because I laid down a Method, *Page 249*, for correcting the whole Work of a Survey, either County or City, by Astronomical Calculation, in which the Sun's Declination is required; but, as I conclude, that those who understand so much Astronomy as to correct their Survey by these Directions, need not be taught the Use of these Tables of Declination; I think a further Explanation altogether needless.



APPEN.

Fig: 82.

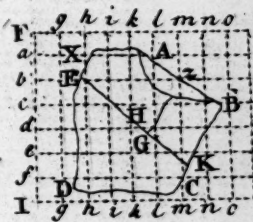
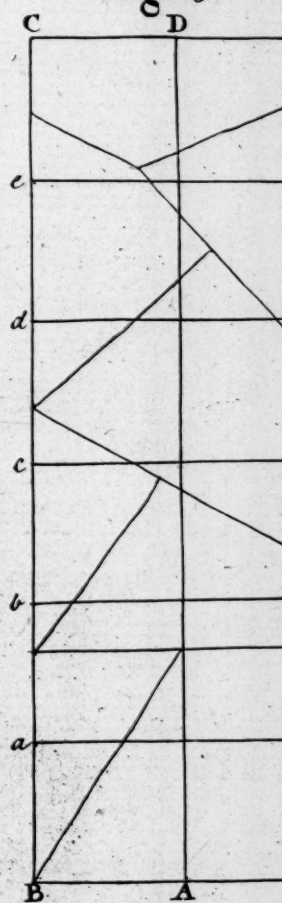
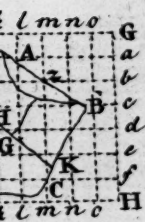


Fig: 85.



82.



85.

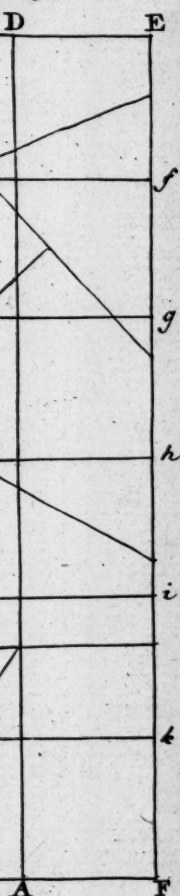


Fig: 83.

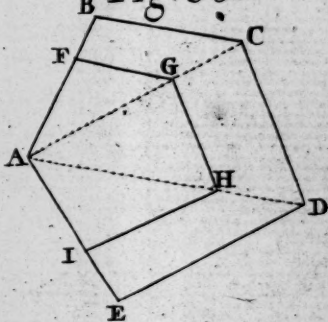


Fig: 86.

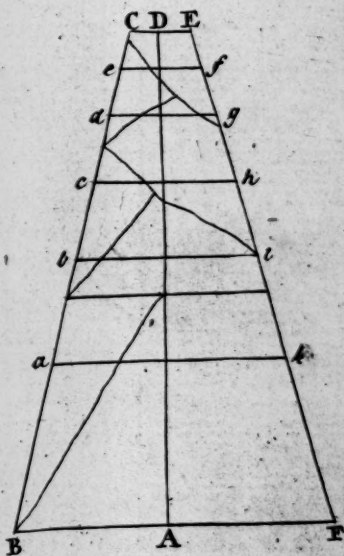
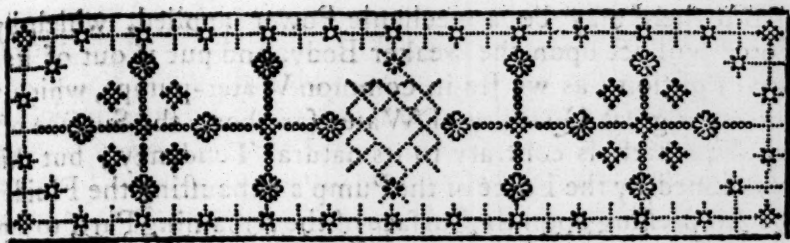


Fig: 84.





A P P E N D I X.

Of LEVELLING, CONVEYING, *of* WATER, &c.



CONVEYING of Water depends upon level-
ling, because all the Surface of the same Body
of Water, or other Liquids, inclines to be
equally distant from the Center of the Earth,
except by the Pressure or Agitation of some
other Accident that affects it; as a Stone,
thrown into the Water, does by a sudden Dis-
turbance of the Fluid, cause the circumjacent Parts to fly off in
concentric Circles; but, when that is over, the Surface of
the Water inclines to reduce itself to its first natural horizontal
Position: And the like may be said of a Liquid that is disturbed
by Wind, or any other Accident; its Surface is rough, in Pro-
portion to the Opposition it meets with, and the Quantity of
Water acted upon. And thus the vast Ocean is more affected
and disturbed, its Waves more swelling and boisterous than those
of a River, because a greater Quantity of Water; and a River,
whose Current tends to Windward more than that which tends
to Leeward, though with the same Gale of Wind, because it
meets with more Opposition; but every Quantity of Water, whe-
ther greater or less, inclines, as soon as the Disturbance is removed,
to reduce itself to its former horizontal, or rather globular Form.

T

But

But there may be a mechanic Power applied, which by its Force will act upon the weaker Body, and put it out of its natural Position; as we see in common Water-pumps, which will convey a great Quantity of Water far above the Surface of the whole, which is contrary to its natural Tendency; but this is occasioned by the Force of the Pump's exhausting the Fluid, and the Air pressing upon the Surface of the remaining Part, to supply the Deficiency, produces that Effect, as may be thus proved.

If the Reservoir of Water, that supplies the Pump, was so closely luted, that the circumambient Air could not penetrate to supply the Place of the exhausted Fluid, the Pump could not perform its Office; notwithstanding that there was Water enough to supply it, and the Pump stood deep enough in that Water to act upon it.

But setting aside that Part of the *Hydrostatics*, my Business now is to treat of the Possibility of conveying Water from any one Place to another Place assigned; and this may be brought under two Heads,

First, To convey Water in close Pipes, and to give Reasons for the Tendency of Water upwards, as we see the New-River Water conveyed by Pipes into the several Apartments in London, tho' two or three Stories high.

Secondly, To convey Water by an open Brook, Rivulet, or other Aqueduct, when the Water is not confined to Pipes, but lies open, as the New-River from Ware to London; or as any other River in its natural Course.

As a Preparative to either of these Methods of conveying Water, we are to consider the Level; for (as I said before) the Surface of the same Body of Water inclines to be equally distant from the Earth's Center; and it would be in vain to attempt to convey Water from one Place to another, without finding first, whether there is a Descent from the former Place to the latter, which is best performed by an Instrument called a *Level*, of which there are several Sorts; of some of which take the following brief Description.

Of a WATER-LEVEL.

A Water-level is a round Tube of Brass, or any solid Metal or Matter, of about three Feet long, or more or less at Pleasure. The Ends of it are turned up at Right-angles, to receive two Glass Tubes (one at each End) to be placed perpendicular to the Body of the Level, and to be luted close to the Brass so as

to retain the Liquid therein contained without leaking; and then pour into it so much common Water, or rather some other Liquid, of a Colour that may more easily be distinguished from the Colour of the Glass: fix it upon a proper Pedestal, with a Ball and Socket, &c. the Liquid in both Ends will be horizontal from the Principle above asserted, to wit, *That all Parts of the Surface of the same Body of Liquid, incline to be equally distant from Center of the Earth.*

Of the AIR - LEVEL.

There is another sort of Level called an Air-level, which is a strait cylindrical Glass Tube, filled within a Drop, with any Liquid that will not freeze, for which Reason Spirits of Wine is most frequently used, and from thence some call it a *Spirit-level*: This, when so filled, is hermetically sealed at the End at which the Spirit was poured in, or at both, if it was open at both the Ends; and when horizontal, the Bubble, or Place possessed with Air, will be in the Middle; but if never so little declining from an horizontal Position, the Bubble flies towards the upper End, because any Quantity of Air is lighter than the same Quantity of the Liquid contained in the same Level; but when the Bubble is in the Middle, the Level is truly horizontal; and consequently, having proper Sights fixed at each End, perpendicular to the Tube, and of equal Length; any thing seen through these Sights (whether small Holes, or cross Threads, &c.) when the Bubble is in the Middle of the horizontal Tube, is truly level or horizontal.

Of the PLUMB - LEVEL.

This is so common amongst Carpenters, Bricklayers, &c. that it needs not a large Description, it being composed of a Piece of Board or Plank, of two, three, four, five, or six Feet Length at Pleasure, and another Piece fitted into it at Right-angles, or perpendicular to the former, and set upon the Middle of it: At the Top of which a Line is fixed, and a Plumbet at the End of that Line, with a Hole for the Plumbet to play in without Interruption; and then, when the Base of the horizontal Part is so situated, the Plumb-line falls just in the Perpendicular; the said Base is then in a true horizontal Position; and, as the Plumb-line deviates from the Perpendicular, the Base declines in the same Proportion from an horizontal Position.

I might instance in several other Sorts of Levels, but they that know the Use of these, may with a little Application know the

Use of all Instruments designed for that Purpose : But it is necessary to give the following Caution in the Use of Levels, *viz.*

The Earth is a Globe, and, as I have said before, the Surface of any Body of Liquid inclines to be in every Part of it equally distant from the Center of the Earth.—Therefore,

The less Distance you take in Levelling, the truer your Level is; for although the Surface of the Sea seems at a small Distance, to coincide with, or insensibly to differ from a Right-line, yet, if continued, it is evident, that as the Tangent-line cannot coincide with the Circle, but only touch it in one Point, so the Level being a strait Tube, when placed horizontally, the Middle of it is nearest the Center, and most affected with the Gravity that acts upon the Object that is in the Tube. As for Instance;

Every thing that we call perpendicular to the Surface of the Earth, must have an equal Tendency to its Center : Thus, for Illustration, suppose *DHIK* be the Circumference or Surface of the Earth, *A* its Center ; let *DB*, and also *EC* be built perpendicular (as the common sound Way of Building is) the Lines *CE* and *BD* tend equally to the Earth's Center, and terminate therein ; and therefore cannot be parallel, because they are at some Distance at the Surface, and terminate in one Point at the Center *A*.

Hence, though *DB* be equal to *EC*, and consequently *AB* equal to *AC*, yet the Line *FC* drawn at Right-angles with *AC*, and the Line *BG* drawn at Right-angles with the Line *AB*, will intersect each other in *L* ; so that if from *C* you would behold *B*, it would be supposed much lower ; that is, the Line *BD* would be thought shorter by this Way of Levelling, than the Line *EC*, because the Point *B* falls under or below the visual Line *CF*, which by this Method, must be deemed horizontal, because perpendicular to the Line drawn from *C* to the Center of the Earth.

But by the same Rule *BG*, being drawn perpendicular to *AB*, will represent *EC* shorter than *BD*, because the Point *C* appears below the Line *BG* : and although this cannot vastly err in the small Space that comes within the Reach of any one Observation, yet it is sufficient to prove that no two Walls, &c. can be parallel to one another, if they are both perpendicular, and tend equally to the Center of the Earth ; and consequently, that the shorter Distances we take in Sight-levels, the nearer we approach to the Truth.

From what has been said, the different Ways proposed for conveying the Water, *viz.* by close Pipes, and by open Rivers

or Rivulets, is to be performed as follows; setting aside the mechanic Force before hinted, which raises Water above its natural Tendency, as the Works at *London-Bridge* supply Places in the City that are situated very much above the Surface of the River *Thames*.

And first of conveying Water by its own Gravity, without any mechanic Force added to it, when it is to be carried in close Pipes.

Suppose it was proposed to convey Water from the Head of a Fountain at G, to a House at L, and to carry it up two Pair of Stairs in the same, when conveyed by Pipes under-ground to the Foundation; and it is required to know, whether it is possible to be done, and if it can be done, to know how it is to be performed.

And first, in order to know, whether it is possible to convey Water from any Spring, &c. to any other Place proposed, take this general Rule by the Way.

If the Place to which the Water is to be conveyed, is lower or nearer the Centre of the Earth than the Head of the Spring, it is certain the Water will incline to the lowest Place by the Hypothesis before asserted: But if the Place proposed be higher or further from the Centre of the Earth than the Head of the Spring, the Impossibility of its being conveyed is proved from the same Foundation; and this is to be determined by the following Directions.

Place your Water-level at some convenient Distance from the Spring-head, in a Right-line towards the Place to which the Water is to be conveyed; as at 30, 40, 60, or 100 Yards distance from the Spring-head. Then have in Readiness your two Station-staves divided into Feet, Inches, and Parts of Inches, from the Bottom upwards: Being thus provided, cause one (whom you may call your *first Assistant*) to set up one of the said Staves at the Spring-head, and require another (which you may call your *second Assistant*) to erect the other Staff beyond your Instrument, at 30, 40, 60, or 100 Yards foward, towards the Place to which the Water should be conveyed. These Station-staves being erected perpendicular, and your Water-level about the Mid-way precisely horizontal, go to the End of the Level; and looking thro' the Sights, cause your *first Assistant* to move a Leaf of Paper up and down your Station-staff, till thro' the Sights, you see the very Edge thereof; and then, by some known

known Sign or Sound, intimate to him that the Paper is then in its true Position: Then let this *first Assistant* note against what Number of Feet, Inches, and Parts of an Inch, the Edge of the Paper resteth, which he must note down in a Paper: Then your Water-level remaining immoveable, go to the other End thereof, and looking through the Sights towards your other Station-staff, cause your *second Assistant* to move a Leaf of Paper along the Staff, till you see the very Edge thereof through the Sights; and then (by some known Sight or Sound) cause him to take notice what Number of Feet, Inches, and Parts of an Inch, are cut by the said Paper, which order him also to keep in Mind, or note in a Paper, as your *first Assistant* did.

This done, require your *first Assistant* to bring his Station-staff from the Spring-head, and cause your *second Assistant* to take that Staff, and carry it forwards towards the Place to which the Water is to be conveyed, 30, 40, 60, or 100 Yards, and there to erect it perpendicular, as before, letting your *first Assistant* stand at the Staff, where your *second Assistant* before stood: Then in the Mid-way between your two Assistants, place your Water-level exactly horizontal, and looking through the Sights thereof, cause your *first Assistant* here to move a Paper up and down, and when you give them a Sign to note what Number of Feet, Inches, and Parts of an Inch, are cut by the Paper, and note them down: Then going to the other End of the Water-level, looking through the Sights, and cause your *second Assistant* to move a Paper along the Staff, and note the Feet, Inches, and Parts of an Inch, as before.

Then cause your *first Assistant* to bring away his Station-staff, and cause your *second Assistant* to take it, and carry it 30, 40, 60, or 100 Yards forwarder, towards the Place to which the Water is to be conveyed: And, leaving your *first Assistant* at the Place where your *second Assistant* last stood, place your Water-level again in the Mid-way between your two Assistants; and looking through the Sights, as before, cause each of them to move a Leaf of Paper up and down their Station-staves, and note down in their several Papers the Number of Feet, Inches, and Parts of an Inch cut, when you looked through the Sights of your Water-level.

In this Manner you must go along from the Spring-head to the Place unto which you would have the Water conveyed; and if there be ever so many different Stations, you must, in all of them, observe this manner of Work precisely: So by comparing the Notes of your two assistants together, you may easily know whether the Water may be conveyed from the Spring-head

head to the desired Place or not, though there be many Hills between.

Here note, That in your Passage between the Spring-head and the appointed Place, from Station to Station, you must observe this Order, otherwise great Error will ensue, viz. That your *first Assistant* must, at every Station, stand between the Spring-head and the Water-level; and your *second Assistant* must always stand between your Water-level and the Place to which the Water is to be conveyed: Thus, by observing the Order in your Work, you shall have no Confusion, neither shall one of your Assistants take more Pains than the other.

Having thus orderly proceeded from the Spring-head to the Place appointed, call both your Assistants together, and cause them to give in their Notes of Observations at each Station, and add them together severally: Then, if the Note of the *second Assistant* exceed (or be greater than) the Note of the *first Assistant*, take the lesser out of the greater, and the Remainder will shew you how much the appointed Place, to which the Water is to be brought, is lower than the Spring-head.

The first Assistant's Note.			
Station.	Feet.	Inch.	Parts.
1	15	3	$\frac{1}{2}$
2	2	1	$\frac{1}{4}$
3	1	6	0
Sum	18	10	$\frac{3}{4}$

The second Assistant's Note.			
Station.	Feet.	Inch.	Parts.
1	3	2	$\frac{3}{4}$
2	14	0	$\frac{1}{4}$
3	3	11	0
Sum	21	2	0

By this Table you may perceive, that the Notes of the *first Assistant*, collected at his several Stations, being added together, amount to 18 Feet, 10 Inches, and $\frac{3}{4}$ of an Inch; and the Notes of your *second Assistant* at his several Stations, being added together, amount to 21 Feet and 2 Inches: So the Number of the *first Assistant's* Observations, being taken from the Number of the *second*, there will remain 2 Feet, 3 Inches, and $\frac{1}{4}$ of an Inch; and so much is the Place, to which the Water is to be brought,

brought, lower than the Spring-head, according to the strait Water-level; and therefore the Water may easily be conveyed.

Note, That H represents the Instrument or Water-level, and K the Station-staves; G the Spring-head, and *Fig. 95.* L the Place to which the Water is to be conveyed,

Here *Note,* That when you have called your two Assistants together, and examined there several Notes, and added them together; if then you shall find the Sum of your *first Assistant's* Notes to be greater than the Sum of your *second Assistant's* Notes, that then it is impossible to bring the Water from that Spring-head to the intended Place: But if the Sums of the Notes, of your Assistants do exactly agree, there is then a Possibility of effecting it, if the Distance be but short, though with more Charge and Difficulty.

• When Water-levels are made with Telescopes four or five Feet long, then great Distances may be seen very distinct; • and in such Cases, Allowances may be made for the Curvature • of the Earth, as before hinted, which must be as in the following Table.

Distances in Chains	Allowances in Inches and Parts.
5	,031
10	,124
15	,279
20	,496
25	,772
30	1,116
35	1,519
40	1,984
45	2,511
50	3,088
55	3,751
60	4,464
65	5,239
70	6,076
75	6,948
80	7,936
85	8,959
90	10,044
95	11,191
100	12,400

• Or, by the Pen, thus; multiply
• the Square of the Chains by 124,
• and divide the Product by 100000.
• So, if it be required to know what
• Allowance must be made, when the
• Level is 40 Chains distant from the
• Station-staves: Work thus,

$$\begin{array}{r}
 40 \\
 40 \\
 \hline
 1600 \\
 124 \\
 \hline
 6400 \\
 3200 \\
 \hline
 1600 \\
 100000 \overline{) 198400} (1,984
 \end{array}$$

• That is, almost two Inches. And
• in like Manner, if the Station-staff
• were distant from the Level 80
• Chains, or one Mile, the Allowance
• is 7 Inches $\frac{2}{5}$.

• And

‘ And by these Allowances must each Observation be lessened in the Accounts of both your Assistants. These Rectifications give us the true Level ; but for the Current of the Water another Allowance must be made, according and proportionable to the Velocity required in that Current.

‘ *Lastly*, Observe, that of these three Allowances, that, for the Error of the Level, which is the first, is always proportional to the Sum of the Distances ; and the second, which is for the Curvature of the Earth, is as the Sum of the several Distances ; and the last, which is for the Current, is like the first, and is as the Sum of the Distances themselves.’

Note 2, That the most approved Authors, concerning this Particular, do aver that, at every Mile’s End, there ought to be allowed $4\frac{1}{2}$ Inches more than the strait Level, for the Current of the Water.

Note 3, If there be any Hill lying in the Way between the Spring-head, and the Place to which the Water is to be conveyed ; you must then cut a Trench by the Side of the Hill, in which you must lay your Pipes equal with the strait Water-level, with the former Allowance. And if in this Case there be a Valley, yet laying the Pipes underground, though it be a great Descent to the Bottom of the Hill, and a great Ascent to the Top of the other, yet the Water will flow in the Pipes to the Top of the other Hill, by reason of the Pressure of the Descending Water, provided that the Top of the Hill to which the Water is to be conveyed, lie below the horizontal Line of the Place from whence the Water is to be brought.

Note 4, That in conveying of Water to an appointed Place, it is not always convenient to bring it from the Spring-head by the nearest Distance, or in a strait Line, but by a crooked or winding Way ; and you ought also to lay the Pipes one up and another down, where they will have too violent a Current.

To know whether it is possible to convey Water by an open Rivulet, and to perform it, if it is practicable.

To effect this, take two Boards of about two Foot over, or thereabouts, of what Form or Shape is not material, the larger the better ; about the Midst of each of which Boards bore a Hole, into which put the Ends of two Sticks, setting them up in their Holes perpendicularly : The Sticks must be exactly of one Length, and have at the Tops of each of them Marks of white Paper, or the like ; and the Sticks must be of such Length,

Length, that when the two Boards with the Sticks in them are floating upon the Water, the Marks of Paper at the Tops of them may be seen at a competent Distance from the Pond. The two Sticks being thus seated in the Water, and in a Right-line (as near as you can guess) towards the Place to which the Water is to be conveyed; then, at as great a Distance as you can conveniently erect a third Stick upon the Land, setting a Mark thereupon in an erect Level with the two former, which are in the Water; removing the Marks that are either upon these in the Water, or that upon the Land, which you last erected, higher or lower (as Occasion offers itself) till you have brought them all three to a right Level: Then taking exact Notice how high the Marks are above the Top or Surface of the Water, go on with a fourth, a fifth, a sixth, &c. Staff, so long as you go in a Right-line; for having placed two Marks in an exact Level, it is easy to find as many more as you please; and when you are to alter your Course, that is, you are to vary from a strait Line, you may make use of your Instrument, and proceed in all respects as is before directed: And according as you find the Ground at the Place to which the Water is to be conveyed, either higher or lower than at the Pond, you may determine the Possibility or Impossibility of its being conveyed thither.

How to try whether Water may be conveyed from a Spring or Head to a designed Place, though at a considerable Distance.

At the Spring-head begin to make a little Trench, about three or four Poles long, towards the Place to which the Water is to be conveyed; whether this Trench be straight or crooked is not material in this Case. Then let so much Water run as will only fill this Trench, letting none run over at either End of the Trench; but when either End is full up to the Brink, then stop the Course of the Water. Now, if you find the Trench dry, or shallower of Water at that End which is next the Head, than at that End farthest from the Head, it declares the Ground to be falling. Then do the like for three or four Poles further, still making the Water to follow you. And having filled the Trench, so that the Water may stand level at both Ends thereof, then at both Ends erect two Staves of about four Foot long, at each End of the Trench, being both of equal Height from the Surface of the Water. This done, go on in the same Line about 10 or twelve Poles farther, where set up a third Mark so that you stand behind it, and looking to the middlemost Mark, you see

all the Tops, or all the Bottoms, according to which you measured your equal Heights, may all agree. And then, if that Stick, which is below the Mark, be longer than the other two, it shews a Descent; but if it be shorter, it shews the Ground is higher: But if any rising Place of Ground be in the Way, you may easily find the Height of its Rise by setting up a Stick, and measuring it, as before is shewed.

If it was required to be known, whether it is possible to convey Water from the Spring-head at A, in the Side of the Mountain G, to the House at F, near the Bottom of the Mountain I; Fig. 96.

First, find the Possibility or Impossibility of it, as before directed, and, if it is found practicable, convey the Water by an easy Descent, from the Spring-head at A, along the Side of the Mountain to B; but there being a great Valley BE, between the Mountain G and the Mountain I, consider whether the Hills, at the End of that Valley, be as high as to continue the Course of the Water with a Descent from B, and yet allow it a sufficient Descent to the House, or Place at F; and, if so, continue the Course of the Trench or River, by which the Water is to be conveyed from B behind the Mountain G, till it comes to turn towards the Mountain H, which is supposed to lie behind, or more remote than the Mountains G or I, and so convey it along the Side of the Hill or Mountain H, from C to D, and so much further as that Mountain extends behind the Mountain I; and so winding it about along the back Part of the said Mountain I, bring it about to E, and so by an easy Descent from E to the House at F.

But if there had been no Mountain H, or other Ground so high as the Trench CD, it must in that Case have been conveyed from B to E in a Trough large and strong, in proportion to the Quantity of Water to be conveyed: as the *New-River*, by which the Water is conveyed from *Ware* to *London*, is by a large leaden Trough, supported by strong Timber, and secured by strong Planks, and from thence called the *Boarded River*.

Again; If, notwithstanding the sufficient Descent from the Spring-head to the Place that the Water is to be conveyed to, some Hill shall interpose, of a continued Extent, and whose Top is above the Level with the Head of the Spring, it must, in such Cases be conveyed by a Passage cut in the said Hill, so deep as to answer the proper Descent from the Spring-head, and may either be left open, if the Ground is of no great Value; but if in valuable Ground, or in a Town, or the like, it may be arched

arched over, as the *New River* is, where it is conveyed through, or rather under *Ipsington*.

Note, Four Inches and an half Descent is allowed to be enough to convey Water a Mile.

Note also, That when Water is confined in Pipes, if it flows down a steep Hill till it comes at the Bottom, it will rise in the Pipes perpendicular, or otherwise, so high as to be near a Level with the Place it first proceeded from, whether a Spring or Reservoir of Water, contrived to supply the Place appointed.

How to cleanse Water, when the Course or Stream is foul and muddy.

Make a Trench three or four Perches long; or, if longer, it is better, and let it be a Foot and a half, or two Feet deep, viz. of Depth, and also of Breadth proportionable to the Quantity of Water to be conveyed by it, in order to be cleans'd; fill this Trench about a Foot thick from the Bottom, with Hurlock, or Chalk Rubbish, cut in Pieces as fit for a Lime-kiln; after that, fill it three or four Inches higher with Pebble-stones; and then fill it to the Top with Gravel or Earth, and the Water, by draining through that Passage, shall be so cleansed and purified, if foul, or so softened, if it is hard, as frequently Spring-water is, that it will be fit either for Washing, Brewing, or any of the Uses of common soft Water.

Of the Flowing of dry Grounds.

It is to be supposed in this Case, that there is a Quantity of Water sufficient for this Purpose, above the Level of the Ground to be overflowed, and this may sometimes be procured different Ways.

As first, when Land lies near some River, that has a considerable Fall or Descent; then, although that Part of the River, that lies nearest the said Land, has its Surface so far below the Surface of the Land, that it cannot be brought directly from that Part of the River to the Land; yet by going higher up the River, especially above a *Fall*, if there be any such Thing, it often happens that Water may from thence be conveyed with an easy Descent, according to the foregoing Rules, to the highest Part of the Land proposed; as for Instance: It would be impracticable to overflow any of the Land that confines the River

Let,

Lea, by the Water of the said River, yet the *New River*, tho' taken out of the River *Lea*, and confined more horizontal, and to an easier Descent; its Surface lies not only much above that of the River *Lea*, but above the Land by which the River *Lea* is bounded, when at a considerable Distance from *Ware*; so that it might be capable of overflowing the said Grounds.

Sometimes Water is forced by Mills, or other Engines: Sometimes a Spring happens on the Side, or towards the Top of a Hill. But, by whatsoever Means the Water is procured, at or above the Level of the Place to be flow'd, the Practice is as follows:

Having by *Drains* and *Dams* brought the Water to the highest Part of that Ground which you would flow, you must cut a little Trench as level as you can guess; (by an Instrument or the bare Eye) which Trench let be about nine Inches broad, and seven or eight deep, and fifteen or sixteen Feet in Length at the first, laying the Turfs which you cut out close to the Trench, on the lower Side thereof, with the Grass-side downwards. This done, let the Water into the Trench, allowing it to run over at the lower End thereof. And thus may you stop the Water with a Turf, and cause the Water to run over in any Part of the Trench.

Now, in the making of this Trench, if you find that the Water will not follow you, you must, with a Spade made crooked at the End, sink it deeper, and cast out the Earth; and, in your going on, go deeper and deeper, as you shall find Occasion, yet not deeper than the Water may just follow you; thus proceeding till you come to the further Side of the Ground. And in your Passage, according as you find the Ground to fall, you may make cross Trenches about four or five Poles one from the other, the same Way as is formerly said, till you have made Drains enough.

In case you are to carry your Water over some Ditch, Brook, or Valley, you must then make a Case of Boards nailed together, making a Trough thereof, through which the Water may run.

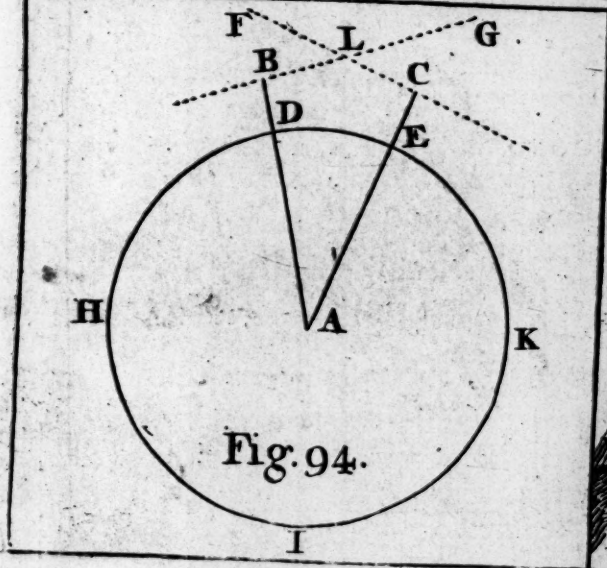
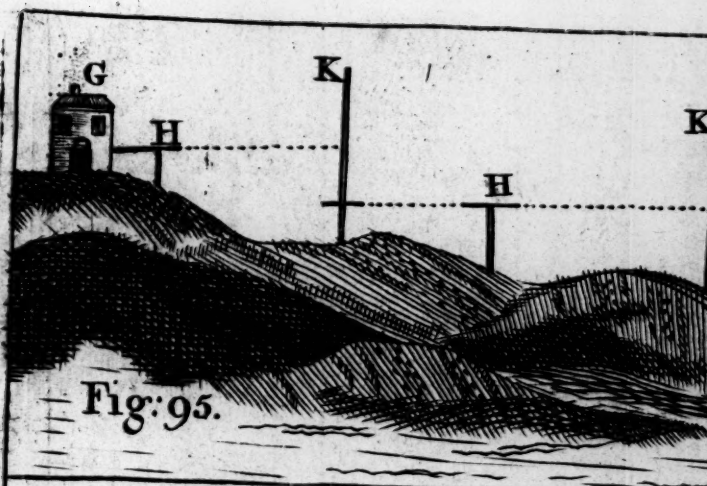
Of Draining wet Land.

If this wet Land lie adjacent to a navigable River, where the Tides ebb and flow regularly, the most proper Method of draining that Land is to make Trenches from that Place that is to be drained, to the said River, leaving a free Passage from the wet Ground to the said River, when the Tide is so low that the Surface of it is below the Water that is in the Ground to be drained,

drained, for then the Water will descend towards the said River and drain the Ground; always taking care, either by Sluices or falling Ports, to keep the Water from flowing out of the River into the said Ground, when the Tide is so high, that the Surface of the Water in the River is higher than in the Ground, as it was the Method of draining the Ground that was overflowed between *Dagenham* and the *Thames*, by *Dagenham Breach*.

But if there is no Opportunity of a Descent into a navigable River, or a low Ground, &c. but that some mechanic Engine or Machine is necessary: The Description of such Engine, and the Manner of using them, is too copious a Subject to be handled in this Tract of Surveying.





To front the last Page of the Appendix

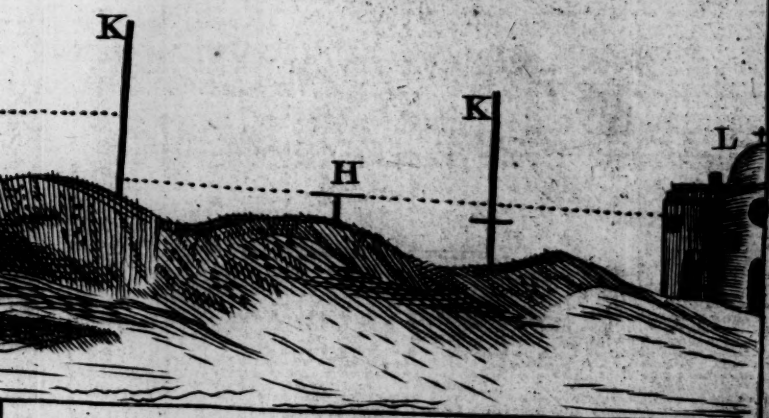
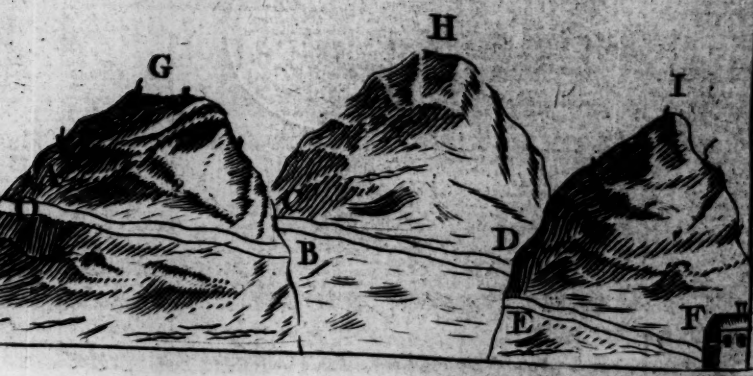


Fig.96.



Geodæsia Accurāta:

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T H E
P R E F A C E.

Courteous Reader,

IT would not only be needless, but even also ridiculous, for Men to praise an Art that all Mankind know they cannot live peaceably without : Nor can it be less valuable for its Antiquity ; seeing it is nearly as ancient as the World : For, how could Men set down to plant, without knowing some Distinction and Bounds of their Land ? But Necessity being the Mother of Invention, we find the *Egyptians*, by reason of the *Nile's* overflowing, which either washed away all their Bound-marks, or so covered them with Mud, that they were constrained to bring this measuring of Land first into an Art, and honoured much the Professors of it.

In brief, the great Usefulness, the pleasant and delightful Study, and the wholesome Exercise attending it, tempted so many to apply themselves, that at length, in *Egypt* (as in *Bermudas*) every Rustic could measure his own Land : Nor could it be limited or confined within those Bounds, seeing afterwards it was brought into *Greece* by *Thales*, and was for a long Time called *Geometry* ; but that being too comprehensive a Name for the Mensuration of Superficies only, it was afterwards called *Geodæsia*, or the Art of measuring Land, whose Honour amongst the Ancients in general was so great, that there needs no better Proof than *Plato's* ; and not only *Plato*, but most, if not all the learned Men of those Times, refused to admit any into their Schools that had not been first entered into the Mathematics, especially Arithmetic and Geometry.

From hence we may see, why those great Monuments of Learning, built upon such solid Foundations, continue unshaken to this Day ; which cannot be otherwise, seeing the Wisdom of those Designers appears evidently in chusing Geometry for their Ground-plot ; especially, when we consider that the ancient

Romans so valued and esteemed this kind of Learning, that they concluded that Man to be incapable of commanding a Legion, that had not at least so much Geometry in him, as to know how to measure a Field: Nor did they indeed, either respect Priest or Physician that had not some Insight in the Mathematicks. To which we may add, for the Honour of our own Island, that there has not likewise been wanting Worthies, both ancient and modern, who have encouraged and promoted this noble Science.

But, as it is needless to light a Candle to see the Sun by, so I shall not only desist from all other historical Observations of this kind but even also from enumerating the Particulars contained in this *Enchiridion*, seeing the Title, the Index, and the Book itself will sufficiently manifest the Performance, which I have endeavoured to render so easy, plain, and concise, as well as complete, that I hope, neither the ingenious Practitioner, nor yet the industrious Learner, will miss of their Expectation; especially if Accuracy and Exactness can recommend it to Favour.

I must grant, that tho' all Treatises of this kind require Practice and Experience, for their Life and Improvement; yet, nevertheless, it cannot be denied, but that plain Definitions, Rules, Figures, Examples, Explanations, &c. are not only previously necessary, but even also the best Preparative to the practical Performance of Surveying by the Chain only: All which considered, I thought an Essay of this kind, appearing so methodically complete, as well as plain and concise, would be no ways unacceptable to the Public, much less derogatory to the Works of any Author of this kind already published, seeing, Emulation in Arts and Sciences, when modestly performed, without Scurility and Reflection, must of necessity tend to the Advancement of Learning and the public Benefit.

To conclude, as the Reasons for this Essay are plainly exhibited in the *Argument*; so if its Performance should likewise happily meet with due Acceptance and Approbation from the Public in general, as well as each candid and judicious Reader in particular, it will be no small Satisfaction to,

Your Humble Servant,

W. H.



THE A R G U M E N T.

IN this Enchiridion, I shall shew my Reader, with all imaginable Plainness and Brevity, how SURVEYING may be performed by the CHAIN only, without the Trouble of the other graduated Instruments, preceding, viz. the Surveying-wheel, the Theodolite, the Circumferentor, the Semicircle, the Plain-table, the Protractor, and the Sector : To which, I may now add the Perambulator, a new Mathematical Instrument, adapted to the Surveying of Land, as well as the plotting of remote Objects, and taking Distances and Altitudes both accessible and inaccessible.

Now although all and every of the abovesaid Instruments (excepting the last) are particularly described and applied by the Author in the preceding Part; and indeed (amongst many others) are esteemed the best; yet it is observed, that in Practical Surveying, most of them require likewise the Help of the Chain for taking Lengths, &c. which shews it to be herein, not only an Instrument, particularly, but even (almost) universally useful; and, as such, it is here selected, and recommended to Use beyond all others, upon the following Reasons and Considerations.

1. True : For what Surveyor can be sure, even with the best Theodolite, or other Instrument, as above, to take the four Angles of a Trapezium, so exactly, as to be just equal to four Right-angles; or if not, to which Angle will he impute the Error : Now, in this Case, I suppose all will allow, that to take with the Chain one Diagonal (or both, if you please, tho' not always necessary) is the surest way to correct the Work; and if so, the Angle will hereby be determined, because the three Sides of each Triangle is given; and consequently, the Instrument for taking Angles is rendered thereby altogether useless, and the Errors contracted by the Use of Instruments are rectified by the Chain only.

2. It is the most easy both to learn and to practise; since it requires not a Course of Mathematics, nor a thorough understanding of Euclid's

clid's Elements: For he that can but add, subtract, multiply, and divide, is sufficiently qualified with Arithmetic, to attain and practise the Method of Surveying; especially when we consider, that the Chain is so common and conspicuously easy of itself, that even any Person of an ordinary Capacity, may soon attain unto the practical Part of Surveying by it: Whereas, on the contrary, the other Instruments aforesaid require, not only the best Explanation, but even also an acute Genius and Apprehension, for the ready understanding them in their Use and Application.

3. It is the cheapest way; seeing it requires no Assistant with the Surveyor, but only one to lead the Chain; whereas, on other Cases, there must be (besides the Surveyor, and him that leads the Chain) one Man, or more to attend with the Instruments, which herein may be very numerous and chargeable, tho' needless: For, as Mr. Love and others well observe, what need is there for a Horse-load of Brass Circles and Semicircles, heavy Ball-sockets, wooden Tables and Frames, and also three legged Staves, cum multis aliis; unless to amuse the ignorant Countryman, to make him more freely pay the Surveyor.

4. It is the most expeditious; there being no Time spent in fixing the Instruments, or waiting for the settling of the Needle and Compass; but the Chain generally kept going, excepting when the Surveyor is writing.

If any thing has even the Colour of an Objection, it may be, that for want of Instruments to take Angles, we shall be obliged to take all the Diagonals or cross Lines in Trapeziums; or irregular Polygons, in the Field.

To which I answer:

If that is not done in the Field, it must be done at home; for Polygons must be divided into Triangles (or managed some way equivalent to it) before they can be cast up; so that there is no Time lost, except, doing that in the Field which might be done in the House; and if that is an Inconveniency, it is infinitely recompensed by the Exactness of the Work: For a small Error in an obtuse Angle (of which Polygons generally consist) may cause a great Error in the Base of one of the Triangles, which being measured in the Field, is wholly avoided.

From what has been observed upon this Head, we may rationally infer, that when Angles are needful to be taken in the Field, they must likewise be laid down in the Work at Home; but when otherwise, Diagonal Lines are taken in the Field by the Chain only, then all Protraction by the Semicircle, or any other Brass Instrument,

is intirely needless, because Angles are herein neither regarded nor required.

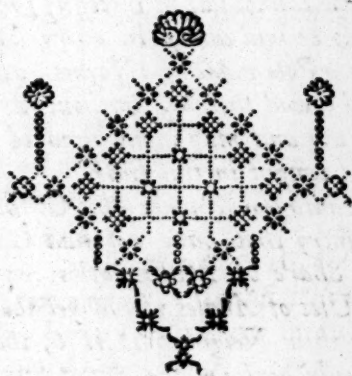
And yet, lest this Compendium should be thought imperfect, I have shewn the Learner, how any acute or obtuse Angle may be occasionally taken by either the Chain or Pole, according to the best Method extant; and have also inserted concise Tables, by the Help of which, and measuring three or four Poles on a Right-line, you may by Inspection only, at one View, determine exactly the Quantity of an Angle, either internal or external; especially, when the Field or Enclosure is so large, and consequently the Diagonal lines therein contained so extensively long, that you are dubious of carrying the Chain in a straight Line from one regular Point to another; then, in this Case only, it may be needful to measure all the Angles and Sides round the Field, without regarding the Diagonal Lines above-said, according to the Directions following.

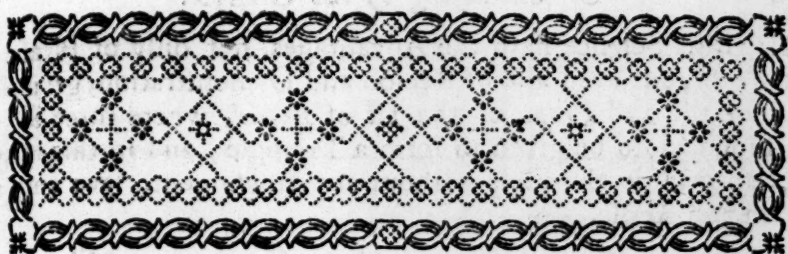
5. It is most convenient; for its being (when folded) so portable, as to be contained in your Pocket; which several of the other Instruments, as above-said, require a Person to be the Bearer: Yea, not only so, but it sometimes happens, that the Surveyor living so remote from any City or great Town where those Instruments can be had, that if there was no Way to do without them, he must sometimes postpone his Work most Part of a Summer for want of them, and perhaps lose the doing of it at last; whereas, put Case at worst, if all Instruments and even the Chain itself fail, he must have but a slender Share of Ingenuity, that cannot (upon Necessity) make himself a Chain, seeing there is Wire to be had almost in every Market-town; where, likewise, he may have a Pole made by a Foyner, according to the Directions herein contained; and thereby, without any other Instruments, may be enabled to survey and map many hundred Acres of Land, according to the Method taught in this Book.

6. It is very advantageous, upon this Consideration; that there are many in the Country that have not had Opportunity or Ability to acquire so large a Share of Mathematics, as to be versed in the Nature and various Uses of Angles; and yet would be glad to be useful to themselves or their Neighbours, if by the Country Education they have had, they could perform the Art of Surveying with as much Safety, Truth, and Expedition, as those that have had a great Charge of Education bestowed upon them: For, indeed, it often happens, that those who are born to (or have acquired) such plentiful Fortunes, as to allow them a liberal Education, and a genteel Maintenance, are thought above the Slavery of an actual Surveyor; and therefore, ingenious Persons, tho' of lower Circumstances, and whose creditable Living depends upon their Industry, are the likeliest Persons (if they can be thoroughly qualified) to engage in this laborious Undertaking, and for the Use of such this Enchridion is chiefly intended, though,

as to the Truth of the Performances, it is offered to the Examination of the able Mathematicians.

But, secondly, There are many industrious and ingenious Men, that understand the Use of Instruments, and live where they may be had, who cannot spare five, six, or eight Guineas for a Theodolite, or fifteen or sixteen for a new invented one; besides Plain-tables, Semicircles, Protractors, &c. and yet could spare Money to buy a Chain, or (where they cannot be had) Wire to make one; and also a handsome strong Pole: And these are all the Instruments necessary in Surveying, according to this Essay; which, if well understood, is sufficient to make any Person, tho' but of a common Capacity, to be a complete Master of the Art of Surveying, which is the Design of the Book, and of the Author,





OF
SURVEYING
BY THE
CHAIN.

CHAP. I.

*The Description of the Chain, Pole, Arrows,
and Field-Book.*

SECT. I.

Of the CHAIN.

I.  SURVEYING is a Science so excellent, that many of our modern Worthies, viz. Noblemen, Clergymen, Gentlemen, &c. affect the Study thereof; because its Exercise is so pleasant, wholesome, and innocent, as well as insinuating: For we seldom find a Man, that has once entered himself into the Study of Geometry or Geodæsia, can ever after lay it wholly aside: So natural is it to the Mind of Men, and so pleasingly inticing, that the *Pythagoreans* thought the Mathematics to be only a Reminiscence, or calling to mind, Things formerly learned: So that we may safely say, none but unadvised Men ever did, or do now speak evil of it.

II. This Science hath the Advantages, not only of Pleasure and Profit, but even also of Reason and Demonstration, grounded upon the 22d *Prop.* of the 1st of *Euclid*, where three Sides are supposed to be given to form a Triangle; and in this Essay is performed in the Field without any Instrument, save only a Chain and a Pole.

III. The Chain here recommended to Use, is that of Mr. *Gunter*; because it appears to be commodiously contrived for measuring, and expeditious for casting up what is measured by it.

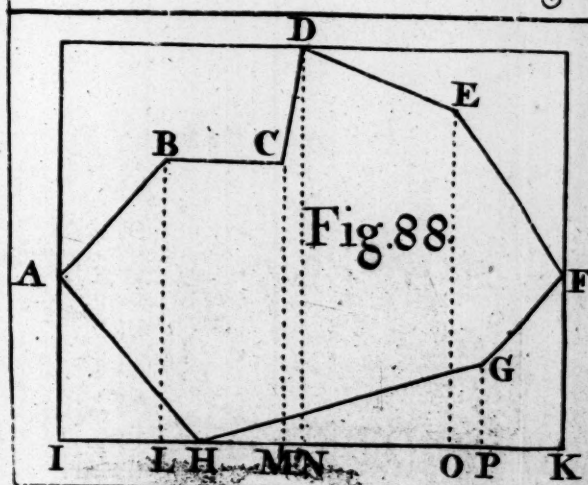
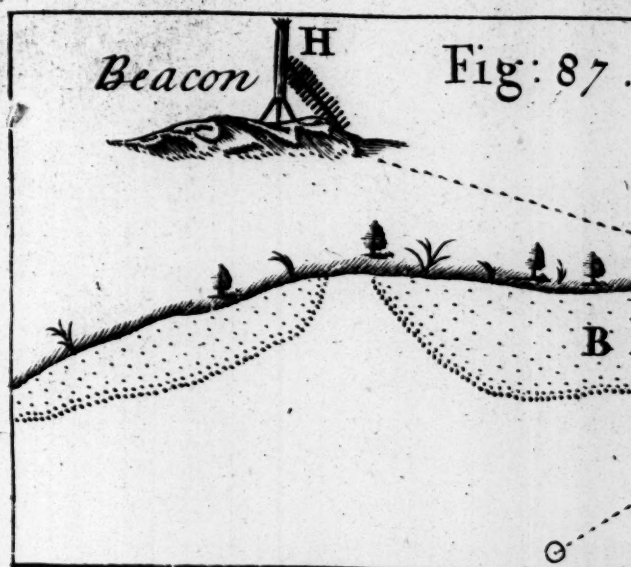
IV. Its Length contains 4 Statute Poles or Perches; each Pole or Perch containing $5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ Feet, and each Foot 1.515 Links, and each Link 7.92 Inches: So that the whole Chain contains 22 Yards; 66 Feet; 100 Links; and 792 Inches. Hence it is, that every Square Chain, *viz.* every Piece of Ground, which contains a Chain in Length, and the same in Breadth, comprehends within the Limits of that Superficies 16 Square Poles or Perches; and there being 160 Square Poles in an Acre, it follows, that 10 Square Chains, to wit, 10 Chains in Length, and one in Breadth; or 5 Chains in Length and 2 in Breadth (or any superficies equal to that) is just an Acre.

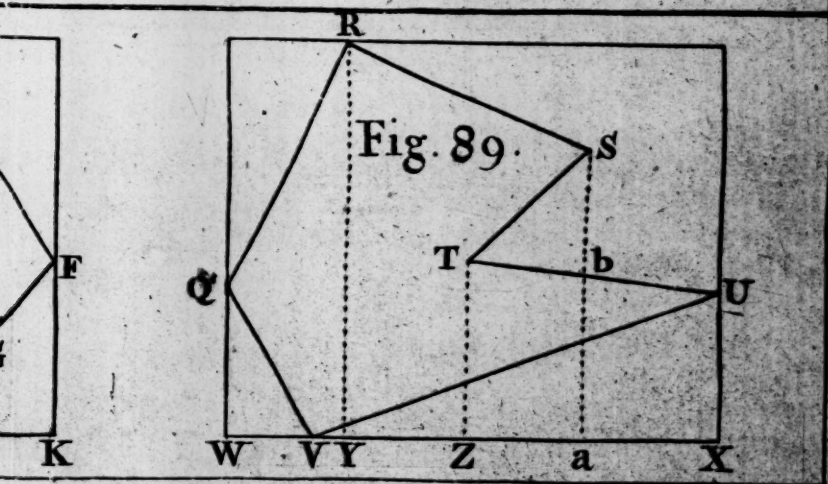
V. As the Chain is equally divided into 100 Links, it follows, that every 25 Links in Length, is equal to one Pole or Perch, being the 4th Part of the Chain; every $12\frac{1}{2}$ Links (*viz.* 8 Feet 3 Inches) is equal to half a Perch, or $\frac{1}{2}$ Part of the whole Chain; every 10 Links (*viz.* 6 Feet, 7.2 Inches) is equal to the 10th Part of the Chain; and every 5 Links (*viz.* 3 Feet, 3.6 Inches) is exactly equal to the 20th Part of the Chain, &c.

First, For the ready counting the Links of the Chain, there ought to be these Distinctions; namely, in the Middle thereof, which is at two Poles End (*viz.* 50 Links) let there be hanged a large Ring, with a red Rag tied to it: So is the whole Chain, by this, divided into two equal Parts.

Secondly, Let each of these two Parts be divided in two other equal Parts by two other like Rings, distinguished from the other, as well as themselves, by Rags of contrary Colours: So shall the whole Chain be divided into four equal Poles or Perches, each Perch containing 25 Links.

Thirdly, At every ten Links, let be fastened a lesser Ring than the former; or rather a Piece of Brass, on which is either stamped





Stamped, or some ways distinguished, the Numbers 10, 20, 30, 40, &c. the 50th Link, or Middle of the Chain, being marked with a large Ring, or round Piece of Brass, as abovesaid; then it follows, that those above 50 Links are known by their Complements to 100; which, likewise, may be marked and numbered orderly on the reverse Side of the Brass; that which is 30 Links from one End, must on the contrary Side of the Brass, be noted 70 from the other End; and then 10 from one End, is 90 from the other, &c.

Lastly, At every fifth Link (if you please) may be fixed other Marks: So by this Means, you shall most easily, readily, and exactly, count the Links of your Chain without any Trouble. The Chain being thus distinguished, it matters not which End be carried forward, because the Notes of Distinction proceed alike on both Sides from the Middle of the Chain: But it is very convenient, and I always use it to tie at my middle Ring a large red Rag, and at the End of every 25 Links, a Rag likewise of a contrary Colour to any of the former, tho' somewhat lesser; because in long Grass, or the like, the Rings are not to be seen; therefore the Rags being fixed and distinguished as before, will be of great Advantage to you in counting.

VI. In measuring with this Chain, you are to take notice only of Chains and Links: As saying, Such a Line or Side of any enclosed Piece of Ground, &c. measured by this Chain contains 99 Chains 37 Links, you may express more briefly thus, 96.37; because the Links are always signified to be so many decimal Parts of a Chain, which may likewise be expressed without the Points, in Links only, thus, 9637; and these are all the Denominations which are necessary to be taken notice of in surveying of Land.

S E C T. II.

Of the Pole, or Off-set Staff.

THIS Instrument is frequently used with the Chain, by which its Length, or any Part of it, may be truly and exactly examined. — It is also singularly useful in taking Angles and Off-sets in the Field, as well as measuring all short Distances, such

such as are contained in Yards, Gardens, or the like: And in making of it, may likewise be rendered useful in measuring the Works of Bricklayers, Carpenters, &c. which sometimes fall in the Way, where you are surveying.

2. Its proper and convenient Length does usually contain 6 Feet, 7 2 Inches, which is exactly equal to one tenth of the whole Chain, composed of 10 Links, each Link containing 7.92 Inches, as above, by which Division it falls readily into Decimals; seeing 8 Poles is 80 Links, and 17 Poles is one Chain 72 Links, &c. without further Trouble of reckoning.

3. Although this Pole may be made any other Length at Pleasure, that is moderate or reasonable, yet this of ten Links, as above, is the most convenient, seeing it may be equally divided into Links with Brass Nails numbered thus, 1, 2, 3, 4, 5, 4, 3, 2, 1, the 3 from one End being 7, from the other, &c. And that it may be no less useful for the measuring Bricklayers and Carpenters Work, &c. it may likewise be equally divided into Feet, &c. by Notches, contrary to the Division of Links by Brass Nails, as above.

S E C T. III.

Of the Station-staffs and Arrows.

I. **T**HAT the Surveyor may proceed orderly and methodically in taking his Dimensions in the Field, whether Sides or Angles, it is necessary that he should likewise be provided with three Station-staffs and ten Arrows.

2. The three Staffs for the proper Use, ought to be strong and straight, each of them containing about 5 Feet in Length, but not exceeding six, having their small Ends sharp-pointed for sticking them easily and readily into the Ground, and their Tops displayed with white Linen Rags or Paper, to render them more conspicuously evident when erected at their proper Distances or Situations; in order that he who leads the Chain, as well as he that follows it, may see them (when conveniently fixed and erected) interposing between the proper Object from whence you last departed, unto that in the next Angle or Corner, which the Chain must be led unto in a direct or straight Line, without deviating to either the Right or the Left-hand.

3. You

3. You must also be provided with ten Arrows or small Sticks, each containing about 12 or 13 Inches in Length, but rather longer when the Ground is soft and the Grass long. Now, that these Arrows or small Sticks may be conveniently useful, it is likewise needful that their Tops may be marked with Bits of red Cloth, and their small Ends pointed with iron Ferrils, that the Ground may be more easily and readily pierced at the End of each Chain, or any Part of it by him that leads it.

S E C T. IV.

Of the Field-Book.

1. **H**AVING in the preceding Sections described the Chain, Pole, Station-staves and Arrows, with their particular Uses in the Field, we shall now shew the Nature and Quality of the Field-book, and the Method of entering immediately into it, with a black or red Pencil, all the Dimensions taken in the Field, with the greatest Accuracy and Exactness imaginable.

2. Let your Field-book be reduced into four Columns of an equal Breadth, and let the first Column on the Left-hand have written upon its Top, *Changes or Decades*; implying, so many Tens or Changes must be contained in that Column, as Times the Arrows are occasionally spent or used in the Field by him that leads the Chain: Now, for the more regular and exact Protraction of the Work, when you return home, you are to observe, that every Change in this Column, containing ten Chains in Length, must be lineally and particularly noted by a Dot or Point, from the Left-hand to the Right, and their Total (at last) expressed in Figures, in such order, that the odd Chains and Links which the Line or Side in the second Column (noted by its proper Letters) concludes with, may stand agreeably in a straight Line with those Changes or Decades belonging to the same, tho' standing in the first Column. But when it happeneth that there is occasionally taken a diagonal Line, extending to any particular Mark assigned, and proceeding from any angular Point, which the said Line or Side begins and ends; then, for Order and Distinction sake, the said Points and Figures may stand perpendicular to those in the straight Line from whence it
pro-

proceeds, after it is separated by a Division-stroke, according to the Scheme following.

3. The 2d or next Column towards the Right-hand, must have written on its Top, *Side-lines, and Diagonals*: and underneath, orderly descending downwards, alphabetical Letters, in Pairs, viz. AB, BC, CD, &c. signifying, that the first Line or Side taken in the Field must be noted in Capitals AB, the 2d BC, the 3d CD, &c. and so each Side round the whole Field, orderly increasing with the next alphabetical Letter. But when it is needful to take a diagonal Line cross-ways, unto any opposite Point assigned from any Angle or Corner, signified by the first and last Letters of the Pair, which represent the Line measured; then may the said Diagonal Line be always noted *dl*, and may immediately succeed the Dimensions of the Side-line, from whence it proceeded. As for Instance, admit the Line or Side AB, is found to be 69.87 Chains, and the Diagonal Line (*vid. dl*) proceeding from one of the Angles or Corners at B = 96.59 Links; then must they be placed in the two first Columns of the Field-book, thus:

Changes. Chains. Links.

Changes. Chains. Links.

6 — 9 — 87 — and 9 — 6 — 59.

according to the following Scheme, which we see.

4. The third Column towards the Right-hand, must have written on its Top, *Angular Quantities or Chord Lines*, and underneath, orderly descending downwards, $\odot A$, $\odot B$, $\odot C$, &c. signifying, that when the Field is measured round, those Angles above are orderly measured immediately after their Sides, &c. and their particular Quantities (in Degrees, &c.) discovered by the Table, according to the Chord Lines (in Links and Tenths) found in the Field, which are to be exactly noted down in your Field book, in order to Protraction: As for Example, the Sign $\odot B$, in the third Column of the Field-book, implies, that the Angle there found is an acute Angle (noted thus $\angle$) containing 61.0 Deg. equal to the Chord-line of 101.5 Links found in the Field. Also, the Angle $\odot C$ (noted thus $<$) is an obtuse Angle, whose Quantity by the Table, is found to be 104 Degrees, agreeable to its Chord or Subtense of 61.6 Links found in the Field. See the Scheme.

5. In the 4th or last Column of the Field-book, must be orderly inserted such Things remarkable as you meet with in your Way, viz. Houses, Ponds, Brooks, Mills, Trees, or the like. As for Example, in measuring the Side AB, at the first Angle (*viz. $\odot A$*) I find a Farm-house, which I accordingly note

down against the same, under the 4th Column under *Things remarkable*: And at the Angle C, in the Beginning of the Line or Side CD, I find a large Tree, which I likewise note down, &c.

The SCHEME or FIELD-BOOK.

Lines or Angles. Particulars Totals.	Side Lines and Diagonals.		Ang Points. D. M. Lin. 10 ^{ths} Angular Quantities or Chords.	Things remarkable.
	Litr.	Sig. Ch. L Ch. L.		
9				
6	AB=9.87	dl=6.59	⊙ A = ∠ 61—0 = 101.5	A Farm House at ∠ A
	BC=		⊙ B	
9	CD=8.64	dl=3.97	⊙ C = < 104.0 = 61.6	A large Tree at ∠ C, &c.
	DE=	⊙ D=	⊙ D=	⊙ D=

OBSERVATIONS.

The Reader will find, that the Form or Method of this Field-book, and its Description, is not only plain and concise, as well as complete, but even also entirely new, as being the Product of my own Invention; by observing, that there are scarce two Surveyors in *England*, that have exactly the same Method for their Field Notes; some of them being too intricate and confused, and others too numerous and tedious; which induces me to recommend to Practice the Use of this Form, with some useful Remarks thereupon, seeing it is so particularly adapted to the Use of the Chain only.

1. Note,

1. *Note*, In measuring by the Chain only, or taking exactly the Dimensions of any Field or enclosed Piece of Ground, it is most methodical to begin at some remarkable Place, *viz.* House, Style, Gate, Tree, &c. and from thence proceeding orderly on the Right-hand, you may measure round the same, by taking the Dimensions, first of a Side, and then of the interposing Angle between that and the next; which being particularly noted down into your Field-book, as above, you may proceed in this Order, with Sides and Angles interchangeably, until you have surrounded the whole Field, and concluded with an Angle, where you began with a Side; by Means whereof you have got into your Field-book the Dimensions of all the Sides and Angles which encompass the same in Order to Protraction.

2. When you are accidentally obstructed in measuring round the Field, as above; then having taken only a few of the Dimensions of the enclosing Sides, you may then take one or more Diagonals in straight Lines, proceeding from, and terminating at some particular Angle, Corner, or Mark assigned; which must be accordingly noted in your Field-book as before directed; by which Means, your Field or Protraction will thereby discover its true Form or Shape, and consequently, its Area or Content.

3. It may be proper at your Entrance into the Field, to observe (if possible) its Form or Shape; and having with your Pencil drawn (at Adventure) a Figure that may somewhat resemble the same; it will be the only Means to give you such an Idea or Apprehension of it, as to know whether it is proper to measure it by Diagonal Lines, or Angles, or both, as above directed.

4. Although it is convenient in the Field to take your Dimensions in Changes, Chains, and Links, and entering them accordingly in your Field-book; yet, when protracted, it will be necessary to add your Changes and Chains together, in order to render them more agreeable to the Scale of equal Parts, which admits only of Chains and Links.

5. When the Sides and Angles circumscribing or enclosing your Field happen to be many, then must your Field-book contain just as many ruled Lines downwards, as there are Sides contained in your Field, for the more orderly receiving Dimensions taken.

6. As concerning the general Method for taking Angles and Sides, both external and internal, of what Qualities, Quantities, and

and Denominations soever, I shall refer my Reader to the Performance of the same, in a more proper Place; seeing this Chapter is only intended as a needful Preparation.

CHAP. II.

Of the Description, Use, and Application of certain Tables adapted to Practical Surveying.

SECT. I.

Of Tables useful in Surveying.

FOR rendering every thing plain, easy and conspicuous to an ordinary Capacity, I shall here shew in this Chapter, a particular Description of certain Tables, very useful and applicable to the practical Part of Surveying.

TABLE I.

A general Table of long or lineal Measure.

Long.	Link.	Foot.	Yard.	Perch.	Chain.	Mile.
Inch.	7.92	12	36	198	792	63360
	Link.	1.515	4.56	25	100	8000
		Feet.	3	1.65	66	5280
			Yard.	5.5	22	1760
				Perch.	4	320
					Chain.	80

Explanation.

Explanation.

1. By long or lineal Measure in Surveying, you are to understand, that when the Length of any great Denomination is given to be reduced into a lesser Name, and Specie equivalent, then it will follow, that so many Particulars as are contained in that lesser Name, when placed End and End in a straight Line, will be exactly equal unto the Length of the greater, from whence it proceeds; according to those Axioms in *Euclid*, which shews, that when equal Quantities are added to equal Quantities, the Sum of these Quantities will be equal: Also those Quantities that are equal to one and the same thing, are equal to one another.

2. That Quantities may differ in their Name or Specie, yet agreeable to their Dimensions, will appear by the Table above, by observing that 7.92 Inches in Length, is 1 Link; 1.55 Links, or 12 Inches, is 1 Foot; 3 Feet is one Yard; 5.5 Yds. is one Pole or Perch; 4 Poles or Perches is 1 Chain of *Gunter's*, and 80 Chains in Length is one *English* Mile Statute Measure: Hence it is, if you would know how many Feet in Length is contained in one Perch, or in 1 Chain, or in 1 Mile, the Answer for each of them particularly in their Order, are 16.5, 66, and 5280; found readily by Inspection, in looking on the Top and Side of the Table above, for the two Names given and sought, and in the common Angle of meeting is the Answer desired.

APPLICATION.

To find arithmetically, by Multiplication only, the Quantity of any inferior Name or Denomination, agreeable to any Quantity given in the greater.

R U L E.

Look in the Table for the Number of Units or Particulars sought, that is equal to an Unit of the greater, which being multiplied by the given number, the Product is the Answer desired.

Or otherwise thus:

Multiply orderly the given Number by all the particular Values of the intermediate Denominations, and the last Product will be the Quantity sought

Example I.

In one Mile (according to the Table) is contained 63360 Inches: To know how many Inches in Length is equal to 20 Miles

Answer. 1267200 Inches.

Explanation.

In this Example, according to the first Part of the Rule, I multiply the Inches in one Mile (*viz.* 63360) by the Number of Miles given (*viz.* 20) the Product (*viz.* 1267200 Inches) is the Answer desired.

Example II.

How many Gunter's Chains, Perches, Yards, Feet, and Inches, are contained in 20 Miles?

$$\begin{array}{r}
 20 \text{ Miles.} \\
 \times 80 \\
 \hline
 1600 \text{ Chains.} \\
 \times 4 \\
 \hline
 6400 \text{ Perches.} \\
 \times 5 \frac{1}{2} \\
 \hline
 32000 \\
 3200 \times \frac{1}{2} \\
 \hline
 35200 \text{ Yards.} \\
 \times 3 \\
 \hline
 105600 \text{ Feet.} \\
 \times 12 \\
 \hline
 1267200 \text{ Inches.}
 \end{array}$$

Answer. As above.

Explanation.

Here the given Number (*viz.* 20 Miles) is multiplied by the next inferior Denomination (*viz.* 80 Chains) the Product is

X

1600

1600 Chains ; which being multiplied by 4, because 4 Poles or Perches is a Chain, the Product is 6400 Perches ; which being multiplied by $5\frac{1}{2}$, taking in half the Multiplicand after you have multiplied by 5, because $5\frac{1}{2}$ Yards is one Pole or Perch, the Product is 35200 Yards ; which being multiplied by 3, because 3 Feet is 1 Yard, the Product is 105600 Feet ; which being multiplied by 12, because 12 Inches is 1 Foot, the Product or Answer is 1267200 Inches, as before.

Having the Length of any Quantity given in an inferior Name, to find (by Division only) that of the Superior, which shall be equal to the same, though differing in Name and Specie.

R U L E.

This is exactly the Converse of the last Rule ; and, as such, requires that the Quantities given in any inferior Denomination, must be divided by so many Units of that Name, as are equal to an Unit of the greater Quantity sought, in order to find in the Quotient, the Number desired.

Example III.

In an *English* or Statute Mile there is contained 63360 Inches in Length : To know how many such Miles are equal unto 1267200 Inches ?

Answer. 20 Miles.

$$\begin{array}{r} 63360 \) \ 1267200 \quad (\ 20 \text{ Miles.} \\ \underline{12672} \\ \text{Rem. } (0) \end{array}$$

Explanation.

In this Example, or any of the like Kind, it may be observed, that the Number given in the least Name mentioned (*viz.* 1267200) must be divided by so many Particulars of the same Name, as are equal to an Unit of the greater Denomination sought (*viz.* 63360) the Quotient (*viz.* 20 Miles) is the Answer desired ; and consequently, the Converse and Proof to the 1st and 2d Examples.

S E C T. II.

T A B L E II.

Shewing how many Feet and Parts of a Foot, also how many Perches and Parts of a Perch, are contained in any Number of Chains and Links, from one Link to one hundred Chains.

Links.	Feet.	Parts of a Foot.	Perches.	Parts of a Perch.	Chains.	Feet.	Perches.
1	00.66	0.04	1	0.04	1	66	4
2	01.32	0.08	2	0.08	2	132	8
3	01.98	0.12	3	0.12	3	198	12
4	02.64	0.16	4	0.16	4	264	16
5	03.30	0.20	5	0.20	5	330	20
6	03.96	0.24	6	0.24	6	396	24
7	04.62	0.28	7	0.28	7	462	28
8	05.28	0.32	8	0.32	8	528	32
9	05.94	0.36	9	0.36	9	594	36
10	06.60	0.40	10	0.40	10	660	40
20	13.20	0.80	20	0.80	20	1320	80
30	19.80	1.20	30	1.20	30	1980	120
40	26.40	1.60	40	1.60	40	2640	160
50	33.00	2.00	50	2.00	50	3300	200
60	39.60	2.40	60	2.40	60	3960	240
70	46.20	2.80	70	2.80	70	4620	280
80	52.80	3.20	80	3.20	80	5280	320
90	59.40	3.60	90	3.60	90	5940	360
100	66.00	4.00	100	4.00	100	6600	400

*The TABLE explained.**Example I.*

To know how many Feet are contained in 30 of Mr. Gunter's Chains. *Answer*, 1980.

R U L E.

Look under the Title *Chains* for the given Number (*viz.* 30) and right against it, under Title *Feet*, stands the Number of Feet demanded, *viz.* 1980

Example II.

How many Poles or Perches are contained in 80 Chains of Gunter? *Answer*, 320 Perches.

A general Rule for solving Examples by the Table.

Find always the given Number under its Name or Title, and then against it, under the Name enquired for, you will find the Number sought.

N. B. For the ready understanding this Table, the Reader will find it to consist of two Parts, whose Figures are both alike, but different by Points; by reason the first three Columns shew by Inspection only, any Number of Links in Feet and decimal Parts of a Foot; and the other three Columns discovers in whole Numbers, the Feet and Perches that are equal to any Number of Chains from 1 to 100 inclusively.

To find readily by Inspection, or otherwise, the Number of Feet, and decimal Parts of a Foot; also the Number of Perches and decimal Parts of a Perch, that is contained in any Number of Chains and Links.

R U L E.

When it happens that the Number of Chains or Links given, cannot be found at once in the Table, then may they conveniently

niently be found separately with their corresponding Numbers sought ; which being orderly placed and added together, their Sum will be the Number desired.

Or otherwise thus :

Let the Chains and Links given be always made the Multiplier, and the Number of Feet contained in one Chain (*viz.* 66) the Multiplier ; then will it follow, that the Product, after two decimal Places are cut off from the Right-hand by a Point or Comma, for the Links given, will be the Feet and decimal Parts sought ; but when it happeneth, that there are likewise required Perches and Parts of a Perch, equivalent to any Number of Chains and Links given ; then, in this Case, the Number of Poles or Perches contained in one Chain (*viz.* 4) must always be made the Multiplier, in order to find the desired Product, or Number of Perches, &c. sought.

Example I.

In 95 Chains 76 Links, how many Feet and decimal Parts of a Foot ?

	Chains.		Feet. Parts.	
Links.	{	90	=	5940 . 00
	{	5	=	330 . 00
	{	70	=	46 . 20
	{	6	=	3 . 96

Added Sum = 6320 . 16

Answer 6320 . 16 Feet

Explanation.

In this Example, 90 Chains is found by the Table to contain 5940 Feet, and 5 Chains 330 Feet : Also, 70 Links contains 46.20 Feet, and 6 Links 3.96 more, whose Sum by Addition is found to be 6320.16 Feet for Answer, as above.

X 3

Also,

Of Surveying by the CHAIN.

Also, by Multiplication, 95.76 Chains the given Length, being multiplied by the Number of Feet in one Chain (*viz.* 66) the Product is likewise 6320.16 Feet as before.

Or thus,

Chains. Links.

$$\begin{array}{rcl} 95.76 & = & \text{Mnd.} \\ \times 66 & = & \text{Mr.} \end{array} \left. \vphantom{\begin{array}{rcl} 95.76 \\ \times 66 \end{array}} \right\} \text{Factors.}$$

$$\begin{array}{r} 57456 \\ 57456 \end{array}$$

Product - 6320.16 *Answer.*

Example II.

How many Perches and Parts of a Perch are contained in 69 Chains and 57 Links?

Answer, 278.28.

Chains. Perches. Parts.

$$\begin{array}{rcl} \left\{ \begin{array}{l} 60 \\ 9 \\ 50 \\ 7 \end{array} \right. & = & \left\{ \begin{array}{l} 240.00 \\ 36.00 \\ 2.00 \\ 0.28 \end{array} \right. \\ \text{Links.} & & \end{array}$$

$$\text{Added Sum} = 278.28$$

Explanation.

This Example shews from the Table that 60 Chains contains 240 Perches, 9 Chains 36 Perches, and 50 Links 2 Perches; also 7 Links 0.28 Perches. The Sum of whose Particulars are 278.28 Perches, as in the Work.

Also, for Proof, 69.57 Chains being multiplied by 4, the Number of Perches as in a Chain, the Product is likewise the Answer or Number desired as before.

Or thus,

$$\begin{array}{rcl} 69.57 & = & \text{Mnd.} \\ \times 4 & = & \text{Mr.} \end{array} \left. \vphantom{\begin{array}{rcl} 69.57 \\ \times 4 \end{array}} \right\} \text{Factors.}$$

$$\text{Product} = 278.28 = \text{Answer.}$$

S E C T. III.

T A B L E III.

Shewing how many Chains, Links, and Parts of a Link; also how many Perches and Parts of a Perch are contained in any Number of Feet from 1 to 10000

<i>Feet.</i>	<i>Chains.</i>	<i>Links.</i>	<i>Parts.</i>	<i>Perches.</i>	<i>Parts.</i>
1	0	1	515	0	061
2	0	3	030	0	121
3	0	4	545	0	181
4	0	6	060	0	242
5	0	7	575	0	303
6	0	9	090	0	363
7	0	10	606	0	424
8	0	12	121	0	484
9	0	13	636	0	545
10	0	15	151	0	606
20	0	30	303	1	212
30	0	45	454	1	818
40	0	60	606	2	424
50	0	75	757	3	030
60	0	90	909	3	636
70	1	06	060	4	242
80	1	21	212	4	848
90	1	36	363	5	454
100	1	51	515	6	060
200	3	03	030	12	121
300	4	54	545	18	181
400	6	06	060	24	242
500	7	57	575	30	303
600	9	09	090	36	363
700	10	60	606	42	424
800	12	12	121	48	484
900	13	63	636	54	545
1000	15	15	151	60	606
2000	30	30	303	121	212
3000	45	45	454	181	818
4000	60	60	606	242	424
5000	75	75	757	303	030
6000	90	90	909	363	636
7000	106	06	060	424	242
8000	121	21	212	484	848
9000	136	36	363	545	454
10000	151	51	515	606	060

This Table in its Use and Application will evidently appear in the Manner following.

Having any Number of Feet given in Length, to find readily by Inspection, or otherwise, the Number of Chains, Link:, and Parts of a Link; as also, the Number of Perches and Parts of a Perch that is equal thereto, or contained therein.

R U L E.

First, When the given Number of Feet can be found at once in the first Column of the Table on the Left-hand, then the Number of Chains, Links, and Parts of a Link, and also Perches and Parts of a Perch, are immediately found by Inspection only; by guiding your Eye directly forward in a straight Line from the given Number of Feet; you will thereby discover the Answer sought in all its Particulars; according to the first Example.

Secondly, If the Number given cannot be found at once in the first Column of the Table on the Left-hand; then, in this Case, you are to find it in Particulars, which being added together, the Sum is the true Number desired. See Example 2.

Or otherwise thus:

Let, first, the given Number of Feet have annexed unto their Right-hand 3 or 4 Cyphers, with a Point prefixed, which being divided by the Number of Feet in one Chain (*viz.* 66.) the Quotient is Chains and decimal Parts, whose two first Places of Decimals on the Left-hand, must be esteemed so many Links; and the Figure or Figures remaining on the Right-hand, will be so many decimal Parts of a Link, which when multiplied by the Number of Perches in one Chain (*viz.* 4.) the Product (after the whole decimal Parts are cut off by a Point or a Comma) are likewise so many Perches, and Parts of a Perch represented by the said Decimal.—Or having any Number of Feet given, the Perches may likewise be found, by annexing thereto 3 or 4 Cyphers with a Point prefixed, as before; and dividing the same (decimally) by the Number of Feet in one Perch (*viz.* 16.5. Feet) the Quotient is Perches and Parts of a Perch. See Example 3.

Example

Fig: 90.

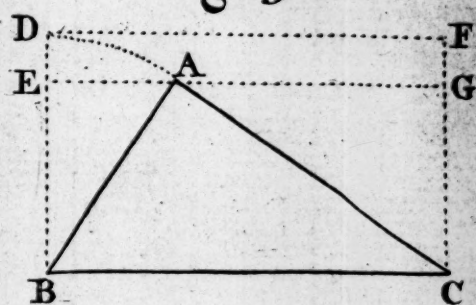
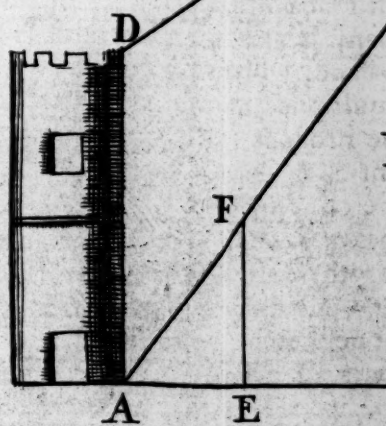
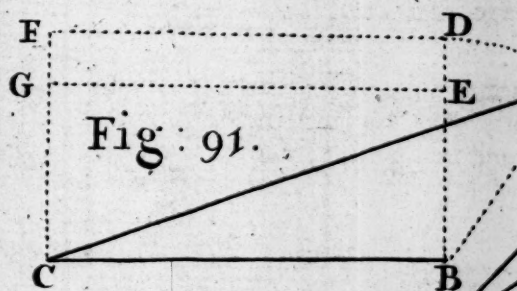
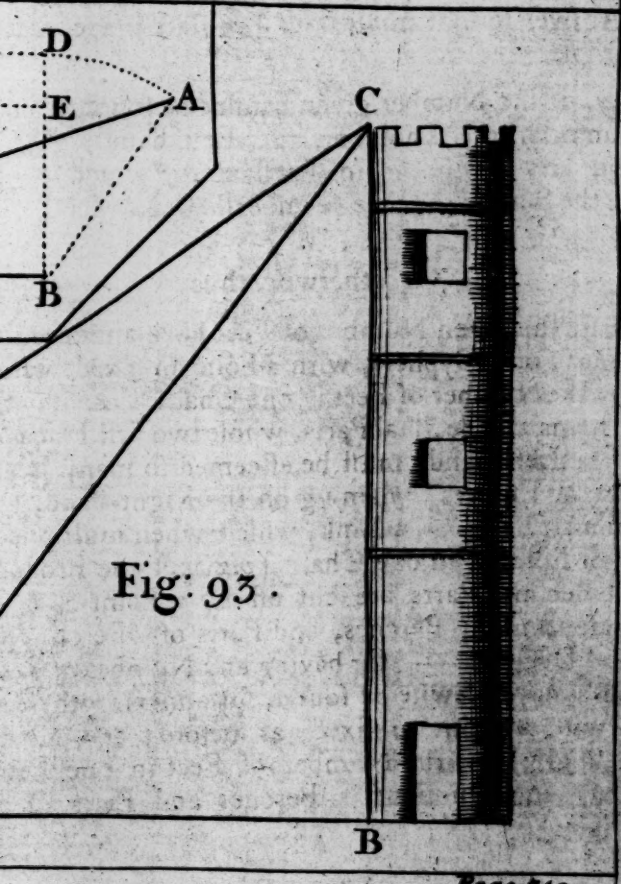
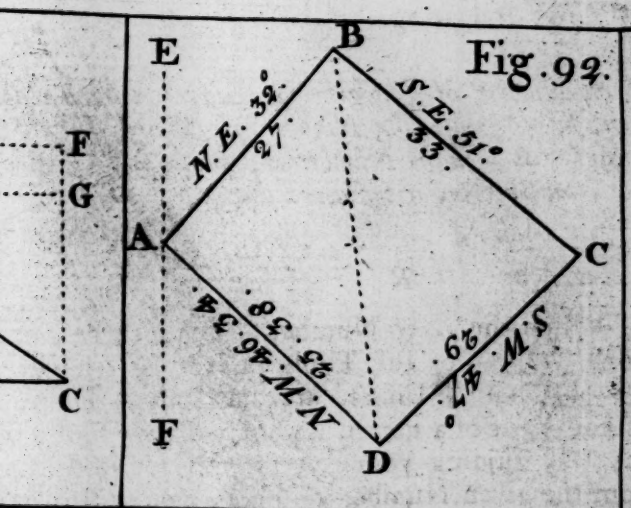


Fig: 91.





Example I.

To know how many Chains, Links, and Parts of a Link ; also the Number of Perches and Parts of a Perch that is equal unto, or contained in 900 Feet.

Chains. Links. Parts. Perches. Parts.

Answer, 13 — 63 — 636 : or 54 . 545, &c.

Explanation.

Having found 900 Feet in the first Column of the Table towards the Left-hand ; then directly forward, you will find the Answer in all its particulars, as above. This is so obvious, that there needs no farther Illustration.

Example II.

In 17546 Feet, how many Chains, Links, and Parts of a Link ; also how many Perches, and Parts of a Perch, are contained therein ?

<i>Feet.</i>		<i>Chains.</i>		<i>Links. Pts. of Lin.</i>		<i>Perc. Pts. of Perc.</i>
10000	=	151	—	51 . 515	=	606 . 060
7000	=	106	—	06 . 060	=	424 . 242
500	=	7	—	57 . 575	=	30 . 303
40	=	0	—	60 . 606	=	2 . 424
6	=	0	—	9 . 090	=	0 . 363
17546	=	265	—	84 . 846	=	1063 . 392

Chains. Links. Pts. of Lin. Perc. Pts. of Perc.

Answer, 265 — 84 . 846 = 1063 . 392 are contained in 17546 Feet.

Or

Or thus :

$$\begin{array}{r}
 \begin{array}{r}
 66 \overline{) 17546.0000} \\
 \underline{132} \\
 .434 \text{ Perch.} = 1063.3936 \\
 \underline{396} \\
 .386 \\
 \underline{330} \\
 .560 \\
 \underline{528} \\
 .320 \\
 \underline{24} \\
 .560 \\
 \underline{528} \\
 .320 \\
 \underline{264} \\
 \text{Remains - (56)}
 \end{array}
 \quad
 \begin{array}{r}
 \text{Cha. Pts.} \\
 265.8484 \\
 \times 4 \\
 \hline
 \end{array}
 \end{array}$$

Explanation.

In Example 2. you may observe, that as the whole Number of Feet given (*viz.* 17546) cannot be found in the Table at once, so it must be found in Particulars, according to the Method exhibited above: Or otherwise, the same, as well as any other Number given, may be effected by dividing the Feet proposed, by the Number of Feet in one Chain (*viz.* 66) the Quotient is the Number of Chains and decimal Parts of a Chain (*viz.* 265.8484 Chains) which being multiplied by the Number of Poles or Perches in one Chain (*viz.* 4) the Product is Perches and decimal Parts of a Perch (*viz.* 1063.3936 Perches) according to Variety 2d, which see.

Example.

Example III.

How many Poles or Perches do 12769 Feet make?

Feet.		Perch. Pts. of Perch.
10000	=	606 . 060
2000	=	121 . 212
700	=	42 . 424
60	=	3 . 636
9	=	0 . 545

$$\text{Sum} = 12769 = 773.877$$

Or thus :

		Perch. Pts.
16,5	) 12769.0000	(773.878
	1155	
	—	
	. 1219	
	1155	
	—	
	.. 6040	
	495	
	—	
	1450	
	1320	
	—	
	1300	
	1155	
	—	
	1450	
	—	

Remains (30)

Ans. 773.877 Perches are contained in 12769 Feet.

Explanation.

As the Table discovers the Particulars of this Example, according to the preceding Work; so their Sum is the Answer desired for Variety first: then for Proof, the Number of Feet given (viz.

umber
ble at
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as any
et pro-
) the
Chain
umber
erches
ccord-

ampli.

(viz. 12769) being divided (decimally) by the Number of Feet in one Perch (viz. 16.5) the Quotient is likewise the Number of Perches sought (viz. 773.878 Perches, as above.

SUPERFICIAL or SQUARE MEASURE.

1. Having by the preceding Tables and their Explanation shewn the Use and Application of lineal or long Measure with regard to Surveying; we shall now likewise explain those in Planometry, or the Measuring the Surface of the Earth, so far as it relates to Land-measure.

2. In Geometry, a Superficies is the second kind of Magnitude, which contains Length and Breadth, but no Thickness; and is produced by the Motion of a Line, as a Line is by the Motion of a Point.

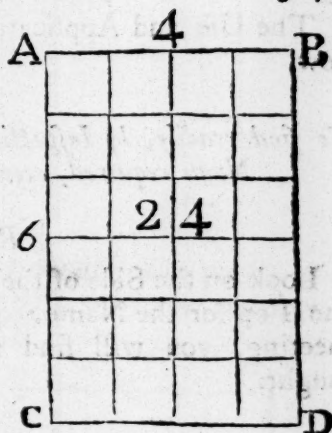
3. The Extremes or Limits of a Superficies are Lines: And under the Definition of a Superficies, may be comprehended all Figures or Forms whatsoever in Surveying, that can be limited or bounded by Lines. Hence it is that the Content of any Superficies is said to be found, when we know how many square Feet, Perches, or Chains &c. are contained therein, according to the Dimensions of its Sides in any of those Denominations given.

4. In Surveying, the Kinds and Names of Superficies are as various as they are numerous, according to the Form or Figure they represent, seeing they may be either regularly or irregularly bounded or limited by any Number of Sides or Lines, whether streight or curved, or both.

5. Amongst the various kinds of Figures in Planometry, none represent to the Eye a square Foot, Perch, Chains, &c. more intelligibly, than a Geometrical Square or Oblong, divided into any Number of equal Parts assigned, according to the following Diagram, viz. Fig. 1.

Inc
1
62.72
144
1296
3600
30204
62726
62726
40144

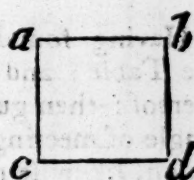
As for Example : Suppose the DIAGRAM, Fig. 1. ABCD was a Piece of Land, and the Length of the Line AB or CD was 4 Perches ; also the Length of the Line AC or BD was 6 Perches ; I say that Piece of Land contains 24 square Perches ; as you may see here divided ; every little Square being a Perch, having a Perch in Length for its Side.



Or thus :

Fig. 2.

A Foot, Perch, or Chain, may be represented by the Line *ab*, in Fig. 2. Now if the said Line *ab*, is supposed to be a Perch long, or $16\frac{1}{2}$ Feet ; it will then follow, that the Line *bd*, and the other two Lines are all of them equal one to another ; and, consequently, the whole Figure *abcd* must rationally be esteemed a square Perch, seeing all Sides are equal.



SECT. IV.

TABLE IV.

A general Table of Square Measure.

Inch.	Links.	Feet.	Yards.	Paces.	Perch.	Chain.	Acre.	Mile.
1								
62.726	1							
144	2.295	1						
1296	20.755	19	1					
3600	57.381	125	2.778	1				
30204	1625	272.25	30.25	10.89	1			
627264.	10000	4356	484	174.24	16	1		
6272640	100000	43560	4840	1742.4	160	10	1	
4014489600	164000000	27878400	3007600	1115126	102400	6400	646	1

The

The Use and Application of this Table will appear as follow.

To find readily, by Inspection only, the Quæſita or Number of any Name required, according to any Denomination given.

R U L E.

Look on the Side of the Table for the Number given, and on the Top for the Name required, then, in the common Angle of meeting, you will find the Square Number for the Name sought.

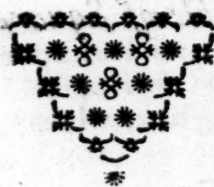
Example.

How many square Feet are contained in one square Chain?

Answer 4356 square Feet.

Example.

Having found Chains, the given Name on the left Side of the Table; and Feet, the Denomination required on the Top thereof: then guiding your Eye downward; you will find in the Angle of meeting 4356 square Feet, for the Quantity sought; which, likewise may be proved by the Pen, by multiplying the Length of one Chain in Feet (*viz.* 66) by itself: the Product will be the same Number of square Feet, as above; The like, upon Trial, will appear in others so obviously, that there needs no other Example.



S E C T. V.

TABLE V.

Shews readily by Inspection, the Length or Breadth alternately, in Perches, Feet, &c. that is needful to compose, or lay out an Acre of Ground in a quadrangular Form.

Breadth.	Length of an Acre.		Breadth.	Length of an Acre.	
	Perches.	Feet.		Perches.	Feet.
10	16	0	28	5	$11 \frac{3}{4}$
11	14	9	29	5	$8 \frac{11}{23}$
12	13	$5 \frac{1}{2}$	30	5	$5 \frac{1}{2}$
13	12	$5 \frac{1}{2}$	31	5	$2 \frac{21}{31}$
14	11	$7 \frac{1}{11}$	32	5	0
15	10	11	33	4	14
16	10	0	34	4	$11 \frac{2}{3}$
17	9	$6 \frac{9}{12}$	35	4	$9 \frac{5}{21}$
18	8	$14 \frac{8}{12}$	36	4	$7 \frac{1}{2}$
19	8	$6 \frac{1}{12}$	37	4	$5 \frac{1}{2}$
20	8	0	38	4	$3 \frac{1}{3}$
21	7	$10 \frac{1}{2}$	39	4	$1 \frac{1}{2}$
22	7	$4 \frac{1}{2}$	40	4	0
23	6	$15 \frac{3}{4}$	41	3	$14 \frac{21}{24}$
24	6	11	42	3	$13 \frac{1}{3}$
25	6	$6 \frac{7}{12}$	43	3	$11 \frac{2}{12}$
26	6	$2 \frac{1}{27}$	44	3	$10 \frac{1}{2}$
27	5	15	45	3	$2 \frac{1}{2}$

The

The Construction and Use of the TABLE.

1. In measuring Land it may be observed that the legislative Power, by 33d of *Edward I.* allows that the Statute Acre, and all greater or lesser Quantities depending upon the same, to lie in any Form, Figure, or Shape soever; provided the said Acre shall contain 4 Square Roods, or 160 Square Perches; each Perch containing 30.25 Square Yards, or 27.225 Square Feet, &c. as in the 4th Table of Square Measure foregoing.

2. You may likewise observe by the 5th or last Table, that if 160 Square Perches is the superficial Content of one Statute Acre, then it follows that if the said Area or Content is divided by any given Side in the same Name, the Quotient is the other Side required, whether it be Length or Breadth.

3. For a right Understanding of this Table, it may be proper to illustrate the same by Examples, Explanations, &c.

Having given alternately either the Length or Breadth of the Statute Acre; to find by Inspection, or the Table only, the other Side required.

R U L E.

Let the Number of Perches &c. for the given Side be found in the Table; then directly against or opposite to it, you will find the Side or Number of Perches, &c. desired, that's needful to form and compose the Statute Acre.

Example I.

To know how many Perches, &c. in Length must be measured for the Comprehension of an Acre, when the given Breadth is 25 Perches.

Perch. Feet.

Answer, 6— $\frac{7}{12}$,

Example II.

Admit an Acre of Ground hath for its given Length 6 Perches $6\frac{7}{12}$ Feet: To know its exact Breadth.

Answer, 25 Perches.

Expla-

Explanation.

These two Examples of themselves are so obvious, that there needs no farther Illustration, save only, having found the given Side in the Table, the other Side desired is immediately found to stand opposite or directly against it.

To find arithmetically the Length and Breadth of an Acre of Ground, when one of them is given.

R U L E.

Let always the Square, or superficial Content of an Acre, be expressed in the same Name of the given Side; then having divided the former by the latter, the Quotient will be the other Side required in the same Name of the Divisor.

Example.

If 44 Yards be given for the Breadth of an Acre; to know its exact Length when one Acre contains 4840 Square Yards.

Answer, 110 Yards for the desired Length.

Explanation.

In this Example, and all others comprehended in this Rule, you must always express the Area or superficial Content of one Acre (*viz.* 4840 Yards) in the same Name of the given Side or Divisor (*viz.* 44 Yards broad) then, having divided the former by the latter, the Quotient is the Length, or other Side desired, as above.

OBSERVATIONS.

1. The superficial Content of one Acre given for the Dividend, must always be expressed in the same Name of your given Side or Divisor; or otherwise, be prepared and reduced into the same, before Division can be performed.

2. When you have found the Length or Side required, in the same Name or Denomination of that which is given, you may, for Proof, multiply the one by the other; the Product, if right, will again produce the Dividend, or superficial Content of one Acre, in the same Name of the Multiplicand and Multiplier.

S E C T. VI.

TABLE VI.

Showing readily the Number of Chains, Links, &c. contained in the Length or Breadth of one Acre of Ground, when either of the two Dimensions are alternately given and required.

<i>Chains.</i>	<i>Chains</i>	<i>Links.</i>	<i>Parts.</i>
1	10	00	.000
2	5	00	.000
3	3	33	.333
4	2	50	.000
5	2	00	.000
6	1	66	.666
7	1	42	.285
8	1	25	.000
9	1	11	.111

The TABLE construed and applied.

As 10 Square Chains make one Acre, that is to say, one Chain in Breadth and ten in Length, or two in Breadth and five in Length, is an Acre, as you may see by the small Table; so it follows, that the Breadth and Length being alternately given in Chains, Links, &c. the other is immediately found by looking on the contrary Dimension, or Side required, you will (by Inspection only) discover how many Chains, Links, &c. must be measured in a Right-line to cut off an Acre of Ground. As for Example, Admit the Length of one Side of an Acre is found to be just 7 Chains; then there must be measured for Breadth on the other Side one Chain 42.285 Links, according to the Table; which being multiplied by 7 Chains, the given Length, the Product is 10 Square Chains *ferè*, or 1 Acre.—The like in all other Examples of this Kind are so evident that there needs no farther Illustration.

To find arithmetically, or by the Pen, the Length or Breadth of an Acre of Ground alternately, having one of them given in Chains, Links, &c.

R U L E.

First, If the given Length or Breadth contains Chains, Links, &c. let them be made a mixed Number, by making the Chains so many Integers, and the Links and Parts of a Link conjoined together, so many decimal Parts of a Chain, with a Point prefixed, for your given Side or Divisor.

Secondly, for your Dividend, let always ten Chains be made the integral Part; then having, after a Point annexed thereto, as many Cyphers, at least, as there are decimal Places in your Divisor, you will thereby discover your complete Dividend, which being divided by your Divisor, the Quotient (in a whole or mixed Number) is the Number of Chains, &c. for the other Side required.

Example.

Admit the Breadth of an Acre of Ground is 1 Chain 66.666 Links: To know what Length the other Side must contain, to lay out an Acre only.

Divisor.	Dividend.	Quotient.
Ch. Pts.	Ch. Pts.	Ch.
1.66666	10.00000	6 = Length.
	9.96996	
	Rests	(4)

Answer, 6 Chains for the desired Length.

Explanation.

In this Example, as well as all others of the like Kind, the superficial Content of an Acre (*viz.* 10.00000) being divided decimally by the given Side or Breadth (*viz.* 1.66666 Chain) the Quotient (*viz.* 6 Chains) is the other Side, or Length required.

To find the Number of Acres equal to any Number of Square Chains given.

R U L E.

Divide always the given Number of Square Chains by 10, the Quotient will be the Number of Acres sought.

Or thus :

Let always the Unit's Place of the Number of Chains given, be cut off by a Point or Comma ; then will it follow that those on the Left-hand of the Point, &c. will be so many Acres ; and the significant Figures remaining on the Right-hand of the Point, will be so many decimal Parts of an Acre.

Example.

Suppose a Piece of Ground found to contain 876 Square Chains : To know many Acres.

Answer, 87. $\frac{6}{10}$ Acres.

N. B. *This Example is so obvious of itself, that there needs no Explanation.*

Having any Number of Square Chains and Links given together ; to find accurately the Number of Acres, Roods, and Perches that are contained therein.

R U L E.

First, For the given Sum, let there be always cut off from the Right-hand three Figures or Places ; that is, two Figures for the Links, and one for the Chains ; then will it follow, that those on the Left-hand of the Stroke, Point, or Comma, shall be Acres.

Secondly, Let the remaining Figures on the Right-hand be orderly multiplied by the Number of Square Roods in an Acre (*viz.* 4.) and afterwards by the Number of Square Poles or Perches in a Rood (*viz.* 40) cutting off always three Figures from the Right-hand of each Product ; you will thereby discover orderly

orderly on the Left-hand of each Stroke, Point, or Comma, so many Square Roods and Perches, as well as the last Remainder, to be so many decimal Parts of a Perch.

Example.

In 6709 Square Chains and 96 Square Links ; how many Acres, Roods, and Perches ?

See the Work.

$$\begin{array}{rcl}
 \text{Acres} & = & 670 \mid 996 \\
 & & \times 4 \\
 \hline
 \text{Roods} & = & 3 \mid 984 \\
 & & \times 40 \\
 \hline
 \text{Perches} & = & 39 \mid 360
 \end{array}$$

$$\begin{array}{rcl}
 \text{Acres.} & \text{Roods.} & \text{Perches.} \\
 \text{Answer,} & 670 & \text{---} 3 \text{ ---} 39 \frac{360}{1000}
 \end{array}$$

Explanation.

Having, according to the Rule, annexed the Links given (*viz.* 96) unto the Square Chains proposed (*viz.* 6709) their Sum, making in the whole 670996 : Now seeing three Figures or Places are cut off from the Right-hand, there appears on the Left-hand of the Stroke 670 Acres, and 996 remaining ; which being multiplied by the Number of square Roods in an Acre (*viz.* 4) and three Figures being likewise cut off, as before, there appears on the Left hand 3 Roods and 984 remaining ; which being multiplied by the Number of Square Poles or Perches in a Rood (*viz.* 40) and the three Figures being cut off, as above, there appears on the Left-hand 39 Perches ; which occasions the Figures remaining on the Right, to be so many thousandth Parts of a Perch, as above.

Having any Number of Acres given, to reduce them into Square Chains: Also, to convert readily Square Chains into Square Links.

R U L E.

First, Acres are immediately reduced into Square Chains by annexing a Cypher unto their Right-hand, which is the same as if they were multiplied by 10.

Secondly, Square Chains are expeditiously changed into Square Links, by annexing unto their Right-hand only four Cyphers, which is the same, as if they were multiplied by 10,000 the Number of Square Links contained in one Chain, the Product is so many Square Links, equal unto the Square Chains given.

Example I.

How many Square Chains are contained in 9120 Acres?

Answer, 91200 Square Chains,

Example II.

In 8074 Square Chains, how many Square Links?

Answer, 80740000 Square Links.

N. B. *That the Converse of the last Rule and Examples, of changing Square Links into Square Chains and Acres, is readily effected by dividing your given Numbers respectively by 10 and 10000, the Quotients severally are Acres and Square Chains. Or otherwise, cutting off from the Right-hand of the Square Chains only one Cypher; and four Cyphers likewise from the same Hand of the Square Links; they will again produce (for Proof) the same Number that was first given.*

To find accurately the Area or Content of any Piece of Ground in Acres, Roods, Perches, &c. measured by Mr. Gunter's Chain, in Chains and Links.

R U L E.

1. Although Fields, Inclosures, &c. may lie in various Forms or Figures; yet when they are protracted, they must, at last, be reduced into Squares or Oblongs, Triangles and Trapezia, in order to find their particular Contents, which admits only of so many Chains and Links given, in two Dimensions of Length and Breadth; seeing half the Base and Perpendicular of a Triangle, or half the Diagonal Sum of the Perpendiculars of a Trapezium, do in effect, reduce each of them into a square or oblong Figure; and consequently, allows the Data of each Triangle, or Trapezium, to be comprehended in two Dimensions, as abovesaid.

2. Having therefore the two Dimensions of Length and Breadth given in Chains and Links, you may place the lesser orderly under the greater (without their separating Points) and then, the one being multiplied by the other, the Product is so many Square Links: Now, seeing the Statute Acre contains 100000 Square Links; instead of dividing thereby, you may only cut off (by a Stroke, Point, or Comma) five Figures, or Places from the Right-hand of your Product; you thereby will discover on the Left-hand of the said Stroke, &c. so many Acres; then multiplying the Figures remaining on the Right-hand, by the Number of Square Roods in an Acre (*viz.* 4.) and cutting off five Figures, as before, you will likewise discover on the Left-hand the Number of Roods which cannot exceed three. Lastly, multiplying the decimal Parts of a Rood left on the Right-hand of the Stroke, &c. by the Number of Square Poles or Perches in a Rood (*viz.* 40) and cutting off five Figures, as above, you will thereby discover on the Left-hand the Number of Square Poles or Perches, which can never exceed 39: Hence you will find on the Left-hand of each Stroke, orderly descending downwards, so many Acres, Roods, and Perches, besides the last Remainder, which are always (when significant Figures) so many decimal Parts of a Square Perch: according to the following Example and its Explanation.

Example.

Admit a Field lying in the Form of a Parallelogram, or long Square, to be one way 75 Chains 86 Links, and the other way

Y 4

63 Chains

63 Chains 94 Links : To know the Content in Acres, Roods, and Perches.

See the Work,

$$\begin{array}{rclcl} & & \text{Links.} & & \\ \text{Length} & = & 7586 & = & \text{Mnd.} \\ \text{Breadth} & = & 6394 & = & \text{Mr.} \end{array} \left. \vphantom{\begin{array}{rclcl} & & \text{Links.} & & \\ \text{Length} & = & 7586 & = & \text{Mnd.} \\ \text{Breadth} & = & 6394 & = & \text{Mr.} \end{array}} \right\} \text{Factors,}$$

$$\begin{array}{r} 30344 \\ 68274 \\ 22758 \\ \hline 45516 \end{array}$$

$$\text{Acres} = 485 \mid 04884 = \text{Product,}$$

$$\qquad \qquad \qquad \times 4$$

$$\text{Roods} = 0 \mid 19536$$

$$\qquad \qquad \qquad \times 40$$

$$\text{Perches} = 7 \mid 81440$$

Acres. Roods. Perches.

$$\text{Answer, } 485 - 0 - 7 \frac{8144}{10000}$$

Explanation.

In this Example the given Length (*viz.* 75 Chains and 86 Links) being conjoined together, make 7586 Links for Multiplcand; and the given Breadth (*viz.* 63 Chains 94 Links) being likewise conjoined, makes 6394 Links for Multiplier : Then Multiplication being performed, their Product is 48504884; from whose Right-hand five Figures being cut off, there remains on the Left-hand of the Stroke 485 Acres, and 04884 remaining; which being multiplied by the Number of Square Roods in an Acre (*viz.* 4) the Product is 019536; from which five Figures being likewise cut off, there remains on the Left-hand 0 Roods; then multiplying the decimal Parts of a Square Rood (*viz.* 19539) by the Number of Square Perches in a Rood (*viz.* 40) and cutting off five Figures, as before, there is found on the Left-hand of the Stroke 7 Perches, besides the decimal Parts of a Square Perch : Hence I observe, that the Area, or Superficial Content desired is 485 Acres, 0 Roods, $7 \frac{8144}{10000}$ Perches,

Perches, as above. In like manner all other Examples of the like Kind are performed, when measured by Mr. Gunter's Chain.

Having the Length and Breadth of your Field given in Perches; to find readily the Area or Content in Acres, Roods, and Perches.

R U L E.

Let the two Dimensions of Length and Breadth given in Perches be multiplied together, their Product is square Perches, which being divided by the Number of Perches in an Acre (*viz.* 160) the Quotient is Acres, and the Remainder Perches; which being divided by the Number of square Perches in a Rood (*viz.* 40) the Quotient is Roods, and the Overplus or Remainder so many Perches: Hence the two Quotients and last Remainder will discover the Number of Acres, Roods, and Perches, equal to the square Perches found in the Product.

N. B. *The Converse of this Rule in reducing Acres, Roods, and Perches into Perches, may be expeditiously performed by multiplying respectively the Acres given by 160, and the Roods by 40; adding unto these two Products the odd Perches given, their complete Sum is the Number of square Perches sought.*

Example.

How many Acres, Roods, and Perches are contained in that Piece of Ground, whose Length is 125 Perches, and Breadth 87 Perches?

Acres. Rood. Perch.

Answer, 67 — 3 — 35.

Explanation.

In this Example the given Length in Perches (*viz.* 125) being multiplied by the given Breadth (*viz.* 87) the Product is 10875 square Perches; which being divided by 160, the Number of square Perches in an Acre, the Quotient is 67 Acres, and the Remainder 152 square Perches; which being divided by 40, the Number of square Perches in a Rood, the Quotient is 3 Roods and 35 Perches remaining: Hence I infer the true Answer is discovered as above.

SECT.

Of Surveying by the CHAIN.

S E C T. VII.

T A B L E VII.

Shews how any Number of square Perches may be readily turned into Acres, Roods, and Perches.

Sq. Perches.	Acres.	Roods.	Perches.
5	0	0 $\frac{1}{8}$	00
10	0	0 $\frac{1}{4}$	00
20	0	0 $\frac{1}{2}$	00
30	0	0 $\frac{3}{4}$	00
40	0	1	00
50	0	1	10
60	0	1	20
70	0	1	30
80	0	2	00
90	0	2	10
100	0	2	20
200	1	1	00
300	1	3	20
400	2	2	00
500	3	0	20
600	3	3	00
700	4	1	20
800	5	0	00
900	5	2	20
1000	6	1	00
2000	12	2	00
3000	18	3	00
4000	25	0	00
5000	31	1	00
6000	37	2	00
7000	43	3	00
8000	50	0	00
9000	56	1	00
10000	62	2	00
20000	125	0	00
30000	187	2	00
40000	250	0	00
50000	312	2	00
60000	375	0	00
70000	437	2	00
80000	500	0	00
90000	562	2	00
100000	625	0	00

The Construction and Use of the TABLE.

Having any Number of Square Perches given, to find by Inspection, or otherwise by Addition, the Number of Acres, Roods, and Perches equal thereto, or contained therein.

R U L E.

Let the Number of Perches given, be found in the first Column on the Left-hand of the Table; then directly against it, you will find orderly in the other three Columns, the Acres, Roods, and Perches that are contained therein, according to Example 1.

But when it happeneth that the Number of square Perches given cannot be found in the Table at once; then, in this Case, you are to find it in Particulars; which being added together, their Sum is the Number of Acres, &c. desired. See Example 2. and its Operation.

Example. I.

How many Acres, Roods, and Perches, are contained in 900 square Perches?

Answer, 5 Acres, 2 Roods, 20 Perches.

Example II.

In 197654 square Perches, how many Acres, Roods, and Perches?

Answer, 1235 Acres, 1 Rood, 14 Perches.

See the Work.

Perches.	Acres.	Roods.	Perches.
100000 =	625	— 0	— 00
90000 =	562	— 2	— 00
7000 =	43	— 3	— 00
600 =	3	— 3	— 00
50 =	0	— 1	— 10
4 =	0	— 0	— 4
Sum = 197654	= 1235	— 1	— 14

Explanation.

The Answer to the first Example is found in the Table at once by Inspection only: And the 2d Example is solved by abstracting from the Table the Particulars of the Sum given, which being added together, the Answer is thereby discovered, as in the Work.

S E C T. VIII.

TABLE VIII.

Shews how to find readily from the Table, by Inspection only, the Number of Acres, Roods, and Perches that is contained in any Number of square Links.

Sq. Links.	Roods.	Perch.
100000	4	00
90000	3	24
80000	3	08
70000	2	32
60000	2	16
50000	2	00
40000	1	24
30000	1	08
20000	0	32
10000	0	16
8750	0	14
8125	0	13
7500	0	12
6875	0	11
6250	0	10
5625	0	09
5000	0	08
4375	0	07
3750	0	06
3125	0	05
2500	0	04
1875	0	03
1250	0	02
625	0	01
468 $\frac{3}{4}$	0	0 $\frac{3}{4}$
312 $\frac{1}{2}$	0	0 $\frac{1}{2}$
156 $\frac{1}{4}$	0	0 $\frac{1}{4}$
78 $\frac{1}{8}$	0	0 $\frac{1}{8}$

The TABLE construed and applied.

To find readily from the Table the Number of Acres, Roods, and Perches, contained in any Number of Square Links.

R U L E.

Look in the Table in the first Column on the Left-hand for the square Links given, and directly forward in the other two Columns towards the Right, you will find the Roods and Perches desired, that is equal to the square Links given: But when the Number of square Links given, cannot be found at once in the first Column, then proceed to find them orderly by Particulars, which being added together, their Sum is the Answer desired.

Example I.

To know how many Acres, Roods, and Perches are contained in 70000 Links?

Answer, 0 Acres, 2 Roods, 32 Perches, see Line the 4th in the Table.

Example II.

In 975703 square Links, how many Acres, Roods, and Perches?

Answer, 9 Acres, 3 Roods, 1 Perch $\frac{1}{8}$.

See the Work.

	Sq. Links.	Acres.	Roods.	Perches.
Particulars.	900000	= 9	— 00	— 00
	70000	= 0	— 02	— 32
	5000	= 0	— 00	— 08
	625	= 0	— 00	— 01
	78	= 0	— 00	— 00 $\frac{1}{8}$
Sum	= 975703	= 9	— 03	— 01 $\frac{1}{8}$

Explanation.

The Answer to each of those Examples above are so obvious, that it would be tautology to grant them any farther Illustration.

N. B. That Propositions 5th and 6th foregoing, will likewise prove Tables 7th and 8th, with their Examples.

TABLE

TABLE IX.

Shews how Angles of any kind may be accurately measured and protracted.

TABLE for Acute Angles.

Degrees.	Tenhs.	Links.	Degrees.	Tenhs.	Links.	Degrees.	Tenhs.	Links.
1	1	7	31	53	4	61	101	5
2	3	5	32	55	1	62	103	0
3	5	2	33	56	8	63	104	5
4	7	0	34	58	5	64	106	0
5	8	7	35	60	1	65	107	4
6	10	5	36	61	8	66	108	9
7	12	2	37	63	4	67	110	8
8	14	0	38	65	2	68	111	4
9	15	7	39	66	8	69	113	3
10	17	4	40	68	4	70	114	7
11	19	2	41	70	0	71	116	1
12	20	9	42	71	7	72	117	5
13	22	6	43	73	3	73	119	0
14	24	4	44	74	9	74	120	4
15	26	1	45	76	5	75	121	8
16	27	8	46	78	2	76	123	1
17	29	6	47	79	7	77	124	5
18	31	3	48	81	3	78	125	9
19	33	0	49	82	9	79	127	2
20	34	7	50	84	3	80	128	5
21	36	4	51	86	1	81	129	9
22	38	2	52	87	7	82	131	2
23	39	9	53	89	2	83	132	5
24	41	6	54	90	8	84	133	8
25	43	3	55	92	3	85	135	1
26	44	9	56	93	9	86	136	4
27	46	7	57	95	4	87	137	7
28	48	4	58	97	0	88	139	0
29	50	1	59	98	5	89	140	2
30	51	8	60	100	0	90	141	4

The Construction and Use of the TABLE.

This Table contains orderly the Chord of every Angle, from 1 to 90 Degrees inclusively, and to Radius 100, the Links in a Gunter's Chain, equal to $\frac{1}{2}$, the Arch of a Circle, or 60 Degrees upon the Line of Chords.

Now, as the Chord of every Arch is always twice the Sine of half the said Arch, it will follow that the Construction of this Table may be performed by taking the Sine of half any Degree proposed; and that doubled is the Chord or Number against the said Degree in the Table.

Example.

I desire to know the Chord, or Number in the Table belonging to 71 Degrees.

Answer, 116.1 Links.

Explanation.

The Half of 71 Degrees is $35^{\circ} 30'$, the nearest Sine of which is $58^{\circ} 0'$ (omitting the rest of the Figures towards the Right-hand, because it is only to Radius 100) that 58 Degree, doubled is 116.1 Links, the Chord of 71 Degrees to Radius 100, as you see in the Table, and so are all the rest found.

N. B. The Reader must observe that the Sines or Tangents spoken of in the Construction of these Tables, is meant only the natural Sine or Tangent, and not the logarithmical or artificial.

The Use of the Table is to take Angles that are acute or less than 90 Degrees; which, to save Repetition, may be best done by Rules, Figures, Examples, &c. as in the Application following.

It may likewise be observed, that the Table is continued but to 90 Degrees; because in obtuse Angles the Chord will not so exactly determine the Quantity of the Angle without some Excess or Defect; nor, indeed, to any reasonable Exactness when it approacheth near 180 Degrees; therefore, to remedy this Inconveniency, and supply the Defect, the next Table is intended for obtuse Angles.

The

The second TABLE, or TABLE for obtuse Angles.

<i>Degrees.</i>	<i>Links.</i>	<i>Tenths.</i>	<i>Degrees.</i>	<i>Links.</i>	<i>Tenths.</i>	<i>Degrees.</i>	<i>Links.</i>	<i>Tenths.</i>
90	70	7	120	50	0	150	25	9
91	70	1	121	49	2	151	25	0
92	69	5	122	48	5	152	24	2
93	68	8	123	47	7	153	23	3
94	68	2	124	46	9	154	22	5
95	67	6	125	46	2	155	21	6
96	67	0	126	45	4	156	20	8
97	66	3	127	44	6	157	19	9
98	65	6	128	43	8	158	19	1
99	64	9	129	43	0	159	18	2
100	64	3	130	42	3	160	17	4
101	63	6	131	41	5	161	16	5
102	62	9	132	40	7	162	15	6
103	62	2	133	39	9	163	14	8
104	61	6	134	39	1	164	13	9
105	60	9	135	38	3	165	13	0
106	60	2	136	37	5	166	12	2
107	59	5	137	36	6	167	11	3
108	58	8	138	35	8	168	10	4
109	58	1	139	35	0	169	9	6
110	57	3	140	34	2	170	8	7
111	56	6	141	33	4	171	7	8
112	55	9	142	32	6	172	7	0
113	55	2	143	31	7	173	6	1
114	54	5	144	30	9	174	5	2
115	53	7	145	30	1	175	4	4
116	53	0	146	29	2	176	3	5
117	52	2	147	28	4	177	2	6
118	51	5	148	27	6	178	1	7
119	50	7	149	26	7	179	0	9

The Construction and Use of the TABLE.

You may observe, that this Table begins with 90 Degrees, and from thence is orderly continued to 179 Degrees, inclusively:

fively : It is likewise deduced from the Table of natural Signs, upon this Consideration, that any Isosceles Triangle (and in this Case, we chuse one whose vertical Angle is obtuse) having its Perpendicular let fall from its vertical Angle, is the Sine of either Angle at the Base, one of the Sides being made Radius.

Therefore, to measure orderly and methodically any obtuse Angle by the Help of the Table, you are to observe carefully such Directions as have relation to the same in the next Chapter following; only here we shall briefly illustrate the same as follows :

To find by the Table or Inspection only, the Number belonging to any Number of Degrees contained in any obtuse Angle.

From 180 Degrees let there be always deducted the Angle given; then will it follow that half the Remainder will be the Degrees that will exhibit the natural Sine or Number sought, agreeable to Radius 100, as in the Table.

Example.

What is the natural Sine, or Number belonging to 140 Degrees?

Answer, 34.2 Links.

Explanation.

Take 140 from 180, there remains 40; the Half of that is 20; and the natural Sine of 20 Degrees, is 34.2 Links, to Radius 100, as above.

N. B. If it had been Radius 1000, the Sine had been 342; but as there is but three Places in Radius, there must be fewer in the Sine, and the Figure cut off is a Decimal, and in this Case is two Tenths of a Link, as the Table expresseth.

The third TABLE, or TABLE for external Angles.

Degrees.	Links.	Tenths.	Degrees.	Links.	Tenths.
1	1	7	31	60	1
2	3	5	32	62	5
3	5	2	33	64	9
4	7	0	34	67	4
5	8	7	35	70	0
6	10	5	36	72	6
7	12	3	37	75	3
8	14	0	38	78	1
9	15	8	39	81	0
10	17	6	40	83	9
11	19	4	41	86	9
12	21	2	42	90	0
13	23	1	43	93	2
14	24	9	44	96	6
15	26	8	45	100	0
16	28	7	46	103	5
17	30	6	47	107	2
18	32	5	48	111	1
19	34	4	49	115	0
20	36	4	50	119	1
21	38	3	51	123	5
22	40	4	52	128	0
23	42	4	53	132	7
24	44	5	54	137	6
25	46	6	55	142	8
26	48	7	56	148	3
27	50	9	57	154	0
28	53	2	58	160	0
29	55	4	59	166	4
30	57	7	60	173	2

The Construction and Use of the TABLE.

This Table finds orderly (in Links and Tenths) the natural Signs or Numbers belonging to their respective Degrees from

1 to

1 to 60, inclusively; and in effect is only a Table of natural Tangents, to Radius of one Chain or 100 Links, and to every Degree corresponding thereto; which is so plain to those that understand the Use of natural Tangents, that no more need be said of them.

2. The Use of this Table, in the Application, will manifestly appear; seeing it is chiefly intended for the taking of external Angles; or, more plainly, Angles whose angular Points come out towards you, according to the Method following.

To find by Inspection, or the Table only, any external Angle, without going into the Field, or being otherwise prevented by any Obstacle or Impediment.

R U L E.

Having by those Directions exhibited in Chap. III. found the Perpendicular or Tangent-line of any external Angle, that will cut the Hedge in some certain Point assigned; and consequently, having discovered by Table III. its natural Tangent in Degrees, &c. and deducted it from 180 Degrees, the Difference or Remainder will be the external Angle sought.

Example.

Admit the Perpendicular or Tangent-line found in an external Angle, should be 107 Links, or 1.07 Chain; to know the oblique external Angle sought.

Answer, 133 Degrees.

Explanation.

The Tangent or Perpendicular Line being measured, is found to be 107 Links; whose nearest Quantity by the Table, is found to be 107.2 Links, and against it on the Left-hand is 47 Degrees; which being taken from 180 Degrees, leaves 133 Degrees for the Angle sought.

C H A P. III.

Of the Quality of Angles, with their Description, Use, and Application to Surveying by the Chain only.

S E C T. I.

Angles particularly defined.

1. **A**NGLES as well as Sides, must first be exactly measured in the Field, before they can be truly delineated and protracted at Home, in order that the Form, Figure, or Shape of the Field may be truly discovered, and consequently, its Area or superficial Content thereby exhibited.

2. Plain Angles as well as spherical consist of three Kinds, viz. right, acute, and obtuse; and with regard to their Position, may be either internal or external; the former is so termed when they are measured within the Field, and the angular Points stand opposite to you, at the Extremity of their Legs or Sides; but the latter are external, when their angular Points stand nearest and out towards you; then, in this Case, you are sometimes obliged to take the Dimensions outwardly, in order to Mensuration and Protraction, as follow.

3. A Right-angle contains the 4th Part of a Circle or 90 Degrees; which always happens when one Line falls perpendicular upon the End or Extremity of another: As for Instance; suppose the Length and Breadth of a Piece of Ground is so equal that each of them contains 1 Chain, or 100 Links, yet so posited, that they become perpendicular to one another, by meeting in an angular Point; I say, that these two Lines or Sides, with regard to their Position, shall form a Right-angle, whose Chord or Subtendant, between the Extremity or End of each Line, opposite to the said Angle, shall just contain 90 Degrees, or 141.4 Links, in Table 1st, and Figure 3d, which see.

4. An Acute-angle is any Number of Degree, &c. less than a Right-angle; because the Extremity of the two Lines or Sides by which it is bounded or limited, is supposed to incline gradually one to another; yet can never meet so close as to make one strait Line, by covering one another exactly, for then they would cease to be angular, because void of Space or Distance.

5. An

5. An obtuse Angle is any Number of Degrees, &c. more than 90, or less than 180 Degrees, or the Semi-circumference of a Circle: For if the Space or Distance between any two Lines or Sides of an equal Length, by which it is bounded or limited, is less than 90 Degrees, it becomes acute; but if just 90 Degrees, it becomes a Right-angle, as above; and if any Distance is greater than 90 Degrees, &c. and less than 180 Degrees it becomes an obtuse Angle, because it contains one Right-angle, and one acute; but it can never exceed two Right-angles, or 180 Degrees; because the two Lines or Sides by which it is bounded or limited, are so far extended, that they then fall into a strait Line, and so must needs cease to be an obtuse Angle; seeing it becomes then the Chord line of a Semi-circle, and consequently, the Diameter or longest Line that can be drawn cross the Circle.

S E C T. II.

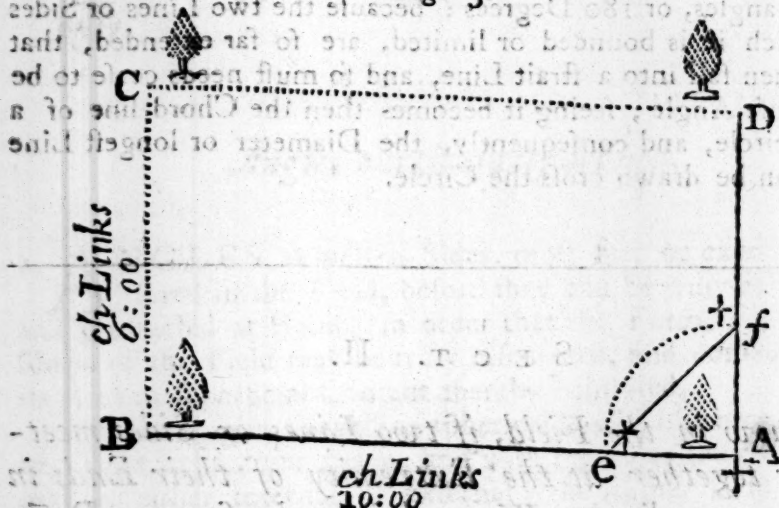
To know in the Field, if two Lines or Sides meeting together at the Extremity of their Ends in a perpendicular Way, does truly form a Rect-angle, or not.

LET any one of the three Station-staffs, as aforesaid, be fixed perpendicularly at the angular Point A in Fig. 3. then it will follow that the Length of the Chain being extended from thence upon each of the two Sides given, that bounds or limits the said Angle, until (by Supposition) it reach to *e* and *f*; where a Station-staff at each of those Places may likewise be fixed: I say the Distance between *e* and *f* being measured in a Right-line, by either the Pole or Chain, will discover the Chord-line or Subtendant in Links and Tenths; which if it happen to be either exactly, or near 141 Links 4 Tenths, equal to 90 Degrees found by Table I. you may then truly infer, that the said Angle, and all others of the same Kind, depending upon the same, are so many Rectangles; but when they appear to be less than 90 Degrees, they can be esteemed no otherwise than acute Angles, belonging to the said Table, as before described.

Example.

Suppose a Field in a quadrangular Form hath its opposite Sides parallel and equal; to know if each of its Angles are rectangular, as well as equal one to another. See Fig. 3. and its Explanation.

Fig. 3.

*Explanation.*

1. In the Figure above it may be observed that the Angle at A, is a Right-angle, equal to 90 Degrees; found in the Field, by extending a Chain containing 100 Links in Length, from the angular Point A, where a Station-staff must be fixed, until it reach to terminate so far as *e* and *f*, upon the two bounding Sides or Hedges BA, and AD; then will it follow that the Chord-line or Subtendant (*viz.* $ef = 141.4$ Links) will be the Measure of that Angle; which being found in Table I. you will find it to correspond exactly with 90 Degrees, as above, which must be noted in your Field-book, as before directed.

2. For Protraction, when you return Home, you are first to draw a dark or obscure Line upon Paper, &c. where your angular Point may be fixed or assigned as at A; then having taken from a Scale of equal Parts, the exact Distance of one Chain to

100 Links; and placing one Foot of the Compasses in the Point A, as above; let, with that Extent, an Arch near so large as a Semi-circle, be described, so that the Extremity of one of its Ends may just touch or bisect the dark or obscure Line given; then taking from the same Scale the exact Number of Links, &c. noted in your Field-book (*viz.* 141.4 Links) and let this Extent be set off exactly upon the said Arch; making such Marks as *e* and *f*, or the like, where the Ends of the Compasses fall: Lastly, from A, through those Marks, draw the Lines AB and AD, which will constitute the Rectangle desired, as true, if not truer, than if it had been taken by the Semi-circle, Circumferentor, or Theodolite.

N. B. *In a quadrangular Field, whose opposite Sides are parallel and equal, if a Rectangle should be therein contained, all the other Angles must consequently be equal to it, according to Fig. 3. above.*

S E C T. III.

How to measure accurately in the Field, as well as protract exactly at home, any acute Angle whatsoever.

R U L E.

1. **T**HE Field Performance begins with the angular Point at A, as before; then, having there fixed perpendicularly a Station-staff, let there be measured from thence a Chain along each Side or Hedge, which, supposing it will reach to *g* and *q*, then with your Chain or Pole measure in a Right-line from *g* to *q*; which allowing to be 86 Links for the Chord-line or Subtendant: Then looking in *Table I.* for the said Number, I find the nearest to it is 86.1 Links, whose Degrees corresponding thereto are 51; which shews that the acute Angle at A is only 51 Degrees; seeing the Fraction need not be regarded, because the Degrees, in this Case, are generally near enough to protract by; especially, if the Field-book do exactly exhibit the same, as before directed.

2. Having found your Dimensions in the Field, as above, your Protraction at Home must likewise begin with the angular Point A, standing in some convenient Part of the obscure Line BA; then having taken from your Scale of equal Parts, the Length of one Chain, or 100 Links, with that Extent let one Point of your Compasses be fixed in the angular Point A, and strike an Arch at adventure, so that it may touch the obscure Line BA. Lastly, the Extent of your Chord-line or Subtendant (*viz.* $gq = 86$ Links) being taken from the same Scale of equal Parts, and applied to the said Arch; so that one Point of your Compasses may rest, as it were, in the Point g , and the other in q ; and having drawn from the angular Point A, through g and q , the Lines BA and CA, which includes the Angle sought BAC, equal to 51 Degrees, or the Chord-line $gq = 86$ Links, as in Table I. and Fig. 4, which see.— In like Manner, may all other acute Angles be so measured and protracted, as abovesaid.

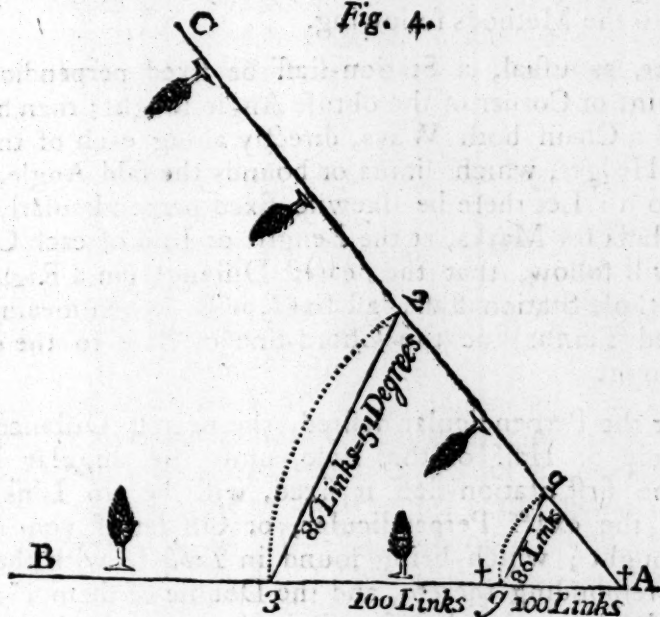
Example.

Admit there is required in the Field any acute Angle, bounded or limited by two Sides or Hedges (*viz.* BA and CA) and allowing there is measured exactly from the angular Point at A, a Chain in Length upon each of the two Sides or Hedges abovesaid, until it terminate with some particular Marks, such as g and q : To know the nearest Distance between those accidental Marks g and q , being the Chord-line or Subtendant of the Angle sought, and consequently the Number of Degrees that is equal to the same.

Answer, 86 Links = 51 Degrees.

Fig.

Fig. 4.

*Explanation.*

In Fig. 4. you may observe that BAC , the acute Angle in the Field, is bounded and limited by the two Hedges BA and CA , and is measured by extending one Chain, whose Length contains 100 Links, from the angular Point A , unto g and q in each Hedge severally; then it follows that the nearest Distance between g and q , being likewise measured by the Chain or Pole, is found to be 86 Links, which being found in Table I. you will find against it 51 Degrees for the Measure or Extent of that Angle.

S E C T. IV.

How to measure in the Field, and protract at Home, any obtuse Angle whatsoever, various Ways.

R U L E.

HAVING already shewn my Reader, how either a Rect-angle or an Acute, may be readily measured in the Field, and afterwards protracted at home; we shall now shew how any obtuse

obtuse Angle may be likewise performed by the Chain only, according to the Methods following.

1. Let, as usual, a Station-staff be fixed perpendicularly at the Point or Corner of the obtuse Angle sought; then having measured a Chain both Ways, directly along each of the two Sides or Hedges, which limits or bounds the said Angle, or is nearest to it: Let there be likewise fixed perpendicularly, two Station-staffs for Marks, at the Length or End of each Chain; then it will follow, that the nearest Distance (in a Right-line between those Station-staffs last fixed, will (when measured in Links and Tenths) be the Chord-line or Base to the obtuse Angle sought.

2. For the Perpendicular desired, the nearest Distance from the Middle or Half of that Base, unto the angular Point, where the first Station-staff is fixed, will be (in Links and Tenths) the exact Perpendicular, or Off-set of your obtuse Angle sought; which being found in *Table I.* with the Degrees corresponding thereto, and the Double of them (because the two Right-angles last found are both equal) being subtracted from 180 Degrees, the Remainder is the Degrees contained in the obtuse Angle desired.

Or otherwise thus:

Let A represent the obtuse Angle required to be taken in the Field by the Chain only; then, in this Case, from A (if the Angle is very obtuse) there must be set off 2 Chains from the Left-hand of A, and other 2 towards the Right-hand, which may be orderly and particularly noted by B and C, or any other Letters or Marks; then fixing one End of the Chain at B, stretch the other directly in a strait Line towards C, making a Mark where the End falls, which may be noted by f, or any other Letter or Number: Measure the Distance from f to A, which suppose to be some certain Number of Links, &c. for the Off-set or perpendicular Line, which being found in *Table II.* you will find exactly opposite to it, the Number of Degrees contained in the obtuse Angle sought at A.

Example.

Admit two Sides or Hedges in a Field should be represented by OQ and OL, in *Fig. 5*: Now it happeneth that QOL is the obtuse Angle sought; and that the versed Sine or Perpendicular,

dicular, commonly termed the Off-set, nor nearest Distance from the angular Point O, unto the Middle of the Chord-line or Subtendant, nm is $Ob = 3.5$ Links: To know the obtuse at O.

Answer, 176 Degrees. See Fig. 5.

Explanation.

1. In *Example I.* and *Fig. 5.* it may be observed, that QO and OL are the two Hedges which include the obtuse Angle sought, *viz.* BOC , whose two Sides or Legs taken in the Field are BO and OC , each of them containing two Chains in Length, in an equal Distance from the Angle at O : Also the nearest Distance between B and C (*viz.* nm) not improperly termed the Chord-line or Subtendant, being measured in the Field, is found to be 3 Chains 97 Links, whose Half, (*viz.* nb or bm) is found to be 1 Chain 98 Links, &c. for the Base of each Right-angled Triangle: Now as $Ob = 3.5$ Links is the Perpendicular or Off-set Line found in the Field, and consequently in *Table II.* it discovers the obtuse Angle sought, *viz.* BOC , to contain just 176 Degrees, as above.

2. For Protraction at home, you are to proceed as follows:

First, Let an obscure Line be drawn, and in the Middle thereof let the angular Point be fixed, and noted by some particular Mark or Letter, *viz.* O .

Secondly, From the said Point let there be a Line perpendicularly erected, and thereupon placed (*viz.* from O to b) the true Extent or Distance of your Off-set Line, found in your Field-book (*viz.* 3.5 Links) taken from a Protractor or Scale of equal Parts.

Thirdly, With the same Extent let a parallel Line be made, and drawn obscurely, but equally distant from that Line, where the angular Point is fixed.

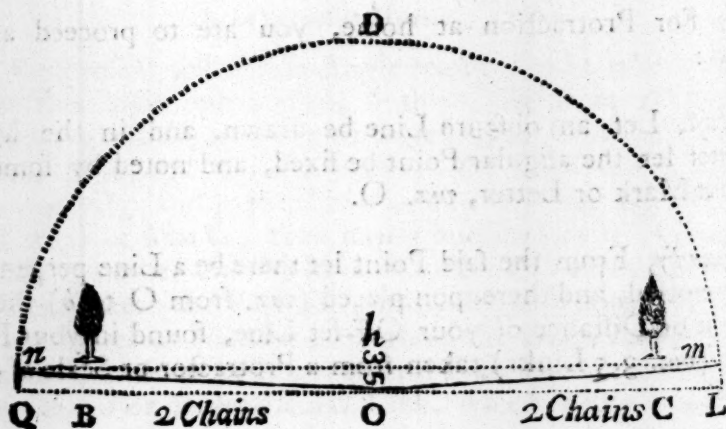
Fourthly, Take in your Compasses from the same Scale of equal Parts, as above, 2 Chains or 200 Links; and having fixed one Point of your Compasses in the angular Point at O , with the other on both Sides of the said Point, bisect the said parallel Line in B and C , as in *Fig. 5.* then will it follow, that

that the two Lines drawn obliquely, from the angular Point at O, thro' the two Points B and C, will both include and discover the Figure and Position of the obtuse Angle sought.

Lastly, The Quantity of the Off-set Line or Perpendicular (*viz.* 3.5 Links) being found in Table II. you will likewise find against it, the Number of Degrees that is contained in the said Angle (*viz.* 176 Degrees :) Also the Chord-line, or Subtendant of the same Angle (*viz.* nm) being taken in your Compasses, and applied to the same Scale of equal Parts, as above, will likewise discover the Base of that obtuse Angle; whose Off-set Line, as before, represents truly the Perpendicular: See Fig. 5. In like Manner all other obtuse Angles whatsoever may be so measured and protracted.

N. B. This Angle is so very obtuse, that by Fig. 5. it appears to want only but four Degrees of two Right-angles, or a Semi-circle. See Reasons for the same exhibited in Sect. V. and Observation 1.

Fig. 5.



SECT.

S E C T. V.

An Obtuse Angle, how protracted by the Line of Chords only.

R U L E.

HAVING shewn in *Sect. IV.* how any obtuse Angle may be truly protracted by a decimal Scale of equal Parts, when the Perpendicular is given in the same Denominations it was measured in the Field; we shall now shew how the same may likewise be truly and readily effected by the Line of Chords only:

First, Let there be drawn upon Paper any obscure Line at pleasure; in the Middle of which, let there be a Point fixed to represent the obtuse Angle sought at A.

Secondly, From your Line of Chords (whether the Radius be great or small) let there always be taken in your Compasses the Extent of 60 Degrees (*viz.* AB or AC.) And having fixed one Point of your Compasses in A, with the other describe the Arch or Semi-circle BDC; so that it may bisect the obscure Line given, on each Side of the angular Point at A.

Thirdly, Having found in your second Table, relating to obtuse Angles, the Degrees corresponding to the given Length of your Off set Line or Perpendicular, in Links and Tenths; then will it follow, that the Extent of a Right-angle or 90 Degrees (*viz.* BD) being taken first from the same Line of Chords, and laid down upon the said Arch, will just extend from B to D: Also the Extent of the acute Angle remaining (which is always the Compliment or Difference between 90 Degrees, and those in the obtuse Angle given) being laid down from D to E, you will thereby discover the Points through which the two Legs or Sides must pass for Bounds or Limits to the said Angle.

Fourthly, Let there be drawn from the angular Point at A, thro' the Point E, the second Leg or Side (*viz.* AE) you will thereby discover the true Figure and Position (*viz.* BAE) of the
the

the obtuse Angle sought, similar to that which was measured in the Field. See Fig. 6.

Or otherwise thus :

1. When the Base, Chord-line, or Subtendant, is only given in Links and Tenths, let there be taken from a decimal Scale of equal Parts, the Extent or Distance of one Inch, by which the Length of one Chain may be represented ; and having fixed your angular Point (*viz.* A.) in the Middle of any obscure Line drawn at Pleasure, let there be an Arch described, so that it may bisect the obscure Line in the Points G and H.

2. From the same Scale of equal Parts, let there be taken the Length or Extent of your Base, Chord-line, or Subtendant, in Links and Tenths, found in your Field-book ; and setting it off upon that Arch, making Marks where the Ends of your Compasses fall, supposing from G to F.

Lastly, From A, thro' the Mark or Point at F, let there be drawn the Line A E or A F, which will likewise constitute the former obtuse Angle sought, *viz.* $\angle GAF = \angle BAE$, as in Fig. 6. which see.

N. B. In all Protractions of this kind, you will find it convenient to plot your Angles with a large Scale, and your Lines with a smaller.

Example II.

There is found in a Field a certain obtuse Angle (*viz.* BA) whose Off-set Line or Perpendicular (*viz.* A b) is 41.5 Links, also its Base, Chord-line, or Subtendant (*viz.* GF) is 18.60 Chains: To know geometrically by Protraction, the Form, Figure, or Shape of the said Angle ; and consequently, the Number of Degrees that is contained therein.

Answer, 131 Degrees. See Fig. 6.

Fig.

2. You may likewise observe, that tho' a right, acute, or an obtuse Angle may be comprehended between any two Sides or Hedges of an unequal Length; yet in measuring or taking Dimensions in the Field, either of the two last must be included in an Isosceles Triangle, whose Off-set Line, or Perpendicular, divided it into two similar right angled Triangles, make your Bases equal, which when conjoined together, becomes the common Base, Chord-line, or Subtendant to the acute or obtuse Angle sought; therefore the Off-set Line or Perpendicular, as abovesaid, must always rise from the Middle of the said common Base; as may be seen in the preceding *Figures*.

3. Although it be needless in surveying by the Chain, only to have any regard to the fractional Part of a Degree arising from its natural Sine in Links and Tenths, especially when under 30 Minutes, or half a Degree: yet, for the sake of the Curious, it may occasionally be found by Analogy thus, *viz.* As 17 Tenths, the natural Sign of one Degree, is to the Minutes in one Degree, (*viz.* 60.00 Min.) so is the fractional Part of any natural Line given in Tenths (*viz.* 3 Tenths by Supposition) to the Minutes corresponding thereto (*viz.* $10.58 = 10 \times \frac{1}{2}$) for Answer. The like in all other Questions of this kind.

S E C T. VI.

To find externally any acute or obtuse Angle, on the Outside of an Inclosure, &c. which by its Complement shall discover the Degrees of the internal Angle required to be measured in the Field.

R U L E.

WHENEVER an outward Angle is required, that shall effectually discover the Degrees in any inward Angle sought, you have no more to do, but to continue a Right-line, the Length of a Chain, from the Extremity of the angular Point which comes out towards you: At the End of which let there

there be fixed a Station-staff perpendicularly; then will it follow, that the nearest Distance from the said Staff to the Hedge, being measured in Links and Tenths; and the Degrees corresponding thereto, being found in *Table III.* you will by them discover the outward Angle required, and consequently, the Complement of the inward Angle sought; which being subtracted from 180 Degrees, the Remainder will be the internal Angle, whether acute or obtuse, that is wanting to be known.

Explanation.

This Rule will evidently appear, by observing *Example 2.* and *Fig. 5.* in *Seet. 5.* wherein it is manifest, that the Length of a Chain being measured outwardly in a strait Line from the obtuse Angle at A, until it determin at H, where must be fixed a Station-staff; and measuring from thence the nearest Distance from H to F, you will find it to contain 82 Links 9 Tenths, which being found in *Table I.* the Degrees corresponding thereto are 49. Hence, I find that the Complement of the obtuse Angle internal, or the measure of the outward Angle, viz. EAC, or FAH, is 49 Degrees, which being subtracted from 180, the Remainder is the obtuse and internal Angle sought, viz. BAE or GAF = 131 Degrees, as above.

Example.

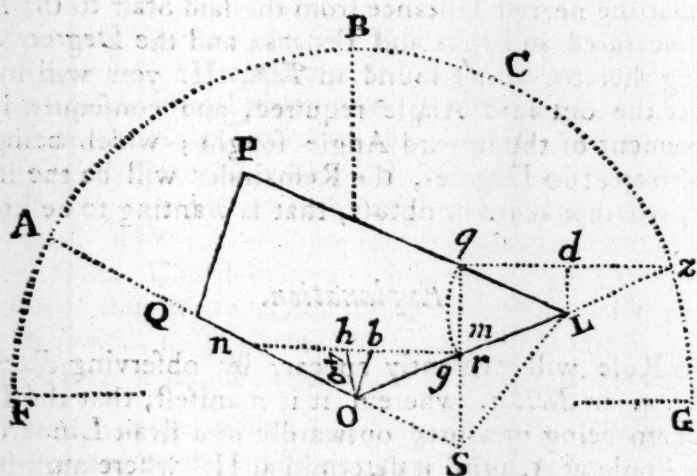
There is a certain Field in the Form of a Trapezium, whose obtuse Angle is vertical, as well as internal; but its Measure cannot be found in the Inside, by Reason of certain Obstacles or Impediments: To know therefore the Figure and Measure of its Complement; and consequently, the Number of Degrees internally contained in the obtuse Angle sought.

Answer, 133 Degrees = $\angle b = 29.9$ Links the Perpendicular = $\angle OL$. See *Fig. 7.*

A a

Fig.

Fig. 7.

*Explanation.*

1. For measuring or taking the Dimensions of the obtuse Angle sought (*viz.* QOL) in Fig. 7. you are to observe, that as the said Angle is internal, and its proper Dimensions cannot be taken in the inside of the Field by reason of certain Obstacles or Impediments, *viz.* Briers, Thorns, Water, &c. so in this Case, you are always to find its Complement (*viz.* OSL) by measuring the Length of a Chain from the obtuse or outward Angle (*viz.* O) until it terminate or end (by Supposition) at S , in such manner, that by certain Marks it may appear to be directly in a strait Line with QO . Again, at the End of the first Chain, supposing at S , as abovesaid, let there be measured from thence, the nearest Distance perpendicularly to r ; which will likewise cut the Hedge OL in the Point r ; then will it follow, that the Length of the perpendicular Line $Sr = 107$ Links, or 1 Chain 7 Links, being found in Table III. or the nearest Number to it (*viz.* 107.2 Links) you will find against it, on the Left-hand, 47 Degrees for the Angle LOS , being the Complement of the obtuse Angle sought; which being subtracted from 180, the Remainder is 133 Degrees, for the internal and obtuse Angle (*viz.* LOQ) desired.

But supposing the acute Angle at L , *viz.* OLP , is required to be taken on the outside of the Field, as before; then, in this Case, you are to proceed from L , in the Right-Line OL , by the Method above directed; and having set off one Chain in a strait Line, from L to q ; then next finding the Point d , which will be exactly in the Middle of qz , you will thereby find

find the Off-set Line or Perpendicular to dL ; which, when measured, is supposed to be 43 Links; and the same being found in Table II. you will find against it on the Left-hand, 129 Degrees for the Angle qLz ; which being taken from 180 Degrees (because zLO or zLg) is a Right-line, there remains 61 Degrees for the inward Angle PLO , or qLg required.

For Protraction.

1. That this may be orderly and methodically, as well as truly effected, you are to be provided with an half Inch Scale, whose decimal Parts or Divisions, may be exactly agreeable to a Line of Chords of the same Radius; which must be used with a Pair of Compasses of a moderate Size, but their Points very nice, small, and smooth: Then you may proceed to Protraction, according to those general Directions exhibited in Sect. IV. which see.

2. Having set off each way, supposing from O to m and n (as in Fig. 7.) an half Inch, representing the Length of 1 Chain for each of the two Legs or Sides; by which the obtuse Angle (*viz.* O) is bounded or limited; then the common Base of your said Angle, known by the Chord-line or Subtendant (*viz.* nm) being drawn, and from its Middle a perpendicular Line being let fall, whose Length (*viz.* $bo = 40$ Links) shall be equal to your off-set Line or Perpendicular, found in your Field-book.

3. By the second Article, as above, you will find that bo is the Sine of the Angle hno , or hmo , if you make Om of On Radius; but Om and On together is the Complement of the Angle O to 180 Degrees; and they being equal, either of them, as supposing Om is half the Complement of that Angle mOn to 180 Degrees: Therefore, whatever the Quantity of the Angle O , the Line Ob (the Distance of the Middle of the Line mn from the Angle O) is the Sine of half its Complement to 180. Hence it is, that the successive Degrees in the first Row of each Column in Table II. being subtracted from 180 Degrees, and half of that Remainder being taken for the natural Sine, is the Number required to be placed in the said Table.

S E C T. VII.

Of the Use of Angles in general.

How to measure and protract by a general Rule, the various Kinds of Angles, both internally and externally belonging to any Field, &c.

R U L E.

HAVING in the preceding *Sections* of this *Chapter*, treated orderly and methodically of the various Kinds of Angles, with their Use and Application to Surveying by the Chain only; illustrated by proper Figures, Rules, Examples, and Explanations; we shall now shew by a general Rule, the Comprehension of the whole, with all imaginable Exactness and Accuracy, as well as Plainness and Conciseness. And,

1. When an Angle is measured in the Field, let it be noted and distinguished by some certain Mark, at which, let there be perpendicularly erected a Station-staff.

2. Having placed upon the said Staff the Ring found at either End of your Chain, let the other be extended both Ways, in a strait Line, along each of the Sides or Hedges which bounds or limits the said Angle, internally or externally taken.

3. Observe carefully at each of the two Places abovesaid, where the Length of the Chain terminates or ends; for there must be likewise erected two Station-staffs perpendicularly.

4. Measure in a Right-line, the nearest Distance between the two Station-staffs last fixed, and let that Distance, in Links and Tenths, be noted in your Field book, for the Chord-line or Subtendant, by which the Base Line of the Angle sought is protracted.

5. From the Middle of the Chord-line last found, let there be likewise measured from thence a strait Line, perpendicularly to the first Station-staff, or Point of the Angle sought; then will it follow, that that Distance, or nearest to it (in Links and Tenths) being found in its proper Table, you have against it, on the Left-hand, the Degrees that the Angle contains, which sometimes is likewise the Complement to 180 Degrees for the internal Angle desired.

Lastly, For Protraction upon Paper, &c. I shall only refer you to that *Section* preceding, which contains a Figure like
unto

unto the same Angle sought, though differing in degrees; where you will find the Method of plotting the same illustrated by Rule, Example, and Explanation.

OBSERVATIONS.

1. If the Angle measured internally by the general Rule above, appears to be at or near a Right-angle not exceeding 90 Degrees, then *Table I.* will truly discover its Quantity.

2. When it happens that the Angle internally measured by the Rule abovesaid is an Acute-angle, containing any Number of Degrees less than 90; then likewise, its Quantity in Degrees is discovered by *Table I.*

3. If the Angle, internally measured, happens to be any Number of Degrees more than 90, and less than 180, then *Table II.* discovers always the Quantity of that Obtuse-angle, according to the general Rule abovesaid.

4. When the Angle measured externally, happens to be either acute or obtuse; then *Table III.* will accurately discover the Degrees contained therein, seeing it is oftentimes the Complement of the Angle sought, according to its proper, &c. in *Seet. VI.* which see.

Questions or Examples propounded for the Exercise and Improvement of the young Practitioner.

Example I.

A quadrangular Field in the Form of an Oblong, or Geometrical Square, hath for the Chord-line, or Subtendant of each of its Angles 141.4 Links: To know the Degrees in each Angle.

Answer, Degrees. See *Fig.* and its Rule.

Example. II.

Admit the Chord-line or Subtendant of an Acute-angle measured in the Field is 111.8 Links: To know the Figure and Quantity of the said Angle,

Answer, Degrees.

Example. III.

Suppose an Obtuse-angle found in a Field hath for its Offset Line or Perpendicular 8.7 Links: To know its Figure by

A a 3

Pro-

Protraction, and consequently, the Degrees contained in the said Angle.

Answer, Degrees. See Fig. 5. and Table II.

Example. IV.

There is found in a Field an obtuse Angle, whose Off-set Line or Perpendicular is 46.9 Links: To know the Figure and Quantity of the said Angle.

Answer, Degrees. See Fig. 6. Table II.

Example. V.

To know the Figure, as well as the Number of Degrees contained internally in that obtuse Angle, whose Complement (being an acute Angle external) hath for its Off-set Line or Perpendicular 107.2 Links.

Answer, Degrees. See Fig. 7. and Table III.

Example. VI.

There is measured externally in the Field an acute Angle, whose Off-set Line or Perpendicular is found to be 17.6 Links: To know the Figure and Quantity of the obtuse Angle internal.

Answer, Degrees. See Table III. and Fig. 6.

CH A P. IV.

Of PRACTICAL SURVEYING accurately performed by the CHAIN only: Or the Method of measuring promiscuously Lines and Diagonals, as well as Angles: by which are represented the Hedges, Fences, Corners, &c. of any Field or Inclosure.

S E C T. I.

Lines, Angles, and Diagonals, explained and defined.

HAVING in the preceding Chapter treated of the various Kinds of Angles, as well as their Use and Application to Surveying; we shall now briefly and plainly shew the Quantity and Use of Lines and Diagonals. And,

1. A Line in the first Magnitude in *Geometry* or *Geodesia*; having Length, but no Breath or Thickness; and is generated by the Motion of a Point, or rather, many Points joined together in Length, by which a Line is said to be produced.

2. Lines consist of two kinds, *viz.* strait and curved.

3. A strait or Right-line is the nearest Distance between any two Points; and by such are represented all Sides or Hedges in Fields, &c. which seem to run parallel to any strait Line taken by the Chain.

4. A curved or crooked Line that is finite, is likewise produced by the Termination of any two Points in a circular Motion, such as the Curve of an Hedge, when it resembles the Segment of a Circle, or an Ellipsis: Therefore it must needs differ from a Right-line in Figure, as well as in Length, especially when it is converted into the same, or so stretched and extended, as that it may appear to be Right.

5. A curved Line that is infinite by its Motion, returneth again to the Place where it first began, and so maketh the Line infinite, and the Ends or Bounds thereof undeterminate, such as the Figure of a Circle, which seldom happens in Surveying.

6. An Angle is the meeting of two Lines in a Point, provided the two Lines so meeting or inclining do not make one strait Line; which are commonly expressed by three Letters, the middlemost of which signifies the Angle meant: As the Angle ABC signifies that the middle Letter, *viz.* B, denotes the Angle belonging to the Triangle ABC, &c.—Hence it is, that an Angle in the Field is occasioned, likewise, by the meeting of any two Sides or Hedges at the Extremity of their Ends, which in their Distance (at some Space from the angular Point) may be either acute or obtuse; the former is when the two Legs or Sides by which it is bounded or limited, are more or less contracted, in such manner, that the Distance between them shall not extend to 90 Degrees, *viz.* a Right-angle, or true Square: And the latter is, when the Legs or Sides forming the Angle, are so far extended from one another, as that their Distance, when measured, shall not exceed 180 Degrees, but be more than 90. See *Chap. III. and Sect. I.*

7. A Diagonal Line is always a strait Line, passing from one Corner to another. Or it is a Right-line, running thro' any inclosed Field, from one Angle to another, in order to reduce the same into Triangles, Trapezia, or some other Figure, whose Superfices being added, shall be exactly equal unto the Content of the Field given: Therefore, whenever a Diagonal

or Diagonals can be conveniently taken in the Field, &c. and accordingly noted in your Field-book, they will truly and readily discover upon Paper, &c. the Form, Shape, or Figure of the same in Protraction.

S E C T. II.

To find accurately the Length of a Right-Line by the Chain in the Field, and to protract the same by the Scale at home, in order that any Hedge or Side may be truly delineated and represented.

R U L E.

HAVING already described in Chap. I. Gunter's Chain, measuring Pole or Rod, Station-staffs, Arrows, and Field-book, with which every Surveyor ought to be provided in order to measure exactly any Right-line, Diagonal, or Angle in the Field, by the Chain only: We shall now proceed orderly and methodically to the practical Performance of the same. And,

I. When you enter the Field, go to some Eminency from whence you can see it all; or if you cannot from any one Place see it all, walk about some Part of it, till you can see the Shape of it; or know how many Sides or Angles it contains; which, when you have discovered, draw upon a Piece of common Paper, with a black Lead or red Pencil, the Form thereof, as near as you can conveniently; but you need not mind to be exact nor at the Trouble of drawing the same by a Ruler, seeing the Design hereof is only to give you imperfectly an Idea or Apprehension of the Figure thereof, particularly the Number of Sides and Angles it contains.

II. Let the Surveyor's Assistant take one End of the Chain, and let it be stretched or extended at full Length, in order to see, when you begin, that the Links be clear; for if they are not, it both makes the Chain shorter, and endangers the breaking of it: Now the Assistant having taken the End of the Chain in his Right-hand, and ten Arrows in his Left, let the Surveyor, who usually follows with the other End of the Chain, extend his End to some remarkable Place, in the Corner, viz. Gate, Style,

Style, Tree, &c. where you intend to begin, supposing at A, as in the Margin.

III. The Surveyor being at A, let his Assistant stretch out the Chain towards the Right-hand, streight along the Hedge toward B, the supposed Mark in the next Angle or Corner, where the first Hedge seems to end, or you intend to measure to; and having the Ring at the End of the Chain put upon one or two of the Fingers of his Right-hand, with one Stick or Arrow, and all the rest in his Left; let him at the End, or full Length of one Chain, stick by the Toe of his Right-foot, the said Arrow; to which, when the Surveyor, or hindermost comes, let him take up: And all the Way afterwards, let the hindermost guide the foremost, and the foremost the hindermost, in a Right-line between him and the Mark at B; and the foremost see the hindermost, in a strait or Right-line between him and the mark at A, where they began; then are they both in the Right-line between A and B.

A

IV. Therefore, proceeding orderly in this Method, as above, till the Assistant or foremost has spent all his ten Sticks or Arrows; which (if right) he will find them in the Hand of the Surveyor or hindermost, who must be careful to place (for every ten Chains) a bold Point in your Field-book, in the Column of Changes, and the odd Chains, or Links (if any happen) in the very next Column succeeding, as before directed: Then proceeding forward, let the Assistant or Fore-man, take again all the ten Sticks or Arrows; and carry them all in his Left-hand, save only one in his Right, as before, together with the Ring at the End of the Chain, upon one or two of the Fingers of the same Hand, as usual; then proceeding, as above, in a strait Line towards B; not looking behind him till he feel the Chain check him; then stick down that Arrow, and away as fast as you can; and as you go, shift another Arrow in your Right-hand, to be ready to stick down at the Chain's End; in the mean time, the Surveyor or hinder Man holding the Chain in the Right hand, let him look that it be not entangled, and go on till he come to the next Stick or Arrow; then clapping the End of the Chain to the Stick, let him take it up with the same Hand that he holds the Chain with, and away after his Leader.

B

V. Now

V. Now the ten Sticks or Arrows being supposed to be run out, and you are not yet at the End of that Station (that is, not come to the Mark at B) let the Assistant or Fore-man run one Chain more, holding still the Ring in his Hand, and at the End thereof set his Toe, there standing still: and let the Surveyor or Hinder-man take up the tenth Stick, and hold that in one Hand, and the other nine in the other, and deliver the nine to the Fore-man setting his Toe to the Fore-man's, and let the Fore-man count the nine, and if they be right, away forward; but if they be not, you must measure that Length again, and seek the Stick; for you know not which of you lost it, and thus go on to the End.

Example I.



Admit the strait Line A B (as above) is allowed to represent a certain Side, or Hedge in a Field; and that each of the first seven Divisions, when measured, contains ten Chains in Length, and the 8th or last Division only 9 Chains 85 Links, which make in the whole 79 Chains 85 Links: To know how the same, or any other of the like kind, may be truly delineated or protracted. See the Margin.

Scales used in Protraction.

R U L E.

In Order to Protraction, or laying down geometrically the Length of any Line measured in the Field, as in the Margin, you are to select a good Scale of equal Parts variously divided into greater and lesser Quantities, corresponding to such equal Divisions, as 10, 11, 12, 16, 20, 24, 30, 40, 48, &c. for plotting all Manner of Figures of several Sizes both greater and smaller, as occasion requires.

Now, although those several Sorts of Scales abovesaid, differ in their Length, for the more commodious laying down Figures of various Kinds and Sizes, whose Areas or Contents, according to the latter, shall be proportionably equal, and all of them com-

comprehended in two Sorts of Lines, viz. the first of equal Parts, for laying down Chains and Links, and the second of Chords, for laying down Angles of several kinds, according to the Method prescribed in *Chap. III.* which see.

But those which had reference to Lines only, we are to observe, that each of them contains ten equal Divisions, which in their Use are proportionably equal one to another; seeing each Line is divided into certain equal Parts, representing usually 8 Inches; and those grand Divisions are numbered with arithmetical Figures, by 1, 2, 3, 4, &c. to the End of the Ruler upon which the Scales are described: And the uppermost large Division is again divided into ten other smaller Parts, every Part containing ten Links of your Chain; each of which smaller Parts you may suppose to be again divided into ten other lesser Parts, representing single Links of your Chain.

There is also another Sort, intituled a Diagonal Scale, more convenient than any of the former, for its Neatness, Ease, and Dispatch, because it comprehends ten equal Divisions to an Inch, and agrees exactly with Mr. Gunter's four Pole Chain: for as the whole Chain contains 100 Links, so each Inch of this Scale contains likewise 100 Parts; by reason whereof, any Number measured in the Field by your Chain to a Link, may likewise be taken from this Scale, and laid down exactly upon Paper.—You may also have reversedly described upon the same Ruler, an Half-inch Scale, divided likewise into 100 Parts; which will be of good Use, when you are required to lay down a smaller Plot; seeing the former, which is twice its Length, is more peculiarly adapted to longer Plots.

Any Distance being measured by your Chain; how to lay down the same Distance upon Paper.

As those Scales which comprehend 10, 11, 12, or 16 Divisions in an Inch, are more particularly adapted to the laying down Figures that require large Paper, &c. so in the last marginal *Diagram*, we have made use of those whose Divisions are lesser, consisting only of 20, 24, 30, 40, or 48 Divisions in an Inch, seeing they are more conveniently agreeable to those Figures that must be comprehended in a lesser Space.

For an Illustration of the same, we will suppose, according to the last Example, that having measured along an Hedge; or the Distance between any two Marks or Places (viz. from A to B) with your Chain, you find the Length thereof to contain 79 Chains 85 Links. Now, to take this Distance from your Half-

Half-inch diagonal Scale of 20 in an Inch ; in order that the same may be protracted, or laid down upon Paper, &c. thus :

First, Draw an obscure Line at pleasure, then place one Foot of your Compasses upon your Scale at the Figure 7, for 70 Chains ; and extend the other Foot to nine of the small Divisions (which represent the odd nine Chains :) Then for the Links, you are to remove your Compasses to the middle Space next that parallel Line, which is noted by the Figure 8 on the top of the Scale ; then will it follow, that the whole Extent from 7 in the large Divisions, unto 9 in the smaller, comprehending likewise in a straight Line (for the 85 Links) 8 parallel Lines and an half, from the Left-hand to the Right, you will thereby receive between your Compasses the true Extent sought.

Secondly, With the said Extent or Distance, let one Point of your Compasses bisect the obscure Line given, at B, where the other is fixed upon the said Line at A ; then having drawn the Line A B, you will thereby, from the diagonal Scale of 20 in an Inch, comprehend upon Paper the Side or Hedge measured, represented by the Line A B, equal to 79 Chains 85 Links, as above.

Thirdly, But if you would have your Line shorter, and yet contain the same Distance, as above ; then, in this Case, you are always to take your Distance from smaller Scales of 24, 30, 40, or 48 in an Inch ; so shall the 79 Chains 85 Links end at C, if taken from the Scale of 24 in an Inch ; or at D, by the Scale of 30 ; or at E, by the Scale of 40 ; or at F, by the Scale of 48 ; either of which Lines will contain 79 Chains 85 Links, and be in Proportion one to another, as the Scales from whence they were taken. And in this manner, may any Number of Chains and Links be taken from any of the Scales abovesaid.

A Right-line being given, to find how many Chains and Links are therein contained, according to the Scale assigned.

Example II.

Suppose A B were a Line given, as before, and it were required to find how many Chains and Links were contained therein, according to the Scale of 20 in an inch.

Answer, 79.85 Chains.

R U L E.

R U L E.

Take in your Compasses the Length of the Line A B, and applying it to your Scale of 20 in an Inch, you will find the extent of the Compasses to reach from 7 (denoting 70 Chains) to the great Division to 9 Chains 85 Links of the lesser Divisions; as before directed: Wherefore the Line A B contains 70 Chains 85 Links; and, as such, is the Converse of Example I. seeing they interchangeably prove one another. The same must be done for any Line, and also by any one of the other Scales.

N. B. Here you must carefully remember, that in laying down of the Length of Lines by your Scales; whatsoever Scale you must continue it to the End; not laying down one Line by one Scale, and another by another. But if you would have a large Work in a little Room; then use a small Scale, as of 24 or 30 in an Inch: But, on the contrary, if you would express every small Particular, then you were best to use a large Scale, as of 10 or 12 in an Inch.

S E C T. III.

To measure in the Field any Diagonal-Line by the Chain only; with its Use and Application.

R U L E.

YOU are to observe, that a Right-line and a Diagonal, differ only in their Position; but not in the Method of measuring, as above; because the former is used when Hedges, Fences, or Sides of Fields, &c. by it are taken and represented: And the latter, when a Line is taken or measured across the Field, from the Angle or Corner to another in order that the Trouble of Angles may be avoided, and that the Figure or Form of the Field may be readily, as well as truly delineated and protracted.

2. It often happens when the Fields, &c. are large, or the ground more high in the Middle than the Sides and Angles, which

which encompasses the Field; by reason whereof, you cannot see from one Angle to its opposite; then, in this Case, it will be found necessary to go somewhere towards the Middle of the Field, or where the Ground is so high, that you can see the two opposite Angles, *viz.* that you departed from, as well as the other you are measuring to; then, in the same Place where your Observation is taken, let a Pole or Station-staff be perpendicularly erected, with a piece of Linen rag, or white Paper, to render it (as well as the other two Poles or Station-staffs, fixed at the two angular Points or Corners abovesaid) so conspicuously plain, that they may all three evidently appear to be in a right or straight Line one to another.

3. Now, that this may be accurately performed, the Surveyor, or him that follows the Chain, must stand at the middle Pole or Station-staff, whilst his Assistant (about the Distance of 60 Yards on each Side thereof) can see from those two Stations, as above, the two opposite Angles, where the other two Poles or Station-staffs are fixed; then it will follow, that the Assistant, by moving gradually towards either Right or Left-hand, until the Surveyor (standing at the middle Pole or Staff as above) is prevented from seeing either the opposite Angles by the Interposition of the other's Body; for there the middle Pole or Station-staff may be perpendicularly erected, seeing directly in a straight Line to the two opposite Angles whose Distance is required.

4. Having thus fixed your three Poles or Station-staffs in a straight Line, as above, you may then begin to measure (according to the Directions of the last Rule in *Sec. II.*) from the first Angle or Corner, to the Pole or Station-staff in the Middle, without regarding it otherwise, than only observing that it likewise be in a Right-line from the Angle you departed from: and from thence, you may proceed in like Manner, until you have arrived at the opposite Angle to be measured to: You will thereby discover the true Distance or Length of the Diagonal-line sought; which must accordingly be inserted in your Field-book, as before directed.

5. For Protraction, you are to proceed exactly according to the 2d Article of the last Rule, by taking from a proper Scale the exact Length, or true Extent of your Diagonal-line found in your Field-book, and laying it down, or making an Arch with the said Extent, from the Extremity or End of your Base-line, which represents the End of your Side or Hedge and consequently, the angular Point you measured from: then it will follow, that a Perpendicular-line being erected from

from the middle of the said Base, will bisect the said Arch, at the Length of the Diagonal-line sought, which will become the Side of an Ifofceles Triangle, and consequently, divide the whole Field either into Triangles or Trapezia, as may be seen in the *Chapter* following.

Example.

There is a certain Field which contains within it a Diagonal-line, whose Length, when measured, is found to be 87 Chains 39 Links: To know how the same may be laid down upon Paper, &c. from a Scale of 20 in an Inch. See *Example I.* and its Illustration in the second Article foregoing.

S E C T. IV.

Angles how measured and protracted from various Scales.

R U L E.

HAVING in the third *Chapter* foregoing, shewn copiously, as well as orderly and methodically, how Angles, in their various kinds may be accurately measured in the Field, and afterwards protracted at home; we shall now briefly shew, how the same may be effected by various kinds of Scales, according to the Form and Size of the Figure, whether bigger or lesser. And,

- I. *How to lay down upon Paper, &c. an Angle, containing any Number of Degrees and Minutes, by the Line of Chords, according to any Length proposed.*

First, Draw a Line at Pleasure, supposing BA; by which is represented the Base-line or first Side of the Angle given.

Secondly, Let a Line of Chords of some moderate and convenient Length, be selected proportionably to the Size of your intended Angle, supposing an Half-inch Scale of 20 Divisions in an Inch: Then,

Thirdly, Extend your Compasses from the Beginning thereof to 60 Degrees always; and with this Distance or Extent, setting one Foot upon the Point A; with the other describe an Arch not too short, supposing C F.

Fourthly, From the same Line of Chords, take always, for your second Extent, the Quantity of the Angle given in Degrees and Minutes (supposing it be 41 Deg. 20 Min.) and with one Foot of your Compasses in the Point C, upon your Base-line, extend the other upwards; so that the said Arch may (by Supposition) be bisected in the Point at E.

Lastly, Thro' the Point E, draw the Line A E, to be continued to D, or any other Length proposed; it will then make the two Lines, viz. A B and A D, to comprehend the desired Angle, viz. B A D, equal to 41 Degrees 20 Minutes, as may be seen by the *Figure*, when protracted according to this Rule.

N. B. If any Angle given or required, happen to be obtuse, or contain above 90 Degrees; then, in this Case, it will be needful to protract it at twice, by dividing the said Angle into any two Parts equal unto it; and then laying them down severally upon the Arch of 60 Degrees, in such manner, that one of the Points of your Compasses (in the second or last Extent) may center exactly where the first Distance or Extent did terminate. As for Instance, if the given Angle were 159 Degrees, take 80 and 79 Degrees, or 90 and 69 Degrees, or rather by taking 60, or twice 60 (if the Angle be so much) and then the Remainder, which according to this Rule, will likewise be illustrated by the *Figure*, when protracted.

II. An Angle being given, to find what Number of Degrees and Minutes it containeth.

This being exactly the Converse of the first Article in this Rule, you will readily comprehend the same, by supposing the *Figure* of the last Article, viz. B A D, were actually given, in order to find the Quantity thereof: To effect which,

First, Let your Compasses be opened and extended to 60 Degrees of your Chord, and placing one Foot in A, with the other describe the Arch C F, as before.

Secondly, Take in your Compasses the Distance of the Angle given, viz. C E, and applying it to the same Line of Chords, or measuring that Extent upon the said Chord-line from the
Begin-

Beginning thereof; you shall find it to reach to 41 Degrees 20 Minutes, which is the Quantity of the Angle required.

Note, The Figures or Geometrical Protraction of these Angles, as well as all others of the like kind, are here omitted, in order that the young Artist, by Projection, may attain unto Proficiency herein.

OBSERVATIONS.

1. The Reader may here observe, how the Line of Chords proportionably agrees with the Diagonal Scales of equal Parts, from whence they are deduced. As for Instance, 60 in the Line of Chords, is always the Radius or Semi-diameter of a Circle, whose Circumference is 360 Degrees; and when made proportionably equal to an Inch-scale of ten Divisions in an Inch, it will exactly prove equal to 3 Chains 10 Links, upon the same Scale, and 90 Degrees from the said Chord will exactly correspond to 4 Chains 40 Links, upon the same Scale of equal Parts.

Also in a smaller Line of Chords made proportionably to an Half-inch-scale of 20 Divisions in an Inch, the former will so agree with the latter, as to be exactly equal to 5 Chains 10 Links; and 90 Degrees upon the same Line of Chords, will agree with 7 Chains 28 Links upon the said Scale.

2. But when Angles are required to be protracted in a small Space, the Chords must then be made proportionably likewise in their Length; and, in this Case, it sometimes will be needful to make use of such a Line of Chords, as that 60 upon the same, will exactly agree with 3 Chains, upon an Half-inch-scale of 20 Divisions in an Inch; and 90 Degrees from the same Chord will be equal to 5 Chains 60 Links upon the said Scale. Also, a Line of Chords may occasionally be made so small as to agree exactly with a Quarter-inch-scale of 40 Divisions in an Inch; then it will follow, that 60 Degrees in the former, will exactly agree with 6 Chains in the latter, and 90 Degrees in the former, will just extend to 8 Chains 50 Links in the latter.

3. From these Observations it appears, that an Angle may be measured or protracted by any of the Scales abovesaid, as well as those Lines of Chords, which are made proportionably equal to the same.

S E C T. V.

Practical Observations for the Use and Application of the Chain. How to rectify accidental or casual Mistakes measured by the Chain: As also, particular Cautions carefully to be observed in the Use thereof.

R U L E.

1. **T**HE Surveyor and his Assistant are to be very careful, that they are not mistaken in the Length of a Chain, either under or over, in their Account; for if they should, the Error would be very considerable; as suppose you were to measure a Field that you knew to be exactly square, and therefore need only measure one Side of it: If, in the Performance, one Chain should happen to be mistaken, and by reason of the same, you reckon but nine Chains, when it was really ten; the Content of that Field would then be but 8 Acres and 16 Perches, when it should be ten Acres just: And if in so small a Line such a great Error may arise, what may you imagine to be in a greater? Therefore, Care and Circumspection herein is needful, according to the general Directions prescribed in *Sett. II.* which see.

2. When you begin to measure, you must be sure that he who leads the Chain shall have all the ten Arrows at the End of every ten Chains, as well as at the Beginning; for if (by Inadvertency) you begin with nine instead of ten, you will err one Chain in ten ever after, until you discover it. Also, if after some time of working, you discover you have lost an Arrow; but know not where, nor when you lost it, you will be obliged to do all the Work over again that you have done that Day; or at least all that you have done since sometime; or since you were at some Place, where you can certainly recollect that you had all the ten.

3. To prevent those Mistakes above, it is needful for the Surveyor to inquire of his Assistant, at the End of every ten Chains, how many Arrows each hath, and if their Sum are not ten, it is a plain Indication, that the Arrow which is wanting, hath been either dropt or left behind; and consequently, the last ten Chains in Length is doubtful: Therefore the same must be re measured again before you proceed.

4. You

4. You are likewise to observe, at the End of every Side or Hedge in the Field so measured, that there be no Mistakes made in numbering the Arrows belonging to the odd Chains, &c. which accidentally are either under or above the last ten. —Now, that this may likewise be prevented, you are to insert orderly in the first Column of your Field-book, intitled *Changes*, a bold Point for every ten Chains, thus found; then it follows, that the odd Chains and Links, with which the Side or Hedge concludes, will stand orderly in the second or next Column, as before directed.

5. In measuring a Right-line in a Field, by which an Hedge or Side is represented, it sometimes happens, that you are obstructed by Briars, Thorns, Bushes, Ponds, or other Things intervening; then, in this Case, you must measure a straight Line at some convenient Distance, parallel to the Hedge from Angle to Angle: But herein you are carefully to observe, that when you have carried this straight Line unto the next Hedge; then having measured from thence (in a Right-line) the exact Distance, unto the Angle or Corner on your Right hand, which must always be esteemed a Part or Beginning of the next Hedge you are going to measure; by which Means, that Space or Distance you measured from the Hedge will be included; especially, if you are careful in measuring exactly your Angles, according to *Chap. III. Sect. 3.* and 4; or occasionally your Diagonal-lines, according to *Chap. IV. Sect. 3.* which see.

6. When it happens that you have not quite a Chain between the last Arrow and the Hedge you are to measure to; let the Assistant or him that leads the Chain, pull the End of it to the Hedge and hold it there, till the hindmost Man (laying down the Chain straight) come to the last Stick or Arrow, and observing which brass Piece is next the same, and consequently how many Links it is from thence to the Hedge; for so many odd Links must likewise be inserted in the second Column of your Field-book. —As for Instance, suppose when you are standing at the last Arrow, you discover by the brass Piece 30 Links, immediately succeeding in the middle Part of the Chain, noted by 50 Links and a red Rag, you will then perceive on the reverse Side of the Piece of Brass, the Number 20, which is the Complement to 100; indicating thereby, that it is just 20 Links from the last Stick to the Hedge; provided the same is neither increased nor diminished by any odd Links, that occasionally may be required to reach or extend exactly to the said Hedge.

The like is to be observed in all other Examples of this kind. See the Description of the Chain, *Chap. I.*

C H A P. V.

Of the Chain and its Use, in actually Measuring, Protracting, and finding the Area or Content of any Field, &c. of what Shape or Quantity soever, that is regular.

S E C T. I.

To measure a Field in the Form of a Geometrical Square, and to protract or lay down the same, on purpose that the Figure may be truly discovered, and the Superficial Content found, according to any Dimensions given.

1. **A** Geometrical Square, (according to Definition) is a quadrangulr Figure, consisting always of four equal Sides, and as many Right-angles; as is the Square A B C D (Fig. 8.) whose Sides and Angles are all equal, containing four equal Sides, which may be attributed either to Inches, Feet, Yards, Perches, Chains, or any other Measure whatsoever.

2. The Superficial Content of such a square Figure is always found by multiplying one of the given Sides by itself; and the Rectangle or Product of that Multiplication, shall be the Content of the whole Square sought, in the very same Name the Side was given in, excepting otherwise reduced.

Example.

Suppose the Square A B C D (Fig. 8.) shall represent a certain Field, or any other inclosed Piece of Ground in the same Form, and that one of the Sides contains 40 Chains 15 Links: To know the Area or Superficial Content.

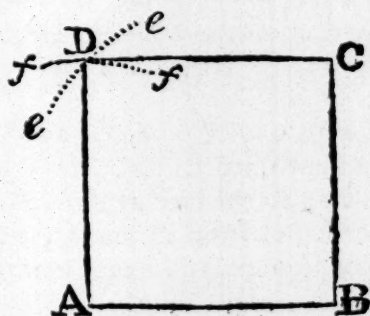
Acres. Roods. Perch. Pts. of Perch.
Answer, 161 — 00 — 32 . 36.

Expla-

40 : 15.

A ————— B

Fig. 8.



Explanation.

In this Example it may be observed, that 4015 Links in Length being squared or multiplied by itself, the Product is 16120225 Square Links; from which five Figures or Places being cut off from the Right-hand (according to *Chap. II. Sect. 6.*) the Figures remaining on the Left-hand are 161 Acres, and 20225 Square Links remaining; which being multiplied by the Number of Square Roods in an Acre (*viz.* 4.) and five Places being likewise cut off, the Product is 0 Roods 80900 Square Links, which being again multiplied by the Number of Square Perches in a Rood (*viz.* 40.) and five Places being cut off from the Right-hand, as before, the last Product is 32.36 Perches, as above. Hence the complete Answer is 161 Acres, 00 Roods, 32.36 Perches, for the true Area or Content sought.

N. B. The Operation of this Example is purposely omitted for the Learner's Exercise and Improvement; which may be readily effected by the Explanation above.

Protraction.

Let the given Line be $AB = 40.15$ Chains and let it be required to make a square Figure, each Side of which shall be the just Length of the said Right-line AB , and consequently,

B b 3

the

the Surface therein contained shall be 161 Acres, &c. as in the Answer above To effect which,

First, Lay down from any Diagonal-scale of a moderate Length (supposing that of 40 Divisions in an Inch) the Length of your given Line, $AB = 40$ Chains 15 Links.

Secondly, Raise a perpendicular Line, of the same Length from the Point B, by its proper Problem; or otherwise by the Chord of 60, laying down upon the same 90 Degrees: Or, *3dly*, the same may be accurately performed by that *Prob.* or *Fig.* in *Chap.* III. *SECT.* 2. which see.

Thirdly, Take with your Compasses the Length or Extent of either of the aforementioned Lines; and setting one Foot in C, describe the Arch *ee*; do the like at A and describe the arch *ff*.

Fourthly, Draw Lines from A and C, into the Point of Intersection; and the Square is so exactly finished, as to comprehend the Superficial Content sought, according to *Fig.* 8. In like Manner, any other Geometrical Square may be thus measured, plotted and cast up; regard being only had to the exact Length of any one of the given Sides.

N. B. *Although Fields, &c. seldom produce a Figure so exact as a Geometrical Square; yet it is frequently found in Gentlemen's Gardens, and other Inclosures near their Dwellings: Now, when ever this Figure is required to be measured, you are carefully to observe, that all the four Angles are right, and each Side equal; then the Figure and Content of the same may be readily found, by measuring (in a streight Line) the Length of one Side only, as above.*

S E C T. II.

To measure, plot, and cast up accurately, any Field, &c. in the Form of an Oblong or long Square.

R U L E.

- 1.** **A**N Oblong or long Square, by Geometricians called a Parallelogram, hath its opposite Sides equal and parallel. Or it consists always of four Right-angles but unequal Sides, two whereof are (oppositely) equal and agreeable in their Length; as the other two are likewise in their Breadth, tho'
op-

opposite one to another; as AB is equal to CD, and AC equal to BD, in Fig. 9.

2. The Area, or Superficial-content of this Figure, is always found by multiplying the given Length by the Breadth; the Rectangle, or Product, will be the desired Answer, in the same Name proposed.

Example.

There is a Field in the Form of an Oblong, or long Square, as ABCD, Fig. 9. Now supposing its Length (*viz.* AB or CD) to be 60 Chains 47 Links, and its Breadth, (*viz.* AC or BD) 30 Chains 64 Links: What is the Content or Superficies?

Acres. Roods. Perc. Parts.

Ans. 185 — 1 — 4.8128

See the Work.

Ch. Lin.

Length = A B or C D = 60.47 = *Mnd.* } Factors.
Breadth = A C or B D = 30.64 = *Mr.* }

24188
36282
181410

Acres = 185.28008 = Prod.

× 4

Roods = 1.12032

× 40

Perches = 4.81280

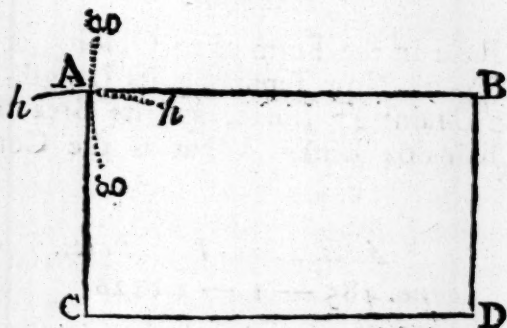
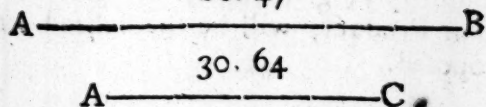
Explanation.

The Work of this Example is so obvious, that there needs no other Illustration than only Chap. II. Sect. 6. which see.

B b 4

Fig.

Fig. 9.
Ch. Lin.
60. 47



Protraction.

The Figure of this Field may be plotted, or laid down, with as much Facility as the last: And,

First, Let the given Length, *viz.* AB or CD = 60.47 Chains, be taken from a Diagonal-scale of equal Parts (supposing that of 40 Divisions in an Inch) and with that Extent let the Base-line, *viz.* CD in Fig. 9. be first laid down: Then,

Secondly, Upon the End of which, at the Point D, let there be erected a perpendicular Line (as before directed) equal in Length to the given Breadth or shortest Side measured in the Field, *viz.* AC or BD = 30 Chains 64 Links.

Thirdly, With the Length or Extent of your longest Line, as above, *viz.* AB or CD = 60.47 Chains, let one Foot of your Compasses be placed in the Point B, and with the other at A, describe the Arch *gg*. Also, with the extent of the lesser Side or Breadth of your Field, from the same Scale, (*viz.* CA or DB) let one Foot of your Compasses be set in the Point C, and with the other describe the Arch *hh*, in the Point A.

Fourthly, Draw Lines from BC unto the Point of Intersection at A; you will thereby discover the exact Figure or Form of the Field that shall comprehend the Superficies desired (*viz.* ABCD) as above.

SECT,

S E C T. III.

To find the Form and Surface of a Field, &c. representing an irregular Oblong; which is not so broad at one End as the other.

R U L E.

1. **T**HE Figure of this Field (*viz* LMON) is by Geometricians termed an oblong angled Parallelogram, of a quadrangular Form, having its two opposite Ends or Breadths unequal, but its Lengths or Sides equal, though not parallel, as the Side MN is equal to LO, the opposite Side or Length; but the End or Breadth LM is unequal to ON, its opposite Breadth or End. See Fig. 10.

2. For the Superficial-content, take the Dimensions of the two Ends, and the perpendicular Length, or nearest Distance between them; then multiply half the Sum of the two Ends, by the said Distance, and the Product is the Answer.—Or, for greater exactness (instead of the arithmetical Mean, as above) find the geometrical Mean between the two Breadths or Ends given, by extracting (decimally) the Square-root of their Product, and multiplying that Root or Mean-breadth, by the given Length or nearest Distance; the last Product is the Area, or Superficial-content sought, in the same Name that was given.

Example.

Admit a Field in the Form of an irregular Oblong hath for its Mean-length (*viz* L r) 64.37 Chains and the two Breadths (*viz* LM and ON) 30 and 46 Chains severally; What is the Superficial-content?

Acres. Roods. Perch. Pa.

Ans. { 1st, 244—2—16.960 the common Way:
2^d, 239—0—12.228 the true Way.

Expla-

Explanation.

In this Example, and all others of the like kind, it may be observed, that when the Arithmetical Mean (*viz.* 38 Chains) or half the Sum of the two Breadths or Ends are taken, according to the first Part of the Rule; the Answer is 244 Acres, &c. which is 5 Acres, 2 Roods, 5.732 Perches more than just, according to the true Way. — Hence it is, that the latter Part of this Rule is preferable to the first, for its Truth and Exactness; seeing the geometrical Mean, or true Breadth (*viz.* 37.14 Chains, &c.) is considerably less than the former; and consequently, the Occasion of the great Difference above; which is always more or less proportionable, as is the Difference between the greater and the lesser Breadths or Ends: Therefore, the Veracity of all Examples of this kind, depends upon the geometrical Mean, according to the latter Part of the Rule above.

Note, The Operation, or Work of this Example is left to the Learner's Performance, in order to the Application of the Rule, &c.

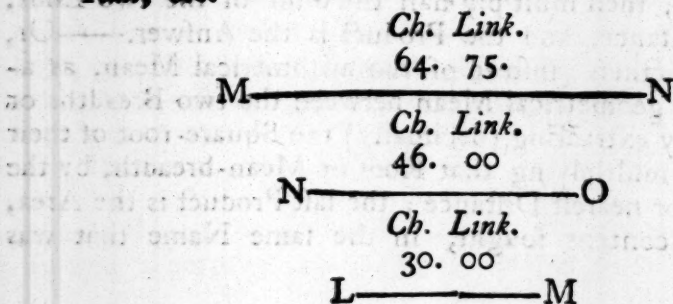
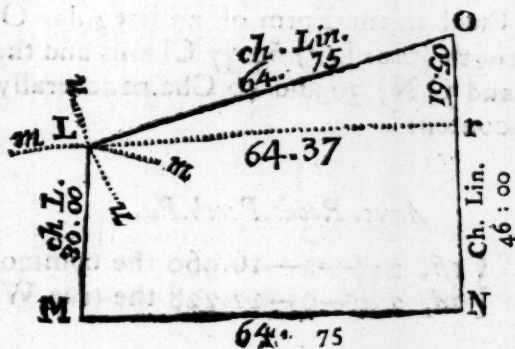


Fig. 10.



Protraction.

That this Figure, and all others resembling it, may be truly plotted, proceed thus :

First, For the Base-line, let there be taken from your Field-book the Length or Extent of any two, the longest Sides given, supposing $MN = 64.75$ Chains; and upon the End thereof, let there be erected Line NO , equal to 46 Chains, given for the greater Breadth.

Secondly, From the same Scale, let there be taken in your Compasses the Length of your Base-line MN or LO (*viz.* 64.75,) and with this Extent, let one Point of your Compasses be placed at the Point O ; then with the other, make the Arch nn .

Thirdly, Take between your Compasses the lesser End or Breadth (*viz.* $LM = 30$ Chains) and with this Extent, make likewise the Arch mm : so as (by Supposition) it may bisect the former in the Point L .

Fourthly, Draw Lines from O and M unto the Point of Intersection at L ; you will thereby procure the exact Figure of your Field.

Fifthly, For the perpendicular Length, or nearest Distance between the given Breadths LM and ON , let a perpendicular Line be let fallen upon the Line NO , bisecting it in r ; then will it follow, that the nearest Distance between the two given Breadths (*viz.* LM and ON) being particularly applied to the same Scale, will likewise discover the true or mean Length sought (*viz.* $Lr = 64.37$ Chains) In like manner all other irregular Oblongs may be thus protracted and measured.

S E C T. IV.

To measure any Triangle in the Field, and to protract and cast up the same at Home, with all imaginable Exactness and Accuracy.

R U L E.

1. **A** Plain Triangle, whether acute or oblique, consists always of three Sides, by which its Surface is comprehended.

2. The Quality and Names of Triangles are various, according to their Form and Position, *viz.* 1st, a Right-angled Triangle; so called, because it comprehends always one Right-angle and two Acute; the longest Side is termed the Hypothenuse; and the other two Legs (accidentally) are either equal or unequal, one of them being always the Cathetus or Perpendicular. 2^{dly}, An Equilateral Triangle is one whose Sides are straight, and Angles all acute and equal. 3^{dly}, An Isosceles Triangle hath always two Sides equal, and consequently, the two Angles opposite to them; the 3^d Angle may accidentally be either acute or obtuse. 4^{thly}, a Scalenum is a Triangle, containing three unequal Sides, and as many Angles, which are likewise unequal, one of them being sometimes obtuse, and the rest acute.

3. Although each of these Triangles, as above, must be exactly protracted at Home, in the very same Form as they appear in the Field; yet before any of their Areas or superficial Contents can be found with Conveniency, they must (if not already) be reduced into the first kind, termed by the *Greeks*, a Right-angled Triangle or Orthogonium, by letting fall a perpendicular Line from each opposite Angle unto the Base, which in this Case, are always the two Particulars given.

4. Having found the proper Data for each Triangle, as above directed; you are to multiply half the given Base by the whole Perpendicular; or one half of the Perpendicular by the whole Base; or otherwise, the whole of the one, by the whole of the other; and taking half, by dividing the same by 2, the Product or Quotient in each of those Varieties, is the superficial Content sought, in the same Name the Dimensions was given in, which usually is Chains and Links, or otherwise Perches.

N. B. *The superficial Content of any Triangle may likewise be found by the three given Sides only, without the Knowledge of the Perpendicular; but as this Method is somewhat tedious, I shall here omit it, seeing the other above is more practicable.*

Example.

There is a scalenous Triangle, *viz.* A B F, whose Base (*viz.* A F) is 60 Chains 12 Links, and Perpendicular (*viz.* B F)

BF) 40 Chains 36 Links: To know the superficial Content?

Acres. Roods. Perc. Pa.

Answ. 121 — 2 — 11.5456 = ABF.

Explanation.

In this Example, as well as all others relating to the measuring of Land, in the Form of any Triangle whatever, it may be observed, that half the Base-line, *viz.* $AF = 30.6$ Chains, being multiplied by the Perpendicular-line, *viz.* $BE = 40.36$ Chains, is exactly equal unto the Product of $AF = 60.12$ Chains, multiplied by half the Length of the Perpendicular, *viz.* $BE = 20.18$ Chains. Again, the whole Length of the Base, *viz.* $AF = 60.12$ Chains, being multiplied by the whole Length of the Perpendicular, *viz.* $BE = 40.36$ Chains, the Product will be double the Area or Content of the Triangle AMNF, whose Half being taken, according to the Rule, discovers the same Answer as before: Hence it evidently appears, that all three Methods, as above, are exactly agreeable; though the last is esteemed the fittest for Surveying, and the other two for Building.

OBSERVATIONS.

1. You may observe, that if the Perpendicular-line BE, had fallen without the Triangle on the Base produced, as MA or NF; yet still the Product made by the whole Length of the Base AF, by the Perpendicular, had been double the Content of the Triangle, as before, and all the proceeding Articles of the Rule would hold good.

2. If through the Middle of the Perpendicular or Sides, in either Example, be drawn a Line parallel to the Base, as GH; then $GH = 30.6$ Chains, by the first Article of the Rule, being multiplied by the Perpendicular $BE = 40$ Chains 36 Links, the Product will be the superficial Content: And this is common in measuring Buildings.

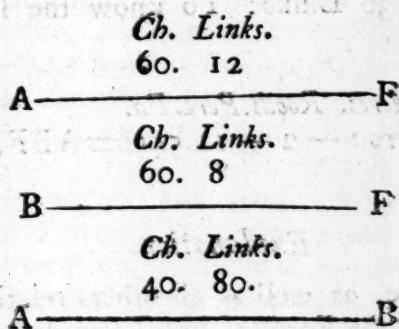
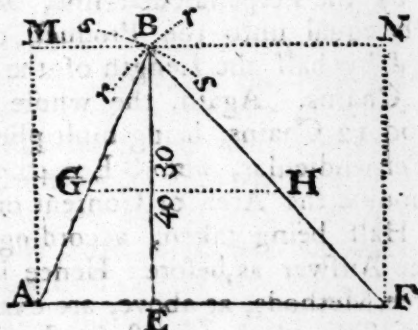


Fig. II.

*Protraction.*

Having measured in the Field the three Sides of any Triangle, according to the general Rule for measuring any right or streight Line in *Chap. IV. Sect. 2.* which see, and the same being noted in your Field-book, as before directed: Then,

First, Take between your Compasses the Extent of the longest Side given (*viz.* $AF = 60.12$ Chains) from any Diagonal-scale of equal Parts, supposing that of 40 Divisions in an Inch; and let it be laid down for your Base-line, as in *Fig. II.*

Secondly, Then having taken likewise the Extent of the next greater Side given (*viz.* $BF = 60.8$ Chains,) with one Foot of your Compasses in the Point F; let upwards the Arch *rr* be described.

Thirdly, For the 3d Extent, let the remaining Side given (*viz.* $AB = 40.80$ Chains) be laid down, by placing one Foot of your Compasses in the Point A, and with the other describe likewise the Arch *ss*; bisecting the former Arch in the Point B.

Lastly,

Lastly, Let there be drawn two Lines from A and F to the Point of Intersection at B; you have therein included the Superficies of the Triangle sought, according to the Dimensions given.—In like manner, all other Triangles whatsoever, may be thus protracted and laid down.

S E C T. V.

How to measure in the Field, &c. a Rhombus, or geometrical Figure in the Form of a Diamond, or Pain of Glass, and to find both the Figure and superficial Content of the same according to any Dimensions given.

R U L E.

1. **T**HE Form of this Figure is more frequently seen in Gentlemen's Gardens than in Fields: However, it is known to be bounded always by four equal Sides, oppositely parallel to one another, comprehending therein four Angles none of them right, but only equally acute and obtuse by Pairs; as they appear diametrically opposite to one another.—Or, a Rhombus, in effect, is only two equilateral, or two Isosceles Triangles, having for their Base, one common Diagonal, by which they are conjoined. See Fig. 12.

2. The Area or superficial Content of any Rhombus, may be accurately found, by multiplying the Diagonal-line or common Base (*viz.* A B) by any one of the two Perpendiculars (*viz.* D E or E C) because they are both equal, and the Content of the whole Figure will be equal to a Square made of the Base (*viz.* A B) and one of the Perpendiculars. Or otherwise, the same Effect will be produced, if the Sum of the two Perpendiculars (*viz.* D E and E C) be multiplied by half the Diagonal-line (*viz.* A E or E B) the Product will be equal to the former Answer.

Example.

Suppose a Piece of Ground in the Form of a Rhombus (*viz.* A D B C A) hath for its Diagonal-line, or common Base (*viz.* A B) 86 Chains, and one of the Perpendiculars

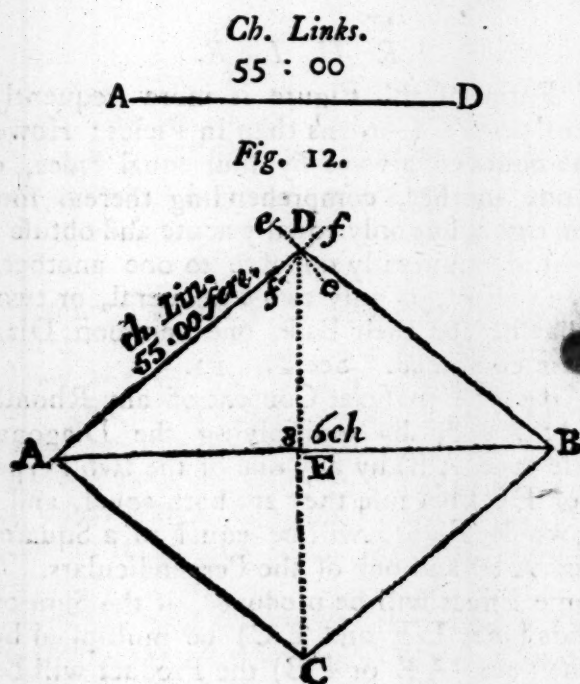
lars (*viz.* DE) 34 Chains; what is the Area or superficial Content?

Acres. Roods. Perches.

Answer, 2924 Square Chains, or 292 — 1 — 24.

Explanation.

This Example is so obvious of itself, that there needs no farther Explication than only the Rule above, by observing, that the Diagonal-line or common Base (*viz.* $AB = 86$ Chains) being multiplied by one of the Perpendiculars (*viz.* $DE = 34$ Chains) the Product is 2924 square Chains for Answer; which may be readily reduced into Acres, Roods, and Perches, by *Chap. II. Sect. 6.* which see. In like Manner, are performed all other Examples, which hath their Dependence upon this Figure.



Protraction.

First, Having measured or taken the Dimensions of the Figure in the Field, I find that the longest Line, or Diagonal contained therein, is $AB = 86$ Chains; and seeing all the Inclosing

closing Sides or Hedges are equal, one of them (*viz.* A D) being likewise measured, is found to be 55 Chains near; which being noted in the Field-book, for the proper Data.

Secondly, To plot this Figure, I take from a Scale of equal Parts the Diagonal-line given, *viz.* 86 Chains; which I lay down from A to B.

Thirdly, From the Scale of equal Parts, I take with the Compasses the Length or Extent of the Side or Hedge measured, *viz.* A D = 55 Chains; and with one Point of the Compasses in A, I describe with the other the Arch *ee*: Again, with the same Extent, I place one Point in B, and with the other Point I describe the Arch *ff*, which makes an Intersection in D. In like manner is performed the same at the Angle C.

Lastly, From A and B, unto the two intersecting Points at D and C, I draw the four Lines, *viz.* AD, DB, BC, and CA, which completes the Figure, and discovers its Form, according to the Dimensions given.

S E C T. IV.

To measure a Rhomboides so exactly, as that its Figure and Superficial Content may evidently appear after Protraction, according to any Dimensions given.

R U L E.

1. **A** Rhomboides differs so much, and no more, from a Rhombus, as a Parallelogram does from a true Square; and, as such, it follows, that the Figure thereof must likewise be bounded with four Right-lines, whose opposite Sides are both equal and parallel, as well as the opposite Angles equally acute or obtuse, by reason whereof, no Right-angle can be therein contained.

2. Having therefore (by Measure) found the Length and Breadth of the said Figure, and consequently the Length of the Diagonal, or longest Line contained therein, you may, by Protraction, soon obtain the true Form or Shape of the Figure, according to the Dimensions given.

C c

3. As

3. As it must have also the same Data as the Rhombus, viz. the Base or Diagonal-line, and Length of one of the Perpendiculars, let fall from an opposite Angle upon the said Base: So that the Rectangle, or Product of the one, being multiplied by the other, is likewise the Area or superficial Content sought; because (in Effect) the Reasons and Definitions are the same.

Example.

Admit a Field, or any other Inclosure, in the Form of a Rhomboides (viz. $ABDC$) hath for the Length of its Diagonal-line (viz. AD) 86.70 Chains, and one of its Perpendiculars (viz. ef) 15.70 Links: To know the Area or Superficial Content?

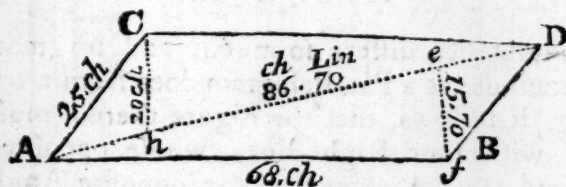
Acres. Roods. Perc. Pts. of Perc.

Answer, 136 — 0 — 19. 04. See Fig. 13.

Explanation.

The Figure being plotted, as follows, there appears for the Length of the Diagonal (viz. AD) 86.70 Chains, and one of the Perpendiculars (viz. ef) 15.70 Chains which being multiplied together, the Product is the desired Answer or the Superficial Content, viz. 13611900 Square Links, or 136 Acres, 0 Roods, 19.04 Perches.

Fig. 13.



Protraction.

First, Having measured in the Field, the Length of the longest Side or Hedge, viz. AB or $CD = 68$ Chains: Also, the Side or Hedge at each End, viz. AC or $DB = 25$ Chains: And lastly, the perpendicular Line, or nearest Distance between the

the two Sides or Hedges, viz. $Ch = 20$ Chains; which being noted in your Field-book, as before directed.

Secondly, From a Diagonal Scale of equal Parts, supposing 40 Divisions in an Inch, let there be taken between your Compasses, the Extent of the longest Side, viz. $AB = 68$ Chains, which being laid down as in *Fig. 13*.

Thirdly, With the Breadth or Extent of the Perpendicular Line, viz. $Ch = 20$ Chains, let there be drawn an obscure Line parallel to AB ; then having taken from the same Scale, the Breadth or Extent of any one of the shortest Sides or Ends, viz. AC or $DB = 25$ Chains; and placing one Foot of the Compasses in the Point A , with the other (upwards) bisect the obscure Line that was made parallel to the Line AB ; supposing it to happen at the Point C .

Fourthly, Having drawn thro' the Points A and C , the Side $AC = 25$ Chains; then from the Point at C let there be laid down the opposite Side $CD = AB$; and from the Point at D , let there be drawn to B , the other opposite Side $DB = AC$; which concludes the Figure, and discovers the true Form thereof.

Lastly, Let the Diagonal-line AD be drawn, whose Extent being applied to the same Scales is found to be 86 Chains 70 Links; then from either, the opposite Angles B or C , let there be a perpendicular Line found, to bisect the Diagonal-line, supposing $fo = e$ 15.70 Chains: You have thereby given the proper Data (viz. the said Perpendicular and Diagonal) for finding readily the superficial Content as above. In like manner all other Figures of this Form may be thus protracted or laid down.

S E C T. VII.

How to measure accurately in the Field, and to protract and cast up at home, any Trapezium whatsoever.

R U L E.

1. **T**HIS Figure contains always four Sides and four Angles, uncertain whether parallel or equal; but is commonly neither, especially, when it appears to be so irregular, as

neither the Sides to be parallel, nor the Perpendiculars equal, yet both Ways agreeable to the general Definition above.

2. The Figure being thus defined, you are to observe, that the common Base is usually the Diagonal, or longest Line that can be contained therein: Or otherwise, it is a Right-line, exactly measured from one opposite Angle to another, which always reduces the whole Figure into two Triangles, whose Areas may be accurately found by their proper Rule in *Chap. V. Sect. 4.* which see: Then it follows, that the Sum of the two Areas so found, is the superficial Content of the whole Trapezium sought.

Or otherwise thus:

The Arithmetical Mean Proportional, being half the Sum of the two Perpendiculars, multiplied by the Length of the common Base or Diagonal, produces the true Content required, in the same Name the Dimensions were given in; because the Trapezium is just equal to half the Square made of the Sum of the two Perpendiculars and the common Base: Or, for the third Variety, the whole Sum of the two Perpendiculars being multiplied by half the Diagonal, or common Base, discovers likewise, the same Product or Answer.—But the Method most fit for a Surveyor is to multiply the Sum of the two Perpendiculars, by the whole Diagonal, or common Base; and then taking half the Product for the true Area or Superficial Content sought.

N. B. That all those four Varieties above, will be found equally true, though one or both of the Perpendiculars fall without; for then the Sum of the Perpendiculars being multiplied by the Diagonal, will produce the double Content.

Example.

There is a certain Field in the Form of a Trapezium, (*viz.* BCDA, *Fig. 14.* whose common Base or Diagonal (*viz.* BD) is 75 Chains; also the great Perpendicular (*viz.* AE) = 29.50 Chains, and the lesser Perpendicular (*viz.* CF) = 25.50 Chains: To know the Area or superficial Content?

Acres. Rood. Perch.

Answer, 206 — 1 — 0.

Expla-

Explanation.

In this Example, above, it may be observed for the first Variety, that the Area of the two Triangles in the Trapezium, viz. BAD and BCD, being particularly found by *Seet. IV.* and added, their Sum is the Area or Superficial Content of the whole Trapezia sought, viz. BADC = 206 Acres, 1 Rood, 0 Perch. Then, for the second Variety, if half the Sum of the two Perpendiculars (viz. AE and CF) = 27.50 Chains, multiplied by the common Base or Diagonal (viz. BD) = 75 Chains, the Rectangle or Product discovers the same Area or Content as above. Again, for the third Variety, if the Sum of the two Perpendiculars (viz. AE and CF) = 55 Chains, be multiplied by half the common Base, or Diagonal (viz. BD) = 37.50 Chains, their Product is likewise the same Content, as above. Lastly, for the fourth Variety, if the Sum of the two Perpendiculars (viz. AE and CF) = 55 Chains, be multiplied by the whole Diagonal or common Base (viz. BD) = 75 Chains, their Product is the double Content, whose Half is the same Area, as before: Which shews, that all the four Varieties above are equally true, and as such, may be indifferently used, though the last is preferable in Surveying.

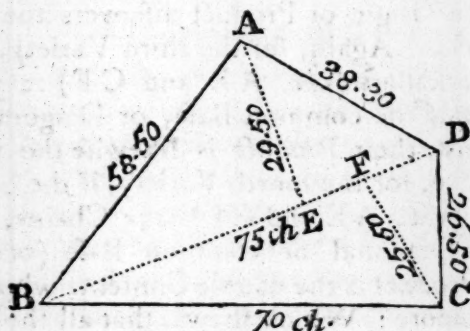
A Trapezium how measured in the Field.

1. As the true Form and Superficies of Figures of this Kind depends upon measuring, or taking exactly with your Chain, the Length of each Side, as well as one Diagonal in the Field; so, in order thereto, it is needful to begin always at some Angle, or Corner, remarkably known by some Gate, Style, Tree, &c. and so proceeding orderly from thence towards the Right-hand, until you have measured round the same, and noted the Length of each Side or Hedge in your Field-book; according to the general Method prescribed for measuring Right-lines and Diagonals in *Chap. IV. Seet. II. III and V.* which see.

2. For the exact Performance whereof, suppose you begin at B, in *Fig. 14.* where having (set for a Mark) a Station-staff with a white Rag, and from thence measuring in a streight Line the Diagonal BD = 75 Chains; as also in like manner, the two Sides or Hedges on the Right-hand of the said Diagonal, viz. BC = 70 Chains, and CD = 26.50 Chains; which being particularly noted in your Field-book.

3. Having thus obtained the Length of your common Base or Diagonal *viz.* BD, and also the two Sides or Hedges on its Right-hand, *viz.* BC and CD; then, likewise, you are to proceed to measure the other two Sides or Hedges, *viz.* DA and AB, which concludes the Measure of the whole Field, containing therein the Diagonal BD. All which must be orderly and exactly placed in your Field-book, according to their true Lengths and Signs; so that the Figure and Surface of the same may evidently appear when truly delineated.

Fig. 14.

*Protraction.*

First, The four Sides, with one Diagonal given, being measured and noted in your Field-book; as above, you are next to find out the true Figure or Form thereof, by taking from a Diagonal-scale of equal Parts (supposing that of 40 Divisions in an Inch) the Length or Extent of the Line or Side BC=70 Chains, which being laid down for the Base or Ground-line, &c. in Fig. 14.

Secondly, With the Length or Extent of your Diagonal-line given, (*viz.* BD=75 Chains, let one Point of your Compasses be placed in the Point B; then, with the other Point opposite to the same, make an obscure Arch at Pleasure; and then, with the Extent of CD=26.50 Chains, taken from the same Scale of equal Parts, let one Point of your Compasses be placed in the Point C, and with the other bisect the former Arch in the Point D.

Thirdly, Through the Points B, D, and C, draw the Triangle BCD, whose Subtendant or Diagonal-line is the dotted Line BD.

Fourthly,

Fourthly, Take between your Compasses the Extent of the given Line $DA = 38.30$ Chains, and having placed one Foot of your Compasses in the Point D, with the other make likewise an obscure Arch, opposite to the Point at D.

Fifthly, For the fourth or last Side, take in your Compasses the Extent of the given Line $BA = 58.50$ Chains; and having placed one Foot of your Compasses in the Point B, with the other describe an Arch, so that it may intersect the other in the Point A; then from the Points B and D, let there be drawn the Sides BA and DA, which completes the Figure of the Trapezium.

Lastly, From each of the two opposite Angles at A and C, let there fall two Perpendicular-lines, *viz.* $AE = 29.50$ Chains, and $CF = 25.50$ Chains upon the common Base or Diagonal-line BD; you will thereby obtain for a proper Data, the three Lines internal that is needful to find out the Superficial Content of the whole Figure as above. In like manner, all other quadrangular Figures, though never so irregular, may be thus measured and protracted.

How a Trapezium, &c. may be accurately measured by Angles and Sides promiscuously, without Diagonals.

1. There is likewise another Method of Measuring any Trapezium in the Field by the Chain only, and protracting the same at home, with all imaginable Exactness and Accuracy, without the Use of any Diagonal-line; and that is, by beginning at some convenient Place in the Inside of the Field, from whence you can measure round the same, by proceeding (if possible) on the Right-hand, where you can come nearest to all your Sides and Angles, in the same Order as they lie; then will it follow, that the first Side or Hedge, being measured as before directed, and then afterwards an Angle, according to its proper Rule in *Chap. III.* you will at last, by surrounding or encompassing the whole Field, discover orderly four Sides or Hedges, and as many Angles, to be accordingly noted in your Field-book, in order, that the same may be truly plotted, as hereafter.

2. Having protracted or laid down, according to your general Rule, all the Sides and Angles in the same Order as you find them in the Field-book, you will thereby discover, as above, the true Form or Figure thereof: Then, next the Area, or superficial Content of the same, will be found, by drawing, first, from one opposite Angle to another, a Diagonal-line,

and letting fall two Perpendiculars upon the same, the proper Data is thereby exhibited, as before.

N. B. *The Method of measuring the Angles in a Field, as above, is not only frequently used, but may also be depended upon for its Veracity.*

To measure a Trapezium from a certain Station in any one of the Sides of it, without the Trouble of Angles.

Let all your four Sides or Hedges round your Field, be orderly and exactly measured and noted in your Field-book, as before, only with this Difference, that any one of the Sides or Hedges, may, in measuring, have a Station-staff erected at or near the Middle of the said Hedge, especially, after you have known, by Measure, the exact Distance from each End of the same, to the said Mark; then it will follow, that two Diagonals being measured from the said Station-staff unto the two opposite Angles or Corners, you will thereby find (in Protraction) the whole Trapezium divided into three Triangles, the Sum of whose Areas is the superficial Content sought. Now, although this Variety may be useful in some Cases; yet either of the other two preceding is more preferable for their Exactness and Facility, especially if Bushes, Ponds, or other Obstructions intervene not; but when otherwise impracticable, then this will be found conveniently needful.

Practical Observations very useful and material in Surveying.

1. Having discovered the Form of your Field, let always a rough Sketch of it be drawn in your Field-book, as near as you can guess, without aiming at Exactness, farther than only having as many Sides and Angles as in your rough Draught, as there are in the Field; by which means, you will thereby form in your Mind, an Idea or Apprehension of the true Draught found by Protraction.

2. When you have several Schemes in one Paper, it is best to keep each Table in your Field-book so methodical, as to contain four several Columns with their proper Titles, for receiving orderly the Number of Changes, Sides, Diagonals, Angles,

Angles, and Things remarkable, occasionally happening; not forgetting, at the same time, to mark every Table with the same Number and Figure you are measuring, or by way of Similitude you can have reference to. But when you are surveying but one single Field, or two, or the like, it is sometimes convenient to make the Schemes large, and write every Number by the Side of its respective Line, as may be seen in some of the Examples.

3. Always choose the shortest Diagonals, or Cross-lines, for they are not only the soonest measured, but less liable to Error: For the shorter the longest Side of a Triangle is, the more acute its opposite Angle, and the less subject to Mistake; but the nearer that the longest Side comes to the Length of the other two Sides, the less Certainty of Truth, as will evidently appear in Practice.

4. In measuring about the Outside of a Field, it is best to go (*cum Sole*) with the Sun; because you have then the Hedge and Angles on your Right-hand so near you, that they may be taken and measured orderly, and entered into your Field-book; but on the contrary, when the Sides, Angles, and Diagonals, are taken and measured in the Inside of the Field, it is then best to accustom yourself to go (*contra Solem*) against the Sun; and then all the Remarks in the Hedge, or beyond it, are on your Right-hand; so that when you look towards the Chain-hand, you also look towards the Hedge; and also, then there is no Charge to the Memory, seeing all the Remarks being to be writ on the Right-hand Side of the Table; but if you have Occasion to go with the Sun, put your Remarks on the Left.

5. When you measure about the Outside of the Field, and find the Angles so irregular, as that they cannot be perceived at a Distance, by reason of Furz-bushes, and the like, you may, as you come at them, hang a Piece of white Paper, or some other Thing remarkable; by the Help of which, you can see them at a small Distance, for Reasons hereafter following, which frequently occur in Practice: Also you will find it needless to have any Marks at the outward Angles, only you must stand at them, till you have set down the Length of the Lines that you measured to them; and so proceeding orderly in this Method, to measure likewise from them to the next Angles.

6. In finding the Area, or superficial Content of any Field, you need not be careful always to put a Dot between Chains and Links; as for Instance, 4.65; but put them all together, seeing the Chain is decimally divided into 100 Links; therefore, it is all one in the Operation to say 4.65, viz. 4 Chains 65 Links, as to say 465 Links; because Decimals do always work

work as whole Numbers : Hence it is, that 5 Places, or Figures, is always cut off from the Right-hand of the Product, because 100000 square Links is an Acre : Therefore, when the first Product exceeds not 5 Figures, you place on the Left-hand of the Point 0, indicating thereby, that the Content is less than an Acre : When the second Product, whose Multiplier is always 4, the square Roods in an Acre, exceeds not 5 Figures, there must be placed a Cypher (*viz.* 0) likewise on the Left-hand of the Point, indicating no Roods : Lastly, when the third Product, when multiplied by 40, the Number of square Perches in a Rood, exceeds not 5 Places or Figures, there must likewise be placed a Cypher (*viz.* 0) on the Left-hand of the Point, to indicate that there is likewise no Perches ; but the Figure or Figures on the Right-hand of the Point, are always here so many decimal Parts of a square Perch. But to be more particular in finding the Area of Superficies, &c. you may have Recourse to *Chap. II.* where the same is illustrated by Tables, Propositions, Rules, Examples, Explanations, &c.

7. As the same Directions are equally useful in Fields of any other Number of Sides, as well as those of four ; so I shall here observe, that all quadrangular Figures, especially those of Trapezias, may be more or less regular, according to the Nature and Quality of the Figure they appear in : As for Instance, that of a Parallelogramic or Parallelopleuron so called by the *Greeks*, because it contains four Sides ; two of which opposite are parallel, and of four Angles, the two at each End being equal, whose Area, or superficial Content, is found by multiplying half the Sum of the two Sides by the Length, the Product is the Answer, in the same Name given. Or otherwise, the Diagonal, or longest Line contained therein, being multiplied by half the Sum of the two Perpendiculars let fall upon it, will exhibit the same Superficies, as above : This is so obvious, that the Figure, with an Example, will illustrate the same, when occasionally performed by the ingenious Surveyor.

S E C T. VIII.

How to measure, protract, and find the superficial Content of any regular Polygon or multangular Figure, consisting of any Number of equal Sides and Angles.

R U L E.

A Regular Polygon hath always its Sides and Angles equal in Quantity and Quality, which are never less than 5; but may be orderly increased to twelve Sides or upwards, each Figure assuming a proper and peculiar Name, agreeable to the Number of its Sides, by which it is distinguished from others: That is, if a regular Polygon assume the Name of a Pentagon, it hath five equal Sides, and as many Angles; a Hexagon, of 6; a Heptagon, of 7; an Octagon, of 8; a Nonagon, of 9; a Decagon, of 10; an Undecagon, of 11; a Dodecagon, of 12, &c. several of which may be seen in Gentlemen's Gardens, as well as in Board-measure, Paving, &c. but seldom or rarely in the Surveyor's Way, to measure them in a large Parcel of Land.

2. Having thus shewn the various Kinds of regular Polygons, with their Definitions both generally and particularly; we shall now shew how all of them may be accurately measured, and their superficial Contents readily discovered.—In order to which, you are first to find the Center, or exact Middle of each Figure, by drawing two Lines from any two Angles that include a Side, to the Center; then find the Content of that Triangle as taught in *Sett. IV.* of this *Chap.* Lastly, multiply the Area of that Triangle, by the Number of Sides, and the Product is the Area, or superficial Content of the whole polygon.—Or, multiply half the Sum of the Sides, by the nearest Distance from the Center to one of the Sides, gives the answer in the same Name that was given.

Example.

S E C T

Admit a regular Polygon should represent a Pentagon of five equal Sides and Angles (*viz.* B C G F E) and that the Perpendicular,

pendicular, or Radius of a Circle inscribed therein, is $AD = 23.20$ Chains; also the just Length of one of the Sides is found to be $BC = 34.50$: To know the Area of one Triangle, and consequently, the superficial Content of the whole.

Sq. Link. A. R. P. Pa.

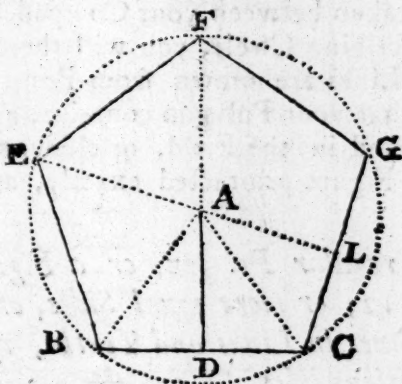
Answ. } $4002000 = 40-0-3.20$ for the Triangle.
 } $20010000 = 200-0-16.00$ for the Pentagon.

Explanation.

In this Example, as well as in all others of the like Kind, it may be observed, that the Particulars of the proper Data are one of the Sides given, *viz.* $BC = 34.50$ Chains, and the Radius of the inscribed Circle, as well as the Perpendicular of the Triangle BAC , *viz.* $AD = 23.20$ Chains, in order to find the superficial Content of the said Triangle, and consequently, the Area of the Pentagon sought.

2. For the Performance of which, according to the general Rule of all regular Polygons, as above, you will find that half the Number of the given Sides (*viz.* 86.25 Chains) being multiplied by the Radius of the inscribed Circle (*viz.* $AD = 23.20$ Chains) the Product is 20010000 square Links, or 200 Acres, 0 Rood, 16 Perches, for the superficial Content of the whole Pentagon sought.—Or otherwise, the same may be effected by multiplying half the given Side (*viz.* BD or $DC = 17.25$ Chains) by the perpendicular Length, or nearest Distance between the Center of the Polygon, and Middle of one of its Sides, *viz.* $AD = 23.20$ Chains, the Product is 4002000 square Links, or 40 Acres, 0 Roods, 3.20 Perches, for the Area of the Triangle BAC ; which being multiplied by the Number of Sides (*viz.* 5 .) the Product is the superficial Content of the whole Polygon, as above.—In like manner, the Superficies of all other Polygons whatsoever may be thus found, regard being always had to the Number of Sides, as well as the perpendicular Distance, or Radius of the inscribed Circle given. See *Fig. 15*.

Fig. 15.



How any regular Polygon may be actually measured and protracted, according to the Number of its Sides.

First, The Figure of the given Polygon (*viz.* BCGFE) appearing to be truly regular, let its Center be exactly found, by measuring a Diagonal-line from the Middle of one of its Sides, to an opposite Angle, *viz.* DF; which being again diametrically crossed by the Diagonal-line EL, you will discover the Point of Intersection to be at A, which is exactly the Center of the Polygon.

Secondly, The Extent or Distance between the Point A, and the Middle Part of any of the Sides, being applied to a Diagonal-scale of equal Parts, discovers the Length of the Perpendicular of the Triangle, or the Radius of the inscribed Circle (*viz.* AD) which must be accordingly noted in your Field-book.

Thirdly, If there is likewise taken between the Compasses, the Distance or Extent between the central Point A, and any one of the angular Points (*viz.* AB or AC) and applied to the same Scale of equal Parts, you will thereby discover the Hypotenuse of the Triangle BAC or DAC, and consequently, the Radius of the circumscribing Circle, *viz.* AB or AC, agreeable to the Notes in the Field-book.

Fourthly, With the last Extent, or Radius of your circumscribing Circle (*viz.* BA) as above, let one Point of your Compasses be placed in the Center, until you have described with the other an obscure Circle, which will pass through all the

the angular Points of the Polygon, viz. B E F G C, as in Fig. 15. which see.

Lastly, The Extent of any of the given Sides of the Polygon (viz. B C) being taken between your Compasses, and measured round the circumscribing Circle, you will thereby find exactly (when the black Lines are drawn from Point to Point) the Number of Sides that your Polygon contains, according to the Dimensions measured in the Field, or elsewhere; and consequently, the same Figure protracted exactly, as above.

How to make a regular Polygon, or a Figure of 5, 6, 7, 8, 9, 10, 11, 12, or more equal Sides, or otherwise, by their natural Sines in Links and Tenths, after an accurate and expeditious Way.

R U L E.

First, Divide 360, the Number of Degrees contained in a Circle, or otherwise, the Links and Tenths of a Chain corresponding to the same (viz. 603.6 Links, &c.) by the Number of Sides you would have your Figure contain; the Quotient, either Way, shall be one Side of such a Figure.

Secondly, From any Line of Chords, or otherwise, from any Diagonal-scale, at Pleasure, take from the former 60 Degrees, and from the latter, 1 Chain, or 100 Links, and with either of those Extents, describe an obscure Circle; which done take likewise from the same Line of Chords, or from the same Diagonal-scale of equal Parts, the Number of Degrees, &c. or Number of Links, &c. that one Side of the Figure is found to contain, as above: Then, beginning at any Part of the Circle, set off either of those Extents round the same; as from A to B, from B to C, and so round till you come to A again: Then having drawn Lines between those Marks, the Figure is thereby completed, according to the Number of Sides you divided by. Thus any regular Polygon whatsoever may be thus readily protracted, though it contains never so many Sides.

N. B. *This Rule above is so plain and obvious, that it may be easily applied to any Figure and Example, according to the Pleasure of the Artist.*

S E C T. IX.

Of the Circle and its Parts: How a Circle, or any Part of it, may be measured, protracted, or cast up with all imaginable Exactness and Accuracy.

The D E F I N I T I O N.

1. **A** Circle is a plain Figure, contained under the Limit or Term of one Line called the Circumference; and of all superficial Figures is the most comprehensive, because it contains in a little Compass a greater Area or Square than any other Form or Figure whatsoever. It is also a Figure most perfect, because its Periphery or Circumference, meeting at the same Point where it first began, must be every-where equally distant from a certain Point in the Middle of its Surface, called the Center, from whence all Right-lines being drawn from thence to the Circumference, must be of an equal Length.

2. As this Figure does frequently appear upon the Surface of Ponds, Basons, Fountains, Board-measure, and Pavements; so a right Understanding of its principal Parts is indispensably needful, seeing almost all other superficies has a Dependence upon the same.

3. The principal Parts of this Figure appear to be ten, viz. 1st, The Periphery or Circumference, which is always an Out-line bounding it, at an equal Distance from the middle Point of its Surface, called the Center, as O: 2^{dly}, The Diameter is always a Right-line, drawn through the Center quite cross the Circle; which divides it into two equal Parts, and consequently, the longest Line in it, as AC or BD: 3^{dly}, The Semi-diameter, or Radius is always the half of the last Line, and may be continued from the Center to any Part of the Circumference, as AO or OE: 4^{thly}, A Chord is any Line shorter than the Diameter, passing from one Part of the Circumference to another, as AE: 5^{thly}, The versed-line, is always a Right-line ascending perpendicularly from the Middle of the Chord-line to the Circumference, as the Line GH: 6^{thly}, An Arch of the Circle as any Part of the Circumference, whether great or small,

as A G or A B E : 7thly, A Semicircle is always the just half of the whole Circle, as A B C : 8thly, A Quadrant is the 4th Part of a Circle, made by two Diameters perpendicularly intersecting each other, as A B O, B O C, &c. 9thly, A Sector of a Circle is a Piece cut out by the Semidiameters, and is bounded by them, or contained between them and an Arch of the Circle, as O B E : 10thly, and lastly, A Segment, Section, or Part of a Circle, is properly a Figure contained under a Right-line and Part of the Circumference greater or less than a Semicircle, Quadrant, or Sector; as A G E is a less Segment of the Circle A B C D ; likewise A D E is the greater Segment of the same Circle. See Fig. 16.

The USE or APPLICATION.

Having in the preceding Article defined all the principal Parts of the Circle, we shall now orderly and methodically proceed to their Use and Application, with all imaginable Plainness and Exactness, as well as Conciseness, according to the Particulars of the following

R U L E.

I. The Diameter being given, to find the Circumference by Analogy, or otherwise:—To effect which, to thrice the given Diameter, add $\frac{1}{7}$ of itself, the Sum is the Circumference sought, according to the common Way of *Archimedes*: Or multiplying the Diameter given by 22, and dividing the Product by 7, the Quotient is likewise the Answer, as above. —Or if the given Diameter had been multiplied by 335, and the Product divided by 113, the Quotient had been also the Circumference sought, according to the Method of *Adrianus Metius*. —Or, lastly, if the Diameter had been multiplied by 3.14159, cutting off (by a Point or Comma) from the Right-hand of the Product the five decimal Places given, the Figures on the Left-hand of the Point or Comma, had been likewise the Circumference sought, according to the Method of *Van Cullen*. The Converse of any one of those three last Methods will likewise discover the Diameter sought, when the Circumference is given, by changing and using the proper Divisors, as above, for Multipliers, as well as, on the contrary, the proper Multipliers for Divisors.

To find various Ways the Area or superficial Content of a Circle, having interchangeably, either the Diameter or Circumference given.

R U L E.

1. The given Diameter being squared or multiplied by itself, and that Product again by 11, and the last Product divided by 14, the Quotient shall be the Area or superficial Content sought, according to *Archimedes*.

2. Let half the Diameter be multiplied by itself, and that Product again by 3 and $\frac{1}{7}$, the last Product will likewise be the Content sought.

3. If from the Square of the Circumference you subtract $\frac{1}{16}$ Part of the Product, and divide the Remainder by 11, the Quotient also will be the desired Content.

Example.

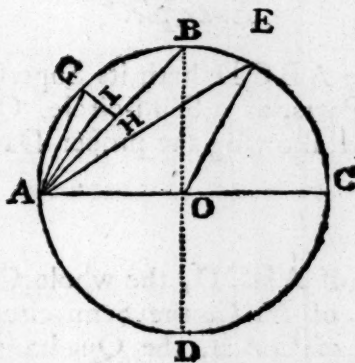
Suppose the Circle ABCD (*Fig. 16.*) hath for its Diameter DB = 28 Chains: To know the Area or superficial Content of the same.

A. R. Perch.

Answer, 616 square Chains = 61—2—16.

Explanation.

Fig. 16.



1. The Diameter given, viz. 28, being squared or multiplied in itself, the Product is 784; this Number being multiplied

plied by 11, giveth 8624, which being divided by 14, the Quotient will be 616 square Chains, and that is the Area of the Circle.

Half the Diameter, *viz.* 14, being squared or multiplied by itself, the Product is 196, which being again multiplied by $3\frac{1}{7}$, the second or last Product is 616 square Chains, as above.

3. The given Circumference, *viz.* 88, being multiplied by itself, the Product is 7744; from which deduct $\frac{1}{8}$ of the said Product, the Remainder is 6776; this Number being divided by 11, the Quotient is likewise 616, for the Area or Content sought. Hence the Truth of the Rule appears in all the three Methods, as above.

To find accurately the Area or Content of a Semi-circle, Quadrant, or other Sectors of a Circle.

R U L E.

That the Area or Content of any of the Particulars above-said, may be exactly found, there must always be given, for the proper Data, the Radius or Semi-diameter of the Circle (being a Particular common to all) together with the Length of the Arch-line belonging to the Semi-circle, Quadrant, and two other Sectors; then it follows, that the Length of any one of those Arch-lines being multiplied by the Radius, or Semi-diameter of the whole Circle, the Product is the Area or Content of each, according to its Name and Quality.

Example.

Admit the Circle ABCD hath its Superficies divided into four such unequal Parts, as a Semi-circle, Quadrant, and two other Sectors: Now allowing the proper Data for each to be as follows: *viz.*

	<i>Chains.</i>
the Length of ABCD, the whole Circumference	= 44
the Length of ABC, the Semi-circles Arch	= 22
the Length of OAB, the Quadrant's Arch	= 11
the Length of OBE, the lesser Sector's Arch	= 4
the Length of OEC, the greater Sector's Arch	= 7
the Le. of AC or BD, the Diameter common to all	= 14

To

To know the Area or Superficial Content of each.

		Chains.			
Answ.	{	ABCD	= 616	= Circle's	} Area.
		ABC	= 308	= Semicircle's	
		OAB	= 154	= Quadrant's	
		OEC	= 98	= Greater Sector's	
		OBE	= 56	= Lesser Sector's	

Explanation.

This Example, as well as all others of the like kind, is so obvious, that there needs no farther Explanation, than only the Application of the Rule itself, by multiplying half the Length of each Arch-line given, as above, by the Semi-diameter, the Product is the Area or Content sought, according to the Name given, whether Semi-circle, Quadrant, &c.

N. B. The Length of the Arch for any Part of the Circle, particularly the Segment, may be found thus: Let GH bisect AE perpendicularly, and draw GA; then from eight times AG take AE, and divide the Remainder by 3, the Quotient will be the Length of the Arch AGE, very near.

To prepare stated Numbers, or proper Multipliers, for finding readily any of the Quæsitæ's above.

R U L E.

1. For the Performance hereof, let the Circumference of a Circle, whose Diameter is Unity, or 1 (*viz.* 3.1416) be orderly divided by $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{11}$, and $\frac{7}{44}$, the Quotients, respectively, are orderly, the Arches of the Semi-circle, Quadrant, and two Sectors sought, according to the Work of the following Example.

2. Let one Fourth or each of those Arches, found as above, be orderly taken and multiplied by 1, the Products respectively, are orderly Areas for that Circle, and its Parts proportionable, whose Diameter is Unity or 1, and consequently, become stated and fixed Numbers, or Multipliers, for any other Circle whatsoever, whose Diameter or Circumference is given.

D d 2

3. Mul-

3. Multiply the Square of any Diameter given, by the said proper Area or Multiplier, for the Semi-circle, Quadrant, or Sector's given, the Product is the Area or Content desired.

Example.

There is a Circle whose Diameter is Unity or 1, which hath the Areas of its Semi-circle, Quadrant, and two Sectors, particularly given, as follow: To know the same in that Circle, whose Diameter is 28.

	<i>Arches.</i>	<i>Areas.</i>		<i>Arches.</i>	<i>Areas.</i>
<i>Ans^w. Given</i>	{	3.1416=.78540	<i>sought.</i>	{	88=616=Circle.
		1.5708=.39270			44=308=Semi-circle.
		0.7854=.19635			23=154=Quadrant.
		0.2856=.07140			8=56=Lesser Sector.
		0.4998=.12495			14=98=Greater Sector.

Explanation.

In this Example it may be observed, that if the several Numbers given for the Areas of the Circle, Semi-circle, Quadrant, and two Sectors, viz. .7854, .3927, .19635, .0714, and .12495, be orderly multiplied by the Square of 28 (viz. 784) the Products are orderly 616, 308, 154, 56, and 98, or near the same, for the particular Areas or Parts proportional of the Circle's Surface sought. In like manner the Parts superificial of any other Circle whatsoever, may be accurately found, according to the Diameter or Circumference given.

Note 1. As one of the given Factors above, contains a certain Number of decimal Places; so there must be always so many cut off (by a Point or Comma) from the Right-hand of the Product; then will it follow, that those on the Left-hand of the Point, &c. will be so many Integers for Answer, and the rest Decimals, or fractional Parts of Unity.

2. As the Operations above, depend upon the exactest Analogies, viz. those of Van Cullen and Metius; so both of them approach so near unto the common Way found by Archimedes, that the Difference between the last and two former, do not exceed $\frac{1}{4}$ of Unity.

To

To find the Area of a Segment of a Circle various Ways.

R U L E.

Although this Problem is something difficult to perform exactly, yet you may come very near the Truth several Ways, by observing the following Directions.

First, Let the Area or Content of the Segment A E G be required. To effect which, draw 1st, the Chord-line A E, and measure the Length thereof, which suppose to be 25.50 Chains; then measure the Versed-line or Perpendicular G H, which suppose to be 7 Chains. Now I say, if you multiply $\frac{2}{3}$ of the Chord-line (*viz.* 17) by the Perpendicular (*viz.* 7) the Product is very near the Area of the Segment A G E = 119 Chains. Or otherwise, if you multiply $\frac{2}{3}$ of the Versed-line or Perpendicular (*viz.* 4.66) by the Chord-line 25.50, the Product is 119 Chains nearly, for the Area of the Segment sought, as above.

Explanation.

1. Two-third Parts of 25 and a half, is 17, which multiplied by 7, the Perpendicular G H produceth 119 Chains, or 11 Acres, 4 Rods, 24 Perches, which is the near Area of the Segment A G E sought. See *Fig. 16*.

2. The Proof of the last Work will appear by observing, that the Area of the whole Sector A O E, is found as before: For the Arch-line A B being a Quadrant, is 22, and the Arch-line B E is 8, which together make 30; the Half whereof, 15, multiplied by the Semi-diameter O B = 14, giveth 210 for the Area of the whole Sector A O E. Now the whole Sector, which contains 210, consisteth of those two Parts, namely, the Segment, A G E, and the Triangle A O E, the Base is 25 and Half; the Perpendicular 7 (it having the Remainder of G H taken from O G = 14) wherefore multiply 25.5 by 3.5, the Product will be 90.25 for the Area of the Triangle. Now it follows, that

The whole Area of the Sector O A E, is	—	210
--	---	-----

The Area of the Segment A G E, is	—	119
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The Area of the Triangle A D E, is	—	90 $\frac{1}{4}$
------------------------------------	---	------------------

The Sum is	—	209 $\frac{1}{4}$
------------	---	-------------------

Which is equal to the whole Sector within less than an Unit.

Secondly, This, likewise, may be performed more easily thus: Draw AG ; then to $\frac{2}{3}$ of AG add AH , and multiply the same by $\frac{4}{3} AD$, the Product is the Segment sought. Or,

Thirdly, The same may be effected more accurately, thus: Find I , the Middle of GH , and draw AI ; then add AG to 4 times AI , and $\frac{1}{3}$ of the Sum multiplied by $\frac{4}{3} AD$ gives the Content of the Segment though it be almost as great as the Semicircle itself) as near as the Proportion of *Archimedes* gives the whole Circle.

S E C T. X.

How to measure, protract, and calculate the Superficial Content of an Ellipsis or Oval, various Ways.

R U L E.

1. **A**N Ellipsis or Oval, is an Egg-like Figure produced from the Section of a Cone, made by cutting through both its Sides, but not parallel to the Base: Hence it becomes so far circular, as to contain different Diameters, the longest whereof (*viz.* EA) is called the Transverse Diameter, and the shortest (*viz.* FG) the Conjugate Diameter. The other Parts remarkable, are the Center and the *Foci*, which last is always a Mean Proportion between the Half-Sum, and Half the Difference of the Transverse and Conjugate Diameters.

2. The Periphery of any Ellipsis may be found by adding once and $\frac{1}{7}$ the Length of the Conjugate Diameter, to twice the Transverse, the Sum is the Periphery nearly. Or, *2dly*, the same may be effected more exactly, by adding unto the Hypothenuse of the Triangle FCA (*viz.* FA) its ninth Part, and multiplying the Sum by 4, the Product is the Periphery sought. Or, *3dly*, (for the sake of the Curious) make, as the fourth Power of AF , to the fourth Power of BC ; so AG to a fourth Proportional; which taken from ten times DC , and the Remainder multiplied by 4, give a Product, which divided by 9,

is the Periphery of the Ellipsis, as near as the Periphery of the Circle given by *Archimedes*.

N. B. That a true Geometrical Oval has its Transverse and Conjugate Diameters so proportionable one to another, as 10 is to 7.68, &c. Hence it is that any Oval, though never so irregular, may be made truly geometrical as above.

3. For the Area, or Superficial Content of this Figure, the common Way, is to multiply the Product of the two Diameters given by 11, and divide the second or last Product by 14, the Quotient is the Content sought: But this Way though frequently used, is not so exact as either of the other two following: Therefore, 2^{dly}, the Product of the Length and Breadth (known also by the Name of the Transverse and Conjugate Diameters) of the Oval, being decimally divided by 1.27325, the Quotient is the desired Area. Or, 3^{dly}, the Product of the two Diameters, as above, being multiplied by .7854, the Product (cutting off four Figures towards the Right-hand thereof) is the Answer by *Euclid*, 1, 2, and 12.

Note, That the Content of Half the Ellipsis may be found by multiplying the Length of the Transverse Diameter by Half the Conjugate, and that Product again by .7854, cutting off orderly the decimal Places from the Right-hand of each Product, gives the Answer.

Example.

A Piece of Ground in the Form of an Ellipsis or Oval, hath for its greatest Length, or Transverse Diameter, (*viz.* E A) 34.50 Chains, and its Breadth, or Conjugate Diameter (*viz.* F G) 24.60 Chains, also each of its Foci (*viz.* D or B) 6.45 Chains: To know the Area or Superficial Content.

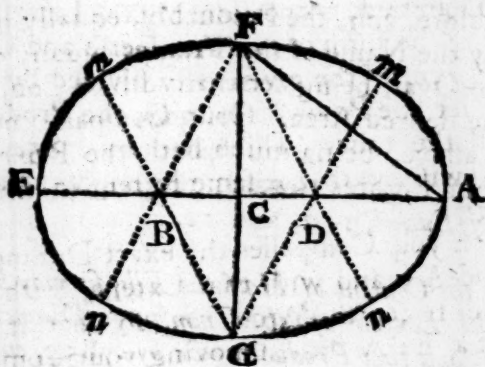
Acres. Roods. Perch.

Answ. 6665689 Square Links, or 66 — 2 — 25 +.

Explanation.

In this Example, and all others of the like Kind, I multiply the Transverse Diameter given (*viz.* $EA = 34.50$ Chains) by the Conjugate Diameter (*viz.* $FG = 24.60$ Chains) and the Product (*viz.* 848.7000) again by .7854; according to the third Method of the Rule, because most exact, and the last Product is 6665689 Square Links, or 66 Acres, 2 Roods, 25 Perches +, for the Area or Superficial Content sought.

Fig. 17.

*Protraction.*

1. When you are required to take the Dimensions of an Ellipsis, or Oval, you are first to measure exactly the Length and Breadth severally in Chains, Poles, Yards, Feet, or any other Name proposed, which must be accordingly noted down in your Field-book, for the Transverse and Conjugate Diameters given, in order to Protraction.

2. The *Foci* of the said Figure must next be found, by subtracting half the Square of the Conjugate Diameter from the Square of the Transverse Diameter, the Square Root of the Remainder will be the Sum of the two *Foci*, whose Half will shew the Distance of each *Focus* from the Middle or common Center of the Ellipsis.

3. From a Diagonal Scale of equal Parts (supposing that of 20 Divisions in an Inch) let there be taken and laid down exactly the Length of the Transverse Diameter given, *viz.* $EA = 34.50$ Chains; which being crossed in the Middle with the

the Perpendicular (*viz.* $FG = 24.60$ Chains) equal to the Breadth, or Conjugate Diameter given, as in *Fig. 17*.

4. Take between your Compasses the Length of one of the *Foci*, and having placed one Foot of your Compasses in the Middle or common Center of the Ellipsis (*viz.* C) with the other bisect the Line EA , on each Side of the Center, supposing these two Bisections should happen at D and B .

5. Draw obscure Lines at Pleasure, each way, upward and downward, on each Side from the Center, C , from the Point F and G , through the two Points of the *Foci*, *viz.* D and B , you will thereby discover obscurely, two parallel Lines, bisecting one another in the Points D and B , and consequently discover the Form of a Diamond in the Middle of an intended Ellipsis, as well as a half Diamond, proportionably, equally on each Side of the *Foci*, bearing the form of two Angles equally obtuse.

6. With the Extent of the given Breadth or Conjugate Diameter (*viz.* FG) let one Point of your Compasses be placed in the Point F , with the other (which hath the Pen) sweep the Arch nn ; and likewise, with the same Extent in the Point G , sweep the Arch mm .

7. Take between your Compasses the exact Distance between E and D , or B and A ; and with that Extent, with one Point in D , let the other sweep the Arch nm , so that it may pass through the Point at E : And then moving your Compasses, but not your Extent, with one Point in B , sweep likewise the Arch mm , so that it may pass through the Point at A ; by means whereof your Ellipses will be perfectly completed, and the true Figure thereof exactly discovered according to the Dimensions given.

OBSERVATIONS.

1. As the common Methods of making Ellipses or Ovals, are not only various and numerous, but even intricate and confused; so I am hereby induced to prescribe this of my own, seeing it is not only plain and easy; but even so universally useful, perfect, and complete, that all others whatsoever may be thus protracted and laid down, according to any Dimensions given.

2. Having found the Area or superficial Content of any Ellipsis, according to the various Methods already prescribed, any of them may be proved by extracting the Square Root of the Product of the Transverse Diameter, multiplied by the Conjugate, gives you the Geometrical Mean, or Diameter of a Circle equal thereto, whose Area being found, will be (within a small Fraction) the same of the Ellipsis.

3. Al-

Although the Figure of an Ellipsis is seldom or never seen in Fields; yet frequently in Broad-measure, Pavements, Flower-beds, &c. then in those Cases, the Area or Content is generally required in Yards, Feet, &c. because the Dimensions are usually taken and measured in those Denominations.

CH A P. VI.

PRACTICAL SURVEYING performed by the CHAIN only: Shewing how Parcels of Land of any irregular Shape or Quantity soever may be actually measured, protracted, and cast up, with all imaginable Exactness and Accuracy.

S E C T. I.

To measure irregular Polygons of any Kind, particularly a Pentagon.

How to survey a Field various Ways, though never so irregular, by going round the same, and measuring occasionally its Sides, Angles, or Diagonals.

R U L E.

HAVING in the III^d, IVth, and Vth Chapters preceding shewn both generally and particularly, the Use and Application of Sides, Angles, and Diagonals, as well as the Method of measuring by them all regular Figures whatsoever, we shall now proceed to those irregular Kinds, which frequently happen in Surveying: And,

I. You may observe, that by the general Name of an irregular Polygon, or Polygram, is understood any multangular Figure whose unequal Sides and Angles are, respectively, never less than five; but may be orderly increased to twelve or upwards; by reason whereof, each Figure obtains likewise a pro-

per and significant Name, according to the Number of Sides, &c. As for Instance, an irregular Pentagon contains always five unequal Sides, and as many Angles; a Hexagon consists of six unequal Sides, &c. a Heptagon of seven; an Octagon of eight; a Nonagon of nine; a Decagon of ten, &c. and in measuring any of those Figures with a Chain, the following Method is the same in all; only for every three outward bounding Sides, there is required a cross Line or Diagonal, to divide the whole inwardly into Triangles.

II. To measure a Field of any Number of Sides, equal or unequal, particularly, one in a Pentagonal Form, by the Help of Diagonal-lines, without any respect to the Angles, or the Quantity of them: To effect which, you are, as before directed, to begin at some Place remarkable, such as a House, Gate, Style, &c. and so from thence proceeding orderly on your Right-hand, and measuring round the Field each Side or Hedge, which being accordingly noted in your Table or Field-book. Then, 2dly, measure the nearest or shortest Distance from the opposite Angle to another, so many diagonal or cross Lines, less by three than the Number of Sides found, by which your Field is bounded or limited; which being likewise noted down, as above, you have all that is necessary for protracting and casting up any Polygon, or many-sided Figure whatsoever, without any Regard or Dependence upon the Angles, which are sometimes (without great Care and Circumspection) subject to Error; whereas this is not: For a small Error in any one Angle, will make a considerable Error in the next Diagonal-line which bounds or limits it.

III. But for the Sake of the Curious, as well as those who affect measuring by Angles, you are always to measure orderly round the said Field the interposing Angle between every two Hedges, by beginning at the very Corner where you intend your angular Point, with the Sign or Character of the same, and sticking down there perpendicularly a Station-staff, through the great Ring of one of the Ends of your Chain, and taking the other End in your Hand, stretch out the Chain in Length, first on the Hedge on your Left-hand, and then again on your Right; each of which Places, where the End of the Chain falls, let there likewise be placed a Station-staff perpendicularly, which must be always noted down in your Field-book or Table by two different Letters: Then it will follow, that the nearest Distance between your two last Station-staffs, being exactly measured in Chains and Links, will be the Base or Chord-line of the Angle sought, which must accordingly be noted down in your Field-

Field-book, before you proceed to measure the Length of the next Side or Hedge.

N. B. The general Method of measuring, or taking orderly and exactly by the Chain, Sides, Angles, and Diagonals, will be found in their proper Sections of Chap. III. which see.

Example.

Admit a Field in a polygonal or pentagonal Form, consisting of five unequal Sides (*viz.* OPQRS, Fig. 18.) whose Particulars appear as follow : To know the Area or Superficial Content.

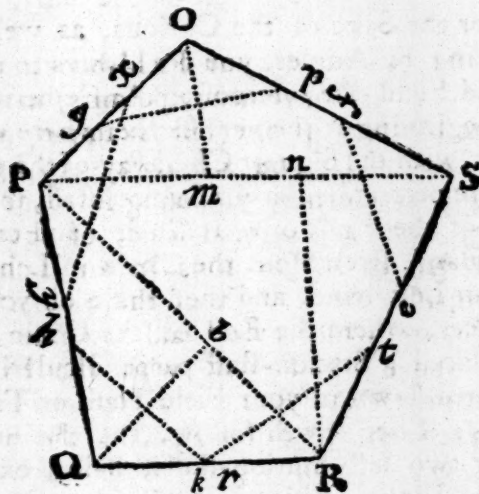
Acres. Rood. Perches.

Answer, 11 — 1 — 18.8

Ch. L.

OP =	8.90	} Sides.
PQ =	12.50	
QR =	9.60	
RS =	13.30	
SO =	12.70	
PS =	17.50	} Diagonals.
PR =	16.80	
Q ^b =	0.75	} Perpendiculars.
R ⁿ =	11.70	
O ^m =	0.65	

Fig. 18.



Expla-

Explanation.

The Field being plotted as follows, the two Diagonal-lines, *viz.* $PS = 17.50$ Chains, and $PR = 16.80$ Chains, reduce the whole Figure into three Triangles, the 1st being POS , whose Base or Diagonal-line is PS , and Perpendiculars, *viz.* $Om = 0.65$ Chains, discovers its Area to be the 0 Acres, 2 Roods, 11 Perches; 2^{dly}, The Triangle PRS hath likewise for its Base or Diagonal-line, $PS = 17.50$ Chains, and Perpendicular-line, *viz.* $Rn = 0.70$, whose Area is 10 Acres, 0 Roods, 38 Perches. Lastly, The Triangle PQR hath for its Base or Diagonal-line $PR = 16.80$ Chains, and Perpendicular $Qb = 0.75$ Chains; which discovers the Area to be 0 Acres, 2 Roods 20.8 Perches. Hence the whole Content sought is 11 Acres, 1 Rood, 18.8 Perches, as in the Example.

Protraction.

To render the Work every Way complete, we shall first shew the Reader how any Field or Parcel of Land, when measured as above, may be exactly protracted or plotted, according to any Dimensions given, whether Sides and Diagonals, or only Sides and Angles: And then, 2^{dly}, proceed to the casting up, or finding the Content of the same, by the general Method as follows:

1st, When the Notes in your Field-book, as before directed, or rather (when Conveniency permits) in little Tables, according to the Method of the last Example, appears to be only Sides and Diagonals, you are always to begin at some remarkable Angle or Corner, supposing it to be at P , in *Fig.* 18. and having, from a Diagonal-scale, of equal Parts (allowing it to be that of 20 Divisions in an Inch) taken between your Compasses the Length of the Side or Hedge (*viz.* $PQ = 12.50$ Chains) which being laid down in a black Line, as in the *Fig.* above; and then, with the Extent of the next Diagonal-line (*viz.* $PR = 16.80$) let one Point of your Compasses be placed in the Point P , with the other describe an Arch at R , which being again bisected with the Extent of the next Side or Hedge orderly following (*viz.* $QR = 9.60$ Chain.) you have thereby obtained the Dimensions of that Triangle

Triangle (*viz.* PQR .) *2dly*, Take between your Compasses the Extent of the other Diagonal-line (*viz.* $PS = 17.50$ Chains) with one Point of your Compasses in P , strike with the other an Arch at S ; which being again bisected with the Extent of the next Side or Hedge (*viz.* $RS = 13.30$) you will thereby obtain another Triangle (*viz.* RPS) which will evidently appear by drawing always, your outward Lines black, and your inward obscure or dotted. *3dly*, Take between your Compasses the extent of the next Side or Hedge (*viz.* $SO = 12.70$) and with one Point of your Compasses in S , with the other describe an Arch at O ; which being again bisected with the Extent of the opposite Side or Hedge (*viz.* $PO = 8.90$ Chains) by placing one Point of your Compasses in P , and bisecting the last Arch at O ; you will thereby discover your third Triangle (*viz.* POS) which perfects the Figure, and discovers exactly the Form thereof to contain, according to the Dimensions given, five unequal Sides outwardly in black Lines, and two Diagonal-lines with three Triangles inwardly. *Lastly*, If from the opposite Angles at O , Q , and R , there be let fall inwardly three Perpendicular-lines, *viz.* Om , Rn , and Qb ; you have thereby obtained all the Requisites for finding the Superficial Content of the said Figure, or any other Polygon whatsoever, according to the Method of the last Example and its Explanation.

Protraction by Angles.

Having in the preceding Article shewn, how any pentagonal Figure, or any other Polygon whatsoever, may be accurately protracted by Diagonals; we shall now shew, how the same may likewise be effected by Angles. And,

1. Let there be laid down, as before, a Side, supposing that of $PQ = 12.50$ Chains, and then the next Angle immediately following (*viz.* the Angle at Q) by taking between your Compasses, from a proper Scale (supposing that of 20 Divisions in an Inch) the Extent of 1 Chain or 100 Links, and placing one foot in the angular Point at Q , with the other describe an Arch at pleasure, supposing it bisects the Side PQ in h .

The Sides as before.

Ch. L.

$$\text{Chord Lines at } \begin{cases} R = 17.00 = kt. \\ S = 15.00 = ef. \\ O = 16.50 = gp. \\ P = 18.40 = xy. \\ Q = 15.50 = hr. \end{cases}$$

2. From the Table following, take between your Compasses from the same Scale, the Chord-line of that Angle found in your Field, and noted accordingly in your Table, *viz.* Angle $Q = 15.50 = hr$; then, with that Extent, and one Point of your Compasses in h , bisect with the other the said Arch in the Point r ; then from the Point at the Angle Q , through r , draw an obscure Line, and set off the next Side QR ; and so proceeding orderly in this Manner, until you have completed the Number of Sides and Chord-lines found in your Table, you will thereby discover the true Form of your Figure, agreeable to that in the Field, whose Dimensions are taken.

3. The Form of your Field being thus plotted, as above in *Fig. 18.* you may with great Ease draw the same Diagonal-lines from one Angle to another (*viz.* PS and PR) and then three Perpendiculars being let fall from those Vertical Angles, which are opposite to the said Diagonals, you have thereby obtained the proper Data for finding the Superficial Content sought, as before.

To find the Area or superficial Content of any Polygon or multangular Figure; or cast up Land of any Figure or Form soever.

1. As all Figures whatsoever must first be reduced into Triangles or Trapezia, before their Areas can be exactly found, according to the Laws of Measuring, or the general and universal Method now in Use, yet, upon strict Observation, it manifestly appears, by *Euclid, Lib. VI. Prop. 2.* that all of them (in Operation) are effectually reduced to rectangled quadrangular Figures, either Squares or Parallelograms, by multiplying the Length by the Breadth, or one Dimension by another, in

in order that the Product or Content may thereby be discovered. Hence it is, that all right-angled Triangles whatsoever, are reduced to Squares or right-angled Parallelograms, by multiplying the Perpendicular of a Triangle by half the Base; because the Triangle is thereby effectually reduced into a Parallelogram as to the Operation, although the Lines be not drawn, nor the Figure actually altered.

2. This being premised, it must needs follow, that any Parcel of Land, though never so irregular, yet after Protraction, Diagonal-lines being drawn therein, the whole Figure must thereby be reduced into Triangles or Trapezia, or both; whose Areas being particularly found and added together, discover the Superficial Content of the whole Figure sought, according to the Method of casting up Triangles or Trapezia in *Chap. V. Sect. 4 and 7*, which see.

Practical Observations very useful in Surveying.

1. As Surveying admits of various Kinds of Figures, so it is needful to observe that any Polygon of what Number of Sides soever, may be divided into a Number of Triangles less by two than there are Sides of the Polygon, and consequently, requires likewise so many Diagonals or cross Lines less by three than the Number of Sides are: Hence it is, that if from the Number of Sides found in your Figure, three be subtracted, the Remainder will be always the Number of cross Lines or Diagonals therein contained. And if two be deducted from the given Number of Sides, the Remainder will be the Number of Triangles that can be found therein. As for Instance, a Figure of four Sides is divided into two Triangles by one Diagonal, and a Figure of five Sides is divided into three Triangles by two Diagonals, &c.

2. In Measuring, or taking Dimensions in the Field, it is always proper to chuse the shortest Diagonals, or cross Lines; for they cannot only be soonest measured, but are even less liable to Error: For the shorter the longest Side of a Triangle is, the more acute its opposite Angle, and the less subject to Mistake; but, on the contrary, the nearer that the longest Side comes to the Length of the other two Sides, the less Certainty of Truth, as may be verified by Examples as well as Demonstration.

3. Although Tables for receiving the Contents of any Field that is actually measured, may sometimes be put against

against the Matter they refer to; yet it is best, in Practice, to have them in a Field-book altogether, according to the Form or Method prescribed in *Chap. I. Sect. 4.* which see.

4. It is also observable in the practical Part of Surveying, that a Piece of Ground being never so often measured, will always differ somewhat (more or less) in its Dimensions, and consequently, in its Area or Superficial Content, by reason the Chain, in taking Lengths, &c. may, more or less (though never so small) deviate from a straight Line to the Right or Left-hand; or otherwise, by the Unevenness of the Ground, may be more or less contracted, &c. yet those Differences (when carefully measured) prove so small and insignificant, that they are never regarded.

S E C T. II.

Of irregular Fields consisting of any Number of Sides and Angles.

How to measure, protract, and cast up any Field, whose Sides and Angles are both many and irregular.

R U L E.

THE general Rule, *Sect. I.* being carefully observed in all its Articles, the Work will be the same as in the five-sided Figure; and this *Section*, for Instance, may serve in all other Varieties whatsoever: And,

I. Having entered your Field, and observed its Form and Shape, with the Number of its Side, and Angles, by walking round the same, and drawing at Random in your Field-book (with a black and red Pencil) all the Sides and Angles therein contained in the same Order or Form as they lie in the Field; you will thereby obtain such a rough and imperfect Draught, as will somewhat resemble the true one, when pro-

E c

tracted.

tracted. But if your Time allow not of so much Exactness, then the same may be otherwise effected, by taking, in like manner, all the Sides and Angles from some elevated Place in the Field, where they can be conveniently seen; which will likewise give you an Idea or Apprehension of the same.

II. A general View of the Field being thus taken, and a Sketch or rough Draught drawn, as above, you may then proceed to measure your Diagonal-lines the shortest Way, so that they may not intersect one another; and having accordingly noted them down in your Field-book, as before directed, you are next to measure orderly the Sides, from any proper Place where you intend to begin; observing always to insert in your Table the exact Length of each Side in Chains and Links found in the Field, in such manner that they may be distinguished from another, by alphabetical Letters in Pairs, for rendering the Work of Protraction more regular and exact.

Example.

A Field or other Inclosure (*viz.* ABCDEFGH, Fig. 19.) of an octagonal Form, whose Dimensions both externally and internally are given in Chains and Links, as follows, *viz.*

Ch. L.		Ch. L.	
Sides	1. A B = 9.30	Sides	5. E F = 10.50
	2. B C = 14.30		6. F G = 13.70
	3. C D = 12.00		7. G H = 14.20
	4. D E = 10.90		8. H A = 15.40

Ch. L.		Ch. L.	
Diagonals	1. A D = 18.30	Perpendiculars	1. C K = 6.20
	2. D B = 23.50		2. D N = 8.80
	3. A E = 22.30		3. F M = 10.00
	4. A F = 21.20		4. L A = 6.70
	5. F H = 23.00		5. A P = 13.40
			6. O G = 8.10

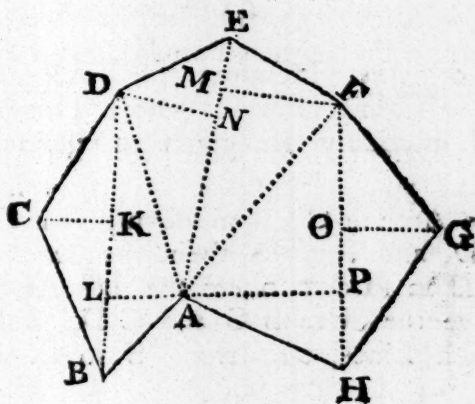
	<i>A. Pa.</i>		<i>A. Pa.</i>
Triangles	1. B C D = 7.28500	Trapezia	1. A B C D = 15.15750
	2. A D B = 7.87250		2. A D E F = 20.96200
	3. A D E = 9.81200		3. A F G H = 24.72500
	4. A E F = 11.15000		Sum = 60.84450
	5. A F H = 15.41000		
	6. F G H = 9.31500		
	Sum = 60.84450		

To know the Area, or Superficial Content.

A. R. Per. Pa.

Answ. 60—3—15.12.

Fig. 19.



Explanation.

In regard that the Field is irregular, that is to say, it is neither Square, Triangle, nor Trapezia, it must, therefore, before it can be measured, be reduced into some of these Forms; which to effect do thus. Draw Lines from one Angle to another, as the Diagonal-lines, A D, D B, A E, A F, and F H; then will the whole Figure be reduced into six Triangles, as in this Figure and Example.—These six Triangles being measured severally, according to *Chap. V. Sect. 4.* and the Contents of them all added together, will shew the Area or

Content of the whole Field as above.—But for an Abreviation of the Work, you need not find the Area of every Triangle, but of every Trapezia, as before taught in *Chap. V. Sect. 7.* by which means, near half the Arithmetical Work will be saved, and an undeniable Proof by this Method exhibited, as above. Or otherwise thus: Add the whole Content of all the Trapezia and Triangles together, and then half the Sum will be the Content sought.

Protraction.

This irregular Field, and all others of the like kind, may be exactly plotted according to the Method following.

1. Draw an obscure Line at adventure, and the beginning at any one Angle, or Place remarkable, supposing the Angle B, and taking (from your Field-book or Table between your Compasses the nearest Diagonal to it, *viz.* $BD = 23.50$ Chains) then, with that Extent, let one Point of your Compasses be placed in the angular Point B, with the other bisect the obscure Line at D.

2. With the Extent of the Side (*viz.* $BC = 14.30$ Chains) let one Point of your Compasses be placed in B, with the other make an Arch at C; then moving your Compasses, with one Point in D, and between them the Extent of the third Side (*viz.* $CD = 12.00$) bisect the last Arch in C; you will then have the Sides BC and CD, which should be drawn in black Lines, *viz.* from B to C, and then from C to D.

3. Take between your Compasses from the same Scale of equal Parts, the next Diagonal-line (*viz.* $DA = 18.30$ Chains) and with that Extent let one Point of your Compasses be placed in D, with the other make an Arch at A; then with the Extent of the first Side (*viz.* $BA = 9.30$ Chains) and one Point of your Compasses in B, bisect the last Arch at A, which being drawn, you have likewise the Side BA.

4. The Extent of the Diagonal-line (*viz.* $AE = 22.30$ Chains) being taken between your Compasses; then with one Point in A, with the other describe an Arch at E; and with the Extent of the fourth Side (*viz.* $DE = 10.90$ Chains)

let

let one Point of your Compasses be placed in D, with the other bisect the said Arch in E, you will thereby obtain the Side DE.

5. From the same Scale of equal Parts, take between your Compasses the Extent of the next Diagonal (*viz.* $AF = 21.20$ Chains) then, with one Point in A. with the other describe an Arch at F, which, with the Extent of the fifth Side (*viz.* $EF = 10.50$ Chains) let the said Arch be bisected, you will also obtain the Side EF, when drawn.

6. With the Extent of the Diagonal-line (*viz.* $FH = 23.70$ Chains) let one Point of your Compasses be placed in the Point F, with the other describe the Arch at H; which being again bisected with the 8th Side (*viz.* $AH = 15.40$ Chains, from the Point at A, you will thereby obtain the Side AH, which must be likewise drawn.

7. Take from the same Scale the sixth Side (*viz.* $FG = 13.70$ Chains) and having placed one Foot of your Compasses in F, with the other describe an Arch at G; which being again bisected with the Extent of the seventh Side (*viz.* $GH = 14.20$ Chains) you will thereby obtain the sixth and seventh Sides (*viz.* FG and GH) which being accordingly drawn, the whole Figure (*viz.* $ABCDEFGH$) is intirely completed, and the true Form of the Field is thereby discovered.

8. Having thus found externally, as above, all your outward Sides or Lines, you are next to find those which are internally, by dotting all your given Diagonals (*viz.* AD , DB , AE , AF , and FH) and then, afterwards, upon three of their Principals or common Bases (*viz.* BD , AE , and FH) there must be let fall, and measured by Application to the same Scale (the six Pependicular Lines from their Vertical Angles (*viz.* CK , LA , DN , MF , AP and OG) so have you all the Requisites of the said Figure, for finding the Area or Superficial Content sought.

S E C T. III.

How to take the Plot of a Field, &c. at one Station, near the Middle thereof, by the Chain only. To measure, protract, and cast up accurately any inclosed Field, whose Angles can be particularly seen from any Part of the Middle thereof.

R U L E.

1. **L**ET the Field given to be measured contain any Number of unequal Angles, suppose seven, viz. A, B, C, D, E, F, G, and $\odot$ the appointed Place from whence, by the Chain, you are required to take the Plot, or rather Dimensions thereof.
2. At or near the Middle of the said Field, viz. the Mark at $\odot$, let there be a Stake put through the Ring or End of the Chain, and make your Assistant take the other End, and stretch it towards the Angle at A, in such manner, that he may appear to be in a straight Line between the Stake and the said Angle; which you may know by your not seeing it through the Interposition of the Assistant's Body; at which Place let there be stuck a Station-staff or Arrow perpendicularly, which must be in a direct Line between $\odot$ and A.
3. In like manner, cause your Assistant to move orderly towards the other Angles, viz. B C D E F G, observing always, at the Length of the Chain, to stick down a Stick exactly in the Line between $\odot$ and B, as at *b*. Afterwards let him do the same orderly at *c*, at *d*, at *e*, at *f*, and at *g*: And if there were more Angles, let him plant a Stick at the End of the Chain, in a right Line between $\odot$ and every Angle.
4. In the next Place, measure the said Line, or nearest Distance between Stick and Stick, as *a b*, so many Chains and Links; *b c*, so many; *c d*, so many; *d e*, so many; &c. and put them down in your Field-book accordingly.
5. Measure also the Distance between $\odot$ and every Angle, as $\odot$ A, so many Chains and Links; $\odot$ B, so many; $\odot$ C, so

to many; $\odot$ E, so many, &c. all which being put down, will appear as in the following Figure and Example.

Example.

Admit an irregular Field in a Heptagonal Form consisting of seven unequal Angles (*viz.* A, B, C, D, E, F, G, *Fig.* 20.) Now allowing the Subtendants or Chord-lines, and the Diagonals or Center-lines to be as follows.

Dimensions given, *viz.*

Ch. L.		Ch. L.	
Subtendants, or Chord-lines.	$\left\{ \begin{array}{l} a\ b=1.15 \\ b\ c=1.06 \\ c\ d=1.30 \\ d\ e=1.00 \\ e\ f=1.20 \\ f\ g=1.09 \\ g\ a=1.50 \end{array} \right\}$	Diagonals, or Center-lines.	$\odot\ A=16.50$
			$\odot\ B=18.10$
			$\odot\ C=15.00$
			$\odot\ D=17.00$
			$\odot\ E=14.06$
			$\odot\ F=19.00$
			$\odot\ G=13.75$

To know the Area or Superficial Content.

Answer, 73 Acres, 2 Roods, 39 Perches.

Explanation.

By observing the Form or Shape of this Field when protracted, you will find it contain as many Triangles internally, as there are Sides outwardly, which may conveniently be reduced into three Trapezia and one Triangle, in order that the Superficial Content of the same may be exactly found, according to the following Particulars.

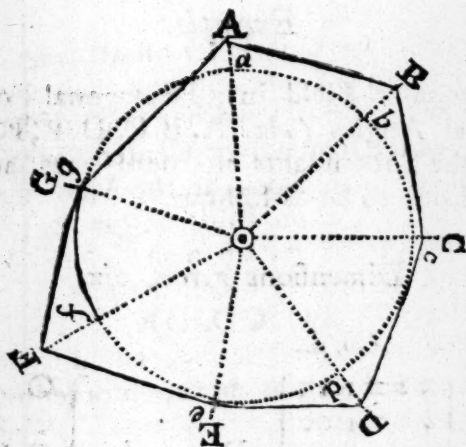
		Ch. L.	
Trapezia	1. $\odot\ A\ B\ C=$	Diagonals, or Bases given.	1. $\odot\ B\ 18.10$
	2. $\odot\ C\ D\ E=$		2. $\odot\ D\ 17.30$
	3. $\odot\ E\ F\ G=$		3. $\odot\ F\ 16.00$
	4. $\Delta\ A\ \odot\ G=$		4. $\Delta\ Side\ A\ G\ 17.40$

		Ch. L.	
Perpendiculars by Pairs.	1.	AC	=24.50
	2.	CE	=22.60
	3.	FG	=21.10
	4.	Δ Perpen.	=12.50

E e 4

Fig.

Fig. 20.



Particular Areas	{	1. $\odot$ A B C	= 23.27500	} Trapezia.
		2. $\odot$ C D E	= 19.54900	
		3. $\odot$ E F G	= 20.04500	
		4. Δ A $\odot$ G	= 10.87500	Triangle.
		Sum = 73.74400		

Acres. Roods. Perches.

Or 73 — 2 — 39 for Answer.

N. B. For the Sake of the Curious, the Superficial Content of this multangular Figure, or any other of the like Kind, may otherwise be found by a certain Method depending upon Theorem VIII. Page 61. of Surveying Improved, which see.

Protraction.

1. To plot those Observations taken in the Field, you are to take from a Diagonal Scale of equal Parts, not too small, 1 Chain, and setting one Foot of the Compasses in any convenient Place of the Paper, as at $\odot$, make the Circle *abcdefg*, obscurely; then taking for your first Subtendant or Chord-line, 1.15 Chains, set it upon the Circle, as from *a* to *b*; and then from $\odot$, through *a* and *b*, draw Lines, as $\odot$ A, $\odot$, B, which be sure let be long enough.

2. In

2. In like Manner, take from the same Scale your second Subtendant or Chord-line, viz. $bc = 106$ Chains, and set it upon the Circle from b to c , and through c , draw the Line $\odot C$, as in Fig. 20.

3. When thus you have orderly set off all your Subtendants or Chord-lines, and drawn Lines through their several Marks, repair to the same Scale, or, if need be, one smaller; and having orderly set off your Diagonal or Center-lines, as you find them in your Table or Field-book, in the Manner following.

4. Upon the Line a , $\odot$ you must set off 16.50 , making a Mark where it falls, as at A , upon the Line $\odot b = 18.10$ which falls at B , and so orderly by all the rest.

5. Draw the Lines AB, BC, CD, DE, EF, FG , and GA , which finishes the Work of Protraction, and discovers the true Form or Figure of your Field, as above, in order to the Superficial Content sought.

Lastly, In like Manner all other multangular Figures, though never so irregular, by which Fields, &c. are represented, may be thus accurately protracted and measured from one Station only, as above; which, if well understood and observed, you cannot miss of Surveying in this Manner by the Chain only, any Field at two or three Stations as well as one, or from an Angle thereof, &c. according to the Directions already given.

S E C T. IV.

How to measure Land bounded by irregular Hedges, or such as curve either inwards or outwards.

HAVING in the three preceding Sections of this Chapter treated orderly and succinctly of all Manner of Fields, &c. whose Sides are many, freight, and irregular; and shewn how they may be accurately surveyed, protracted, and cast up, we shall now observe the Method of measuring a four-sided Figure, as well as those of other Shapes, which (by Supposition) may be bounded with a freight Hedge on one Side; a Hedge curving outwards on the second Side; a Hedge curving inwards on the third Side, and a Brook or River on the fourth Side; which includes all the Varieties that commonly occur in the irregular Bounds of a Survey.

I, It

1. It is convenient in a Field with so much Work in it, to draw the rough Draught pretty large, lest the Letters or Marks upon the Paper should be so curv'd as to be confus'd or obliterated; and having drawn the Out-sides, viz. the Hedges and River, you must, as near as you can, represent their Curvings and Bendings, especially the most material.

2. Let never so many Varieties occur, you are always to proceed orderly and methodically, according to the following Example and its Explanation; only you are to observe, that it is convenient to take all the Off-sets of one Side of the Field from one Right-line, and that drawn as near the Outside of the Field, as possible, that the Off-sets may be shorter, and consequently, both less troublesome and more certain; but yet, not so near the Outside as to leave any Angles or Points of Hedges, Rivers, &c. within.

Explanation.

Suppose a Field was to be surveyed, bounded on one Side by the streight Hedge FE, on the next, by the Hedge curving outwards EH, on the third, by the Hedge curving inwards HG, and 4thly, on the other Side, by the Brook or River GF: Now allowing the Table of Dimensions to be as in the Explanation (viz. as the Table of given Dimensions) to know exactly the Figure or Form of the said Field, and also its Area or Superficial Content.

Acret. Roods. Perches.

Answer, 24 — 2 — 21. See Fig. 21.

Explanation.

1. In order that the Dimensions of this irregular Field may be exactly taken, you are first to observe the Angle in the River at *q*, as well as its flying off at *t*, and *w*, and coming to the Angle *x*, from which may be particularly drawn and represented the Hedge FE streight, and EH curving out, and HG curving in, &c. not regarding the Quantity of the Curve, only leave Room to make the pricked Lines for Off-sets, as *a t*, *b e*, &c. Also draw the straight pricked Line EH, and so far as you can see the Hedge HG in a Right-line as to *m*, continue it by a pricked Line *m o*, to the River at *o*: Likewise, draw the pricked Line *x r*, from the Angle *x* to the Hedge HG, and so as just to touch the Point of the River *q*; and thus have you dis-

discovered the right-lined Trapezium $\ast E H k$, which is the greatest that can be inscribed in the Field $E F G H$ proposed; and by Off-sets from the straight Sides all the Irregularities of the Bounds of the said Field, how difficult or confused soever, may be easily laid down.

2. Having proceeded so far, as above, you are next to begin at the Angle F , and measuring from thence the straight Hedge $F E$, which suppose to be 11 Chains 50 Links, you need only place it in the Table $F E = 11.50$, as you see.

3. This done, proceed in a Right-line towards H , as $H a$ 6 Chains (for where a Hedge curves much alike, you may generally stop at an even Chain) and then by the Means directed in the First Chapter, find the Point a in the Right-line between E and H , and just at the End of the sixth Chain; and finding by the Rules of the same Chapter, the Line that is perpendicular to the Line $E H$, and cuts the Hedge in c ; measure $a c$, which being an Off-set to the Right, place it off on the Right-hand, as in the Table.

4. Go then to b , still taking Care by the aforesaid Means to keep in the Line $E H$; and here, because you perceive the Hedge at c , to be the farthest from you, take an Off-set, whether it be at an even Chain or no; as suppose $a b$ be 5.50, and $b c$ (the Off-set) 3.40; set them down, as in the Table.

5. Then, next supposing $b f$ to be 4.10, and $f g$ 2.40, set them down in the Table, and proceed to $H = 4.20$, which being the Angle and having no Off-set, set it down in the Column of Distances, leaving a Blank in the Column of Off-sets, to shew that you are at an Angle, or at the Hedge.

6. Proceed next to the Hedge $H m G$, if the adjoining Field belong to the same Proprietor, or the like, that you can go into it, you may measure $H b$ and take the Off-set $H m$; and so measure from b to G , as in taking the Off-set off the Hedge $E H$; but if you cannot, or are not allowed to go out of your own Field, as is sometimes the Case, cause your Assistant to go down by the Hedge $H G$, while you stand at the Angle H , and when he comes at the turning at m , let him leave the Hedge, and go directly forward to the River's Side towards O ; and by the Brook's Side, put down a Branch or a Mark in a Right-line, with the Angle H , and Corner m , as you may easily direct him, by waving your Hand to him the Way that he should move, till you settle him in the Right-line: or, indeed, he may direct himself, if he understands it, by seeing the Corner m , and the Surveyor standing at the Angle H ; then measure

$H m$

$Hm = 5.60$, which place in the Table, and measure along the Line mO , till you can see the Angle x , over the Point of the River at q , there place a Mark as at k , and suppose mk to be 4.70 ; put that in the Table.

7. And now for the River GF , let the Surveyor go with his End of the Chain to the Hedge mG , and stand at r ; so that he can by the Point of the River q , just see the Angle x ; then measure $rG = 1.10$ placing it in the Right-hand Column of the Table, and a Blank or Stroke in the Left.

8. Then proceed to take the Distance rk, kq, qs, sn , and nx , and their respective Off-sets ko (there being no Off-set at q , it being at the Point of the River:) Also st , and nw in the same Manner, as you did those from the Line EbH , to the Hedge EeH , always taking care to take your Off-set from the Right-line rqm , and perpendicular to the same, which when done they will appear as in the Table following, viz.

The Table of given Dimensions.

Distances.		Off-sets.	
	Ch. L.		Ch. L.
$FE =$	11.50		
$Ea =$	6.00	$ac =$	1.65
$ab =$	5.50	$be =$	3.40
$bf =$	4.10	$fg =$	2.40
$fH =$	4.20		
$Hm =$	5.60		
$mk =$	4.70		
		$rG =$	1.10
$rk =$	2.60	$ko =$	1.40
$kq =$	2.40	$st =$	2.70
$ps =$	4.00	$nw =$	1.70
$sn =$	4.80		
$nx =$	3.60		
Diagonal $FH = 21.40$			

9. You are likewise to observe, that as the Line is drawn from the Angle at x , to touch the Extremity of the Point q , it must come to the Hedge somewhere, as at r , and rG is measured as an Off-set; but if the said Line had been drawn from the Angle x to the Angle G , it would have lain without q , and

rendered the getting the Point q difficult or impracticable: But it is best to draw the Off-set Lines from Angle to Angle, if it can be done without leaving out any of the intermediate Part of the Hedge or River, as in the Line EH , because if it lay farther into the Field, the Off-sets would be longer than necessary and so both more troublesome and subject to Error.

10. As Fields so irregular as the above, require great Care and Circumspection, in taking exactly all the Distances and Off-sets from the greatest to the least, in order to Protraction, as well as the casting up of the same; so I have here in this Example, been more copious than ordinary, to the Intent, that this Method may become a general Rule, so universally useful in this practical Part of Surveying, as to comprehend all Figures whatsoever, whatever Variety may occur, seeing the Method is once explained, it need not be repeated.

Protraction.

Whenever you are required to plot, or lay down orderly, the Distances and Off-sets found in this irregular Field, according to the Table of particular Dimensions given, as well as any other that is bounded in like manner with curving Hedges, Rivers, Roads, or or any other irregular Limits, you are to proceed as follow, viz.

First. Lay down the Right-lined Trapezium $EHkF$, Fig. 21. by taking the four Sides and the Diagonal FH , as taught in laying down a right-lined Trapezium, Chap. V. Sect. 7. observing, as before, that the Right-lines EH , mo , and Fr , be pricked Lines, because of no Use after the Field is protracted.

Secondly, You are to observe, that the Side EH is obtained by adding together the particular Dimensions of Ea , ab , bf , and fH , their Sum is the Side $EH = 19.80$ —In the same manner is found the Side kF or kx , whose Particulars found in the Table are $kq = 2.4$, $qs = 4.00$, $sn = 4.80$, and $nx = 3.60$, whose Sum is $kx = 14.80$.

Thirdly, Then by the four Sides, viz. $FE = 11.50$, $EH = 19.80$, Hm and mk together in one Right-line, making 10.30 , $kx = 14.80$, and the Diagonal-line FH or $xH = 21.40$, may the Trapezium be protracted, according to the abovesaid Directions.

To find the Area or Content of any Field whose Bounds are irregular, as curving Hedges, winding Rivers, &c. See Fig. 21.

1. Notwithstanding all the Mathematical Rules that can be given, yet Practice and Discretion will have a great Hand in directing you to allow for those Irregularities, as above. For altho' the Content of a Circle is demonstrably proved to be equal to the Content of a right-lined Triangle, whose Base is equal to the Circumference of that Circle, and the Perpendicular equal to the Radius or Semi-diameter of the same; yet Hedges that are not streight are seldom Arches of Circles; that it is needless to adopt any Rules to the measuring them as such; the curving of Hedges and winding of Rivers, &c. being to be accounted for by Methods more mechanic and discretionary.

2. For an Illustration of the same, we will suppose the Segment $EabfHgecE$, Fig. 21. were to be measured. If we should work it as a Triangle, and call EH the Base, and be the Perpendicular, we shall make less of it than it really is; for in so doing we shall suppose the Lines He and eE to be Right-lines, whereas they curve outwards, and inclose more Ground than Right-lines, and therefore serve to make that Segment less than it really is: but to remedy this, it is best at H , upon the End of the Line EH , to raise the Perpendicular HM , so much longer at Discretion than the Line be , as that the Triangle EHM may be equal to the Segment $EabfHgecE$, which will be if you make the Triangle gMH , which you take into the whole Triangle, to be equal unto the Segment contained between the Line Eg and the Curve $gecE$, which is left out of the Field.—For, if the Triangle gMH taken in, be equal to the aforesaid Segment $EgecE$ left out, then is the Triangle EHM equal to the Segment $EabfHgecE$; and measuring the Triangle is measuring the said Segment, the Base of the said Triangle, viz. EH , which is the Sum of Ea, ab, bf , and fH , being given in the Table 19.80, and the Perpendicular $HM = 3.50$, in order to find the Area or Content, as in Chap. V. Sect 4. which see.

3. There being also given the Trapezium $EHkx$, with its Diagonal $xH = 21.40$, the common Base to both Triangles, viz. xkH , and Hex , and their Perpendiculars are found by only taking the nearest Distance from k to the Line $xH = 6.50$ and

and from E to the Line $xH = 10.50$, their Sum, viz. 1700, multiplied by half the common Base 16.70 (the whole Base being by 21.40) gives the Content of the Trapezium, as in *Chap. V. Sect. 7.* which see.

4. The Piece between the Right-line xb , and the River, is reduced likewise into a Triangle by the same means, as that between the Right-line EH , and the Hedge, making the Perpendicular st so much longer than the Off-set st , that the Right-line xt may leave out so much at w belonging to the Field, as it takes in between i and that belongs not to the Field, and then shall the Triangle xst , the equal to the Piece of Ground that lies between the Right-line xq and the River.

5. You have likewise the Triangles qst and OrG , to cast up, which complete the whole Field, and then the casting of it up is all reduced to the Directions of *Chap. V.* and the Products all added together, gives the Content of the whole Field required. See the Work as follows;

OBSERVATIONS.

1. When we speak of the Perpendicular, st being longer than the Off-set st , it is meant, that the Off-set st is to reach but to the Edge of the River; but the Perpendicular st is supposed to extend further, so that the little Triangle taken in between i and t may be equal to the little Slip left out between the Right-line ix and the River at w ; but I let the Letter t serve for both, to prevent confusing the Scheme with Letters. See Article 4. as above.

2. The Base qs and the Perpendicular or Off-set st are both given by Measure in the Table. See Article 5.

3. Sometimes a little Labour may be saved by throwing two Triangles into one; as here the Triangle qst and xst might have been without sensible Error, making qx the Base, and st the Perpendicular, but these Abbreviations of the Work are most naturally acquired by a little Practice.

A Synopsis, or, short View of the particular Contents in Fig. 21.

GIVEN.		REQUIRED.	
Base.		Perpendiculars.	Areas.
Cb. L.		Cb. L.	A. Pa.
Triangles. { 1. EHM 2. qst 3. $\left\{ \begin{array}{l} Fst \text{ or} \\ xst \end{array} \right.$ 4. mGO	EH = 19.80	HM = 3.50	3.46500
	$qs = 4.00$	$st = 2.70$	0.54000
	$xs = 8.40$	$st = 3.00$	1.26000
	$mO = 6.20$	G to $mO = 3.80$	1.17800
Trapezia. EHKx	FH = 21.40	Sum of Perp. } = 17.00	18.19000
			24.63300
			Or, A. R. Per 24--2--11-28 For Answer.

S E C T. V.

To measure, protract, and cast up accurately, any Field, or inclosed Piece of Ground that is inaccessible, by reason of Wood, Copse, Marsh, or any other Obstacle, without going into the same.

1. **I**T sometimes happens that you are required to survey a Parcel of Land, where you cannot have the Opportunity of taking Diagonals, nor perhaps of going along the Insides of the Hedges to measure them; then, in this Case, you may (if you please) make use of Angles, by measuring round the Outside of the same, and finding the Quantity of every Angle externally, according to the Directions prescribed in

Chap. III. Sect. 6, and 7, which see; where by the Use, of the Pole, and the Explanation of Fig. 7. with the Use of the three Tables relating to Angles, you will thereby find all the Angles so plainly and exactly, that nothing of it need to be here repeated: Now the Angles being so obtained, and the Sides measured externally, as above, they may be orderly brought into so small a Table as follows:

2. But to perform the same Piece of Work with the Chain only, and without any regard to Angles, you may with greater Exactness and less Trouble, measure every Side, and place the Numbers in a Table or Field-book, as before directed; but instead of one Column you must have three, *viz.* that on the Left-hand for the Length of every Side; and the other two, one for each Side of a Triangle raised from thence, as may be seen by the Example in the Table that follows.—Now to proceed.

3. Measure outwardly every Side of the Field, and put orderly the Letters and Numbers belonging to each Side down in the first Column of the Table: As suppose you begin at K, *Fig. 22.* and find $KL = 5.50$ Chains, and $LM = 8.60$ Chains; put them down successively in the first Column on the Left-hand of the Table or Field-book, according to the Form following.

4. Having done this, begin again at what Angle you please, and as you began and ended at K, it is the best to begin there again, and measure a certain Number of Chains from K in the Right-line PK, continued somewhere to Q; so that from Q you can fairly see the Angle L (no matter how it bears, or what Angle it makes at Q with the Line PKQ) and then stop; then suppose KQ be 4.00 Chains, set $KQ = 4.00$ Chains in the 2d Column of the Table, against $KL = 5.50$, which in the is first Column.

OBSERVATIONS.

First, It is best, for avoiding Trouble, to make the first Line from the Angle at KQ and LR, &c. an even Number of Chains; for you may stop when you please, as soon as you are got a competent Distance from K, and in Sight of L.

Secondly, the Line KQ must be exactly in the straight Line PK continued, which may be found by setting up your Pole

Pole at K, and looking backward and forward, find the Right-line by the Directions of *Chap. III. Sect. 6.* or it may be proved by looking back from Q, and if you can just see the Angles K and P in one Station, the Point Q is in a straight Line with P and K.

Thirdly, Proceed likewise to measure in a Right-line from Q to L: which suppose you find to be 4.90, set $QL = 4.90$ Chains in the third Column against K L, and K Q, as you see.

Fourthly, In the same manner, measure from L in the Right-line K L continued; as suppose to R, where you can well see the Angle M, which let be $LR = 7$ Chains, set L R 7.00 Chains in the 2d Column against L M, which stands in the first Column.

Fifthly, Again, measure from R to M, which suppose 3 Chains 70 Links, set $RM = 3.70$ Chains, in the 3d Column under $QL = 4.90$ Chains.

Sixthly, Then from M in the Right-line L M continued, measure $MS = 3$ Chains; set $MS = 3.00$ in the 2d Column under L R 7.00, and then measure S N, which suppose 4.37 Chains, set S N 4.37 in the last Column under R M. 3.70.

Note, You need not leave any Marks at the outward Angles, QRS, &c. only stand at them till you have set down the Length of the Lines that you measured to them, as KQ, LR, and MS, &c. and then measure from them to the next Angles, LMN, &c.

Seventhly, According to the foregoing Directions, measure NT in the Right-line MN continued, which suppose 9 Chains, set $NT = 9.00$ in the 2d Column, and measure TO = 11.40 which set in the 3d Column as before.

Eighthly, In like manner proceed from O to V, in the Right-line NO continued, making OV 5 Chains; set OV 5.00 Chains in the 2d Column, and measure from V to the Angle P, which suppose you find to be 5 Chains 78 Links, set VP = 5.78 Chains in the 3d Column, and proceed to measure PW (in the Right-line PO continued) 3.00 Chains, which set down in the 2d Column, and measure WK = 4.36 Chains, which set in the 3d Column; and then is your measuring finished, and the Table will appear as follows:

The TABLE								
Ch. L.			Ch. L.			Ch. L.		
K L	=	5.50	K Q	=	4.00	Q L	=	4.90
L M	=	8.60	L R	=	7.00	R M	=	3.70
M N	=	4.30	M S	=	3.00	S N	=	4.37
N O	=	12.20	N T	=	9.00	T O	=	11.40
O P	=	6.60	O V	=	5.00	V P	=	5.78
P K	=	5.00	P W	=	3.00	W K	=	4.36

Ninthly, From the preceding Articles and Table, as well as Fig. 22. following, you will find, you have six right-lined Triangles raised upon the six Sides of the inaccessible Ground ; and the first Side of each being limited to a Right-line with its adjacent Side, the internal Angles of the Field are also limited and obtained without coming into the Field, or the Use of any Instrument for taking Angles, and done with as much Expedition, because the Chain is kept going, and with greater Certainty of Truth, the three Sides of every Triangle being now given, and the protracting performed by the fundamental Proposition.

Example.

Admit a Wood in the Form of an irregular Hexagon or Figure of six unequal Sides (*viz.* K L M N O P K, Fig. 22.) is required to be surveyed or measured externally, without going into the same : Now allowing the particular Dimensions to be as in the Table above : To know its true Figure or Shape, and also the Area or Superficial Content.

A. R. P. Pa.

Answ. 12—2—39.52.

Explanation.

Having found by Protraction, as follows, the true Form or Shape of the Wood, &c. without going into the same, the

A rea

Area or Superficial Content may be accurately found by drawing therein three Diagonal-lines, because a six-sided Figure, viz. P L, N L, and P N, which reduces it into four Triangles, whose Areas are particularly found, as in the Operation contracted, according to *Sett.* IV. and VII. of this *Chapter*, which contain a sufficient Variety of the Methods of Operation; and this being the same, it is needless to make Repetitions in this, but leave it to the Reader's Practice.

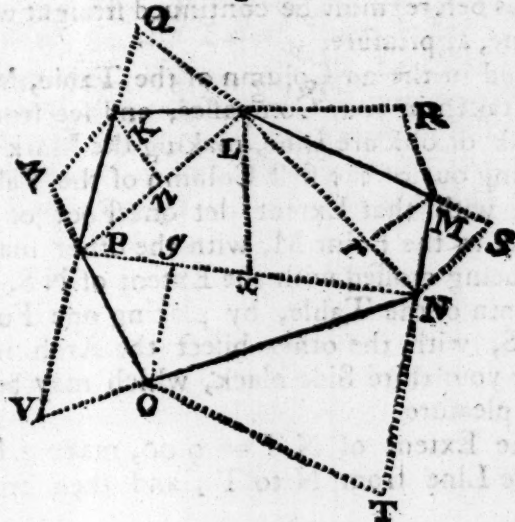
The Operation contracted.

Given.		Sought.
Base.	Perpendic.	Area.
Ch. L.	Ch. L.	A. Pa.
1. PKL = PL = 9.40	K n = 3.80	1.78600
2. LMN = LN = 10.50	M r = 3.40	1.78500
3. PLN = PN = 14.80	L x = 7.00	5.18000
4. PON = PN = 14.80	O g = 5.40	3.99600
Sum =		12.74700

A. R. P. Pa.

Or 12—2—39.52 for Answer. See Fig. 22.

Fig. 22.



F f 3

Protraction.

Protraction.

1. To plot or lay down this Figure exactly, you are carefully to observe the Table of Contents, wherein is contained orderly, the several Dimensions measured in three Columns, viz. in the first is the Length of each Side of the Field or Wood; the second and third are the Lengths of the Sides of the circumjacent Triangles: Or it may be sufficient to find the Points Q, R, S, T, V, and W, without drawing the Lines of the Triangles P W, W K, &c. they being of no Use after the Figure or Polygon is laid down.

2. In order to Protraction you are first to begin at some Side, supposing it to be K L first measured, and having drawn a white Line upon Paper, viz. K L R, at any competent Length at pleasure: then look in the first Column of the Table for the Side K L, which you find to be 5.50; set 5.50 from K to L, and draw the black Line K L, leaving the rest of the Line blank or dotted.

3. Look for L R, which you find in the 2d Column of the Table is 7.00; set 7.00 on the blank or obscure Line L R, from L to R, and mark the Point R: Then take L M = 8.60 from the Table which you find in the 1st Column, and with one Foot of your Compasses in L, make with the other an Arch at M: Then, in the third Column, take R M = 3.70 in your Compasses, and with one Foot in R, cross the aforesaid Arch M in M, and draw L M a black Line, for your second Side, which (as before) must be continued straight with a blank, or obscure Line, at pleasure.

4. Then find in the 2d Column of the Table, M S = 3.00, which being taken in your Compasses, and set from M to S; upon the blank or obscure Line, making the Mark or Point at S: Then taking out of the first Column of the Table the Side M N = 4.30; with that Extent, let one Foot of your Compasses be placed in the Point M, with the other make an Arch at N, which being crossed with the Extent of N S, taken from the third Column of the Table, by placing one Foot of your Compasses in S, with the other bisect the Arch in N; then draw M N for your third Side black, which may be continued further out at pleasure.

5. With the Extent of N T = 9.00, make a Mark upon the last obscure Line from N to T; and then from the 1st Column

Column of the Table, take the Extent $NO = 12.20$, and placing one Foot in N , with the other make an Arch at O , which being again bisected with the Extent of $TO = 11.40$, taken from the 3d Column of your Table; then, with one Foot in T , with the other cross the said Arch in O , and from N to that Crossing, draw the Side NO black, but continuing obscurely the rest of the Line straight, as before.

6. Take next from your second Column the Extent of $OV = 5.00$, which being set off from O to V ; then, from the 1st Column, take the Extent of the Side $OP = 6.60$, and with one Foot of your Compasses in O , make with the other an Arch at P ; which being again bisected with the Extent of $VP = 7.78$, taken from the 3d Column; then with one Foot in V , with the other cross the Arch at P , and to that Crossing, draw in black the Side OP from O to P , which must (as before) be continued straight from P obscurely to W .

7. With the Extent of $PW = 3.00$, taken from the 2d Column, let it be laid down obscurely from P to W ; then, with the Extent of the Side $PK = 5.00$, with one Foot in P , make an Arch at K ; which being again bisected from the Point at W , with the Extent of $WK = 4.36$ taken from the 3d Column; then from P to the said Crossing, draw in black, the Side $PK = 5.00$, and then the Hexagon, or Figure of the Wood, viz. $KMNOP$, is thus finished by Protraction.

N. B. That for every Side taken from the 1st Column of the Table, and laid down in a black Line, there must likewise in a straight Line from thence, be laid down in a blank or dotted Line, the next Extent, taken orderly from the 2d Column, beginning always with the 2d Number in that Column; and then, afterwards the very next Side being taken from the first Column and laid down, let it be bisected always with the Extent of that Number in the third Column, which stands exactly against the last that was laid down from the second.

Reasons why this Method of Surveying by the Chain only may be accurately effected, round the Outside of any Wood, Marsh, or other Inclosure inaccessible; as well as when inwardly performed.

Notwithstanding you may proceed, as before, to continue the Line PK out to Q, and upon it set off $KQ = 4.00$; and then, if $QL = 4.90$ in your Compasses, do not reach exactly from Q to L; there is probably some Error in the protracting; but this is but like a School-boy's proving his Questions in Division or Multiplication; for a little Practice and Care will render that superfluous Work needless in one, as well as in the other: For the Sum of all the Angles being limited to a certain Number of Degrees, it will follow, that if the five first be right, the sixth must be right of course; and if all the external Triangles be laid down true, the internal Triangles of the Hexagon cannot err: For in every external Triangle the three Sides are given, to lay it down by the Problem, therefore the Angles cannot be false; according to *Eucl. Lib. I. Prop. 22.* and in every one of these Triangles, one of their Sides is in the same Right-line with one Side of the Hexagon by Construction: Therefore the two Angles (suppose KLM and RLM) shall be equal to two Right-angles, *Eucl. Lib. I. Prop. 13.* therefore, if the external Angle (suppose RLS) be laid down true, the internal Angle KLM is true, and the same is to be understood of all the rest of any Polygon; all the Angles of the Polygon, being limited by the external Triangles, which are laid down only by this excellent Problem first mentioned, without taking any Angle by the Help of an Instrument.

I have added the above-mentioned Demonstration for the Satisfaction of the more learned Part of Mankind; lest this Method should be disputed, which is so true and expeditious, if well understood, and is grounded upon the fundamental Problem; and so fully answers the Title of being performed by the Chain only.

Observations

Observations very curious and useful.

1. Note, What is meant above by white blank Lines, is Lines drawn upon Paper or Vellum with the Point of a Pair of Compasses only, without Ink; but in drawing or plotting for casting up, it may be done as well upon a Slate, or rather better because the Slate will not shrink; and hence the Perpendiculars, &c. may be truer taken.

2. When you have done protracting, you need not regard any of the Points of the external Triangles, as QRS, &c. nor any of their Sides, viz. RL, RM, SM, SN, &c. they being only helpful in protracting the Polygon, but being no Part of the Figure, as afterwards entirely useless.

3. For the sake of those who would project or prove their Work by Angles, observe, that the Sum of all the Angles of any Polygon whatsoever, is equal to twice as many Right-angles, abating four, as there are Sides in the Polygon, let the Sides or Angles be equal or unequal. Hence, in a Hexagon all the Angles are equal to eight Right-angles, or 720 Degrees, &c.

S E C T. IV.

How a Manor or Lordship, or any Quantity of Ground consisting of several Inclosures contiguous to one another, may be accurately surveyed, protracted and cast up in Particulars, as well as in the Whole.

R U L E.

1. **T**HERE needs not much Time to be spent in this *Section* because let every Field, in any Lordship or Tenement, be of what Shape or Form it will, it cannot miss of some of those already treated of, in this, and the 5th Chapter preceding; and therefore, instead of repeating Directions for doing it, I need generally do no more than refer to the particular

ticular *Section*, where the same is already taught : Only this is to be minded, that here is not only single Fields to be measured, but Care is to be taken, how they join to the adjacent Field, in order to the mapping them true ; but in Land that lies adjoining one Field to another, there is the Advantage to the Surveyor, beyond surveying single Fields, that having measured all the Sides of one, you have commonly measured one Side of one, or more of the rest. As for Instance, If you begin with the Field A, you have, when you have finished that, measured one Side of the Field B, and part of a Side of the Field C, also a Side of the Field D, and a Side of the Field E ; and may, with a little forecast, save the Labour of doing them over again, if you proceed, as follows :

II. Having drawn the rough Sketch of it, as formerly directed, suppose you begin with the Field A, you have the Hedge *a c w* curving inwards, which you are to manage as in the curve Hedge *H m O*, *Fig. 21. Sect. IV.* and the rest, being straight Hedges, their Particulars will appear in the Table on the Left-hand of the Margin, because you are in this Place imagined to have measured against the Sun, seeing all the Remarks in the Hedge, or beyond it, stand conveniently on your Right-hand, therefore you are to place your Dimensions on the Left ; and then,

Ch. L.

<i>a e</i>	=	12.80	
<i>e b</i>	=	13.40	<i>p b</i> . Hedge <i>p b</i> .
<i>b d</i>	=	2.40	
<i>b z</i>	=	5.50	. . Hedge.
<i>z g</i>	=	2.00	
<i>g x</i>	=	2.00	. . <i>x w</i> = 3.40
<i>x e</i>	=	5.10	. . Stile.
<i>c a</i>	=	7.50	
<i>a b</i>	=	19.20	the Diagonal.

First, Begin at *a* and measure *a e*, *e b*, *b d*, &c. and set them down orderly, as you find them.

Secondly, Observe as you go along, that at *b* there is a Hedge in the next Field, on the Right, which mark on the Table on the Right-hand Side, as you see the Hedge *q b*, and regard that no more at present, but proceed to measure *d b*, and *d z*; and at *z* you find a Hedge abutting upon the other Side, which mark as you see in the Table, and so measure *z g* and *g x*, and at *x* take notice of the Off-set, which being on the Right-hand, place

place it on the Right-hand of the Table (as you see) and go to *c*, where there being a Stile, as above; and so proceed to *a*, where there being no Remarks, you have nothing to write on the Right-hand.

Thirdly, When you come to *A*, measure the Diagonal *ab*, and then that Field is done, and also the Hedge *eb* of the Field *B*.

Fourthly, Then go into the Field *B*, and if there is a Stile at *h*, measure from the Angle *b* to the Stile at *h*, and so by *qr*, &c. and when you come at *e* measure the Diagonal *eq*, and this Field is likewise finished; for as it is all straight Sides, there are no Off-sets, to be accounted for: and then the Table will appear as hereunto annexed.

<i>Ch. L.</i>	
<i>h h</i>	= 2.60
<i>h q</i>	= 11.00
<i>q r</i>	= 13.30
<i>r e</i>	= 12.80
<i>e q</i>	= 20.50 Diagonal.

Note, As before hinted, in some Cases, as in this Field *B*, it is most convenient to go with the Sun; for going in at *h*, near the Angle *b*, if you would go against the Sun, you must measure *b e* over again, which was measured before in the other Field *A*; but going with the Sun to *q*, &c. you end at *e*; and then measuring the Diagonal-line *eq*, brings you to the Corner of the Field *C*, where you may go in and begin with that Field.

Fifthly, In proceeding farther, it is now necessary to go against the Sun, as from *q* to *s*, &c. not only for going against the Sun, when it can conveniently be done, but because *qb* is already done in the other Field.

Sixthly, Measure *q G* and the Off-set *GH*, also *GI*, and the Off-set *IK*, as taught in the Section following; and by the same Rule, measure the curve Road *SM t*, *tf*, and *fd*, and bring them into the Table, as before directed in the said Section first named, and the Table for the Field *C*, will stand as follows:

Ch.

	Ch. L.		Ch. L.
<i>q</i> G =	4.50	G H =	2.50
G I =	2.00	I K =	2.70
I S =	7.10		
S L =	7.00	LM =	1.60
L <i>t</i> =	5.00		
<i>t</i> f =	6.80		
f d =	4.70		
d b =	2.50		
b S =	18.80		

Seventhly, Having measured the Field C, proceed next to D, where going into that Field, you observe that *d z* being already measured, you are then to come to *z*, and measure *z k* 4.80 and *k f* = 5.70, and the Diagonal *f z* = 7.60, and place them likewise in the little Table on the Right, as you did the rest; *f d*, and *d z* were measured before.

	Ch. L.
<i>z k</i> =	4.80
<i>k f</i> =	5.70
<i>f z</i> =	7.60

Eighthly, Proceed next to measure in the Field E at *z* or *k*, measure *k* *z*, *z v*, *vw*, and *w z*, and the Diagonal *z v*, and let them be also placed as in the Table adjoining.

	Ch. L.
<i>z k</i> =	4.80
<i>k z</i> =	5.60
<i>z v</i> =	3.40
<i>vw</i> =	10.30
<i>w z</i> =	5.60
<i>z v</i> =	10.60

Lastly, For the Field F, you have already the Sides *t f*, *f k*, and *k z*, and the Partitions of each Side of it being adjusted by Diagonals, the Angles *f* and *k* are determined, and so you have nothing to measure but the Side *t z*, which being a Range of Houses, they must every one be measured separately, and placed in a Table thus:

Ch.

	Ch.	L.
From <i>t</i> to the East End of the House	=	0 . 95
Thence to the East End of the Dairy	=	0 . 70
Thence to the East End of the Barn	=	1 . 20
Thence to the garden Door	=	0 . 90
Thence to the Angle at <i>g</i>	=	1 . 00
From <i>t</i> to <i>g</i> , in all	=	5 . 65

And suppose there be a Pond or Spring in the Field C at O, measure *tO*, which let be 1.50; measure likewise the nearest Distance from O to the Hedge or Wall *ft*, as suppose 0.75, make a Mark or * in the Hedge *ft*, and make a Remark at the Bottom of the Table of the Field C.

$$\begin{array}{l} \text{O a Well } \left\{ \begin{array}{l} tO = 1.50 \\ O* = 0.75 \end{array} \right. \end{array}$$

Also by the same Rule you may insert a House, a Tree, or any thing else that is worth notice, or proper to be expressed in a Map either for Service or Ornament; and by the afore-said Rules, any Land, of whatsoever Form, may be measured in order to be protracted, cast up, and mapped; proper Directions for all which shall follow in their Order.

Example.

There is a certain Manor or Lordship, whose Particulars, when surveyed, are found in the Operation contracted, with its Explanation: To know the Area or Superficial Content of the whole in Acres, &c.

A. R. P.

Answ. 64—3—17. See Fig. 23.

O P E-

OPERATION CONTRACTED.									
	Given				Required.		Fields.		
	Base.		Perpendicular.		Area or Con.				
	Ch. L.		Ch. L.		Acres.	Pts.	A.	R.	P.
Trapezium	1. $aedg$	$ad = 19.20$	from g and e to ad	$= 15.80$	15.16800		A = 16	— 1	— 20
Triangle	2. egw	$eg = 7.10$	$xw =$	$= 3.40$	1.20700		B = 17	— 2	— 4
Trapezium	3. $reqb$	$eq = 20.50$	from b and r to eq	$= 17.10$	17.52750		C = 20	— 2	— 25
Trapezium	4. $bqst$	$bs = 18.80$	from q and t to bs	$= 19.00$	17.86000		D = 2	— 3	— 16
Triangle	5. qSK	$qS = 13.60$	IK =	$= 2.70$	1.83600		E = 4	— 1	— 39
Triangle	6. Stm	$St = 12.00$	LM =	$= 1.60$	0.96000		F = 2	— 3	— 33
Trapezium	7. $dzfk$	$zf = 7.60$	from d and k to zf	$= 7.50$	2.85000				
Trapezium	8. $wzkn$	$zn = 6.80$	from w and k to zn	$= 7.40$	2.51600				
Trapezium	9. $Evnk$	$kv = 6.00$	from n and v to kv	$= 6.60$	1.98000				
Trapezium	10. $fkst$	$st = 6.80$	from k and t to st	$= 8.70$	2.95800				
Sum = 64.66250					= 64 — 3 — 17				

N. B. That the Product of the Base and Perpendicular given for each Trapezium and Triangle, finds it for the double Area, whose half is the true Content sought, as above. See Chap. V. Sect. 4. and 7.

The

The Name and Quantity of Acres in every one of the aforesaid Fields.

	Acres. Parts.	A.	R.	P.
A. Wheat Field	= 16.37500	= 16	: 1	: 20
B. Horse Meadow	= 17.52750	= 17	: 2	: 4
C. Home Close	= 20.65600	= 20	: 2	: 25
D. The Orchard	= 2.85000	= 2	: 3	: 16
E. Calves Close	= 4.49600	= 4	: 1	: 39
F. The Garden, &c.	= 2.95800	= 2	: 3	: 33
Total		= 64	: 3	: 17

The whole is 64 Acres, 3 Roods, and 17 Perches; and if I had continued it to several hundreds of Acres, it would have been but a Repetition of the same Work or Method, which is needless; for in this and the Vth Chapter preceding, you have all the Varieties that can possibly occur in the Practice of measuring or casting up of Land.

Explanation.

1. For the more orderly proceeding, we will begin with the Field A, which being protracted and mapped as follows, it will resemble the Form of a Trapezium, *viz. a c d g*, whose Diagonal $a d = 19.20$, being measured in the Field, and accordingly inserted in the Table or Field-book, with the two Perpendiculars let fall from *g* and *e* respectively upon the said Diagonal-line *a d*, their Sum is 15.80; which being multiplied by half the Diagonal, or common Base, the Product is 15.16800 Acres, for the Content of the Trapezium sought, as in the Operation contracted.

2. The Triangle, *viz. e g w*, belonging to the same Field A, hath its Dimensions found in the Table, *viz. e x = 5.10*, and $x w = 2.00$, whose Sum is the Base $e g = 7.10$, its half being 3.55; which being multiplied by the Perpendicular $x w = 3.40$, the Product is 1.20700 Acres, for the Content of the Triangle sought; which being added to the Area of the Trapezium last found, gives 16.37500 Acres for the whole Field A.

3. Then

3. Then for the Field B, the Diagonal, or common Base, viz. $eq = 20.50$, being multiplied by the Sum of the two Perpendiculars or nearest Distance from b to the Base $eq = 9.00$, and from r to the same Base $= 8.10$. whose Sum is 17.10 , and the Product the double of the Content, whose half is 17.52750 , the Content of the Field B, as above.

4. For the Content of the Field C, you are first to find the Trapezium $bqst$ by letting fall two Perpendiculars from the Angles q and t upon the common Base $bs = 18.80$, which being multiplied by the Sum of the Perpendiculars, viz. 19.00 , and half the Product taken, gives 178600 for the Content of this Trapezium. — Then for the Triangle qsk , the Base, viz. $qs = 13.60$, being multiplied by the Perpendicular $ik = 2.70$, and half of that Product being taken, discovers 183600 for the Content of the said Triangle. — Again, for the other Triangle, viz. stm , the Base $st = 12.000$, being multiplied by the Perpendicular $lm = 1.60$, half of that Product, viz. 0.96000 Acres is the Content of the said Triangle. Hence, the Sum of the three Particulars, namely, one Trapezium and two Triangles found in this Article, viz. 20.65900 , is the Content of the whole Field C.

5. Then for the Field D, the Diagonal or common Base, viz. $zf = 7.60$, being multiplied by the Sum of the Perpendiculars, viz. d and k , to $zf = 7.50$, half the Product, viz. 2.85600 Acres, is the Area or Content of the Field sought.

6. The Field E being a long Slip, with an inward Angle at k , it is best to divide it into two Parts, by a Line drawn cross the Field from k , as the Line kn , and measure it at twice; then it follows, that the common Base or Diagonal-line, viz. $zn = 6.80$, for the Trapezium $wzkn$, being multiplied by the Sum of the Perpendiculars, viz. 7.40 , being w and k , let fall to zn , gives the double Area, whose half, viz. 2.51600 Acres, is the Content of the first Trapezium sought. — Again, the Diagonal or common Base, viz. $kv = 6.00$, for the other Trapezium, viz. $evnk$, being multiplied by the Sum of the Perpendiculars $= 6.60$ (viz. n and e) let fall from their Angles upon the Base kv , the Product is the double Content, whose half is 1.98000 Acres for the said Trapezium sought: Hence the Sum of both viz. 4.49600 , is the Content of the whole Field E.

Lastly, For the Yards, Gardens, and Ground, the Houses stand upon, included in the Trapezium marked F, let the Diagonal $f\mathcal{E}$, be drawn, and upon it let fall Perpendiculars from the other Angles k and t , which when measured, will be found to be 5.00, and 3.70, whose Sum is 8.70; which being multiplied by the common Base (*viz.* $f\mathcal{E} = 6.80$) half the Product is 2.95800 Acres, for the desired Content of the Trapezium F, as above.

A practical and useful Observation.

Note, if a woody, or otherwise inaccessible Piece of Ground, be enclosed on every Side with open and clear Land, you need only measure and plot the circumjacent Land, and the enclosed Piece is also done: As suppose the Field or Piece of Land D, *Fig. 23*, were so inaccessible; yet if the Fields A, C, F, and E, by which it is surrounded, be clear and passable, the Field D is also measured, as soon as they are done; for in the Field A, we marked the Butting of the Hedge kz at z ; and in measuring the Field E, we likewise take the Hedge zk ; and in measuring the Field F, we get the Hedge kf ; and in measuring the Field C, we have likewise the Hedge fd of the Field D; and if the Diagonals of the Fields you measure be carefully taken, it is impossible there can be any Error in their Angles, and if they be right, the Angles also of the inaccessible Piece will be truly adjusted: And therefore, it is necessary when you survey a Lordship or Manour to observe carefully as you go along, what Hedges in the Adjacent Field abuts upon the other Side of the Hedge that encloseth the Field you are measuring, and at what Distances, &c. which being carefully marked in your rough Draught, as you go along, may save a great deal of Labour when you come to survey the next Fields, there being continually, by this means, some one or more Hedges or Sides of the Field done before you come into it, except only the first Field where you begin.

Protraction.

When you have measured your Lordship or Manour, as before directed, and brought the particular Dimensions thereof into a Table, you are next to find the true Form, Figure, or Shape of the same by Projection, as follows: And,

First, Suppose begin with the Field A, as you did when it was surveyed, and its Dimensions accordingly inserted or

G g

noted

noted down in your Table or Field-book; then proceed to protract it, according to the Directions of *Seet. IV.* in this *Chapter*, seeing it is partly bounded or limited by a curved Side or Hedge on the Right-hand: Then it follows, that the Diagonal ad , and the Sides de and ea being given and laid down, you thereby obtain the Triangle ade , according to *Chap. V. Seet. 4.* only with this Difference, that ad , must be made a pricked Line, because it is no Hedge, but only a Diagonal, and of no Use after the Field is protracted.—In like manner, and by the same Rule, lay down the Triangle adg , the three Sides being also given, *viz.* ad , a Side of the other Triangle; also dZ and Zg added together, for the Side dg ; likewise gx , xc , and ca , in one Sum is the Side ga , then is the Trapezium finished, as in the Map, *Fig. 23.* which see.

Secondly, Find the Side de , by adding eb and bd together, their Sum is the Side $ed=15.80$; then upon the Line ed , set off $eb=13.40$ from e to b , and make a Mark at b (thus *//*) for the End of the Hedge of the Field B, when you come into it; also from d towards g set $dZ=5.50$; and because you likewise see in the Table [*Hedge*] make a Dot or Mark at Z ; then from g set $gZ=2.00$, in the Line gea ; and because in the Table, against gx , and on the Right-hand, you see $xw=3.40$, set 3.40 from x perpendicular to gca ; it will extend to w , the Corner of the Field; then from x set $xc=5.10$, and $ca=7.50$ from c to a ; then is $aedZgwca$ the Points the Hedge is to pass through; and because in the Table against $xc=5.10$, you see on the Right-hand [*Stile*] intimating that at c there is a Stile in the Hedge on the Right; mark that, as you see at c , and the Field is finished.

Thirdly, In laying down the Field A, you have also done the Hedge eb of the Field B, and the other three Sides of it, *viz.* er , rq , and qh , which added to hb makes the side qb , and also the Diagonal eq , all which you have in the little Table to Field B; it is to be protracted as a Trapezium by the Directions of *Chap. V. Seet. 7.* which see, in order to prevent any farther Repetition relating to the Performance thereof.

Fourthly, In protracting the Field B, you have also done the Hedge qb of the Field C, and also bd , a Part of the whole Hedge $bdf t$; the Trapezium $qb t S$ may likewise be laid down by *Seet. 7.* abovesaid; and the Off-sets $GHIK$ and LM are to be laid down by *Seet. 4.* of this *Chapter*, to which I refer the Reader to avoid Repetition, and then you have protracted the curving River $qHKSM t f d h q$, which is the Field C.

Fifthly,

Fifthly, In like manner, you have the four Sides and Diagonal of Field D given, to lay it down as a Trapezium, as you see in the Table to Field D ; only observe, that the Side dZ is to be found in the Table of Field A, it being a Part of a Hedge of the same, which was measured before ; as for the same Reason, the Side df , is to be found in the Table to Field C.

Sixthly, By the same *Sec.* VII. protract the Field E, it having no Curves nor Off-sets belonging to it, and the Numbers being put down in the Table referring to it marked E.

Lastly, When this is done, you have also laid down three Sides of the Yard or Garden F, in which you need not mind any Diagonal as tk , or the like ; for if you have been exact in all the rest of the Triangles in each Side of it, the Angles of this cannot miss of being right, the Sides being already laid down, and the Side $t\&$ being included in the Sum of the Particulars of the Wall and Front of Houses, as in the Table of F.

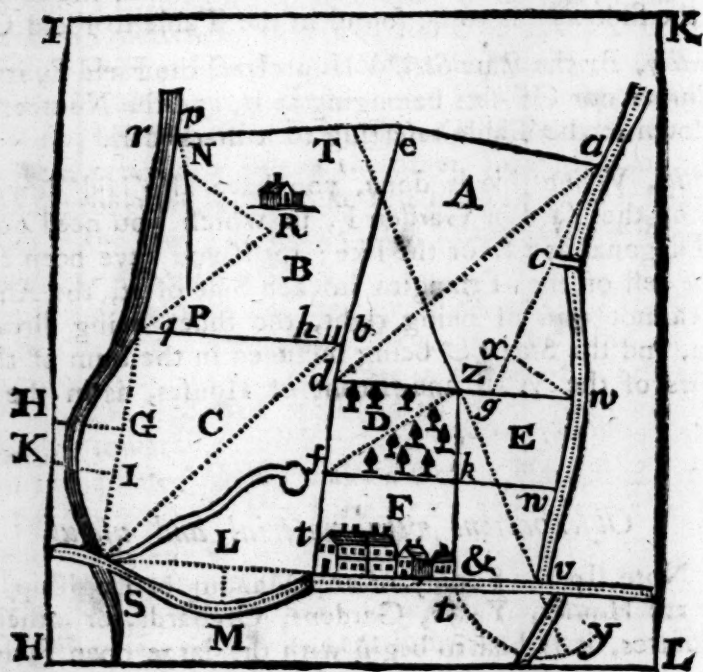
Observations very material and useful.

1. Note, In the Survey of any Manour or Lordship, where there are Houses, Yards, Gardens, Orchards, or other small Enclosures, it is best to begin with the large open Fields, and if the Ground lie contiguous to the House and Gardens on two or three Sides, as here it doth, it is best to measure on every Side of them, and they are then measured without coming into them ; for it often happens, that by reason of Dunghills, Ponds, Gardens, Stables, &c. that may be in the Yard, or behind the House you cannot come within to measure them ; but when the Square, or whatever other Figure it be, that the Yard or Gardens are contained in, is enclosed, and all its Sides measured, in measuring the Ground round it, any Pond, Out-house, &c. may be put in its proper Place, by measuring with a Pole, where the Distances are so small that there is not convenient room to use a Chain.

2. Although the Rules and Examples in this and the preceding *Sections* lead the Way to the Operations, Explanations, and Protractions, which occasion them to be orderly placed in this Essay ; yet nevertheless, the Figure and Protraction of each must likewise be duly regarded, seeing the latter is the Foundation of the former.

The Manour of a certain Gentleman in Kent.

Fig. 23.



S E C T. VII.

How to place any House, Pond, Tree, or any thing remarkable, in its true Situation in a Field to be mapped.

R U L E.

HAVING shewn in the last Section how Fields, Gardens, Orchards, Buildings, &c. may be accurately surveyed, protracted and mapped, we shall now briefly exhibit a Method for

for the right placing any House, Pond, Tree, &c. in the Map. And,

I. In measuring or taking the Dimensions of the Field, wherein the said House, &c. is found, make a Mark as near as you can, in your Table or Field-book, to represent the Place or Situation of the said House, &c. in the Field; then from the said Mark, unto each of the two nearest Angles, let the Distance be exactly measured, and accordingly noted down in your Table.

II. The Dimensions being thus measured, and placed upon the respective Lines or Sides in your rough Draught, or otherwise inserted in your small Table, as above, you are then prepared for Protraction as follows.

Example.

Admit there is found in the Field B, *Fig. 23.* a House, which your Table or rough Draught represents to be at T, but in your Map, the true Situation thereof should be at R: Now supposing NP to be a straight Line, near the River *r q*, and that the nearest Distance between N and R, when measured, is 2.60 Chains, and likewise between P and R = 3.50 Chains: To know exactly the Point R, where the House must stand.

Answ.

See Field B, in *Fig. 23.*

Explanation.

The Rule and Example above is so particularly plain and easy of themselves, that there need no farther Explanation, if the Work of Protraction is but duly observed.

Protraction.

This Example above, or any other of the like kind, may be readily plotted by taking between your Compasses, 2.60 Chains, the measured Distance between N and R, and with one Foot in N, make an Arch at R; then, likewise, taking from the same Scale of equal Parts, the Distance of 3.50 Chains, and, with that Extent, cross the Arch R at R; and at the Crossing of the said Arches, or Point of Intersection, is the

G g 3

Place

Place where the House or Pond is to be : which, by Illustration, will more evidently appear in *Seet. VI. of this Chapter*, and *Fig. 23. which see.*

N. B. *That if the measured Distance had been taken from the two angular Points p and q, instead of N and P, to the Mark at R, the same Effect had been produced.*

S E C T. VIII.

How to enlarge or diminish a Plot given, according to any Proportion required. Or, the Method of reducing a large Plot of Land or Map into a lesser Compass, according to any given Ratio ; or, è contrario, how to enlarge one.

R U L E.

I. **T**HIS useful Problem may be variously performed, as well as generally comprehended in the following Directions, *viz.*

First, According to *Fig. 24.* divide one of the Sides given (*viz.* A B) in Power, as L to K, in such sort, that the Power of A F may be to the Power of A B, as L to K.

Secondly, Then from the Angle A, draw Lines to the Point C and D ; that done, by F, draw a parallel Line to B C, to cut A C in G, as F G.

Thirdly, Again, from G, draw a parallel Line to D C, to cut A D in H.

Lastly, from H, draw a parallel Line to D E, to cut A E in I ; so shall the Plot A F G H I be like A B C D E ; and in Proportion to it, as the same Line L to the Line K, which was required.

II. But

II. But if the lesser Plot were given, and it were required to make a greater in Proportion to it, as K to L; then you are to proceed thus:

First, From the Point A, draw the Lines AC and AD, at any convenient Length you please: Also, increase AF and AI by a blank or obscure Line.

Secondly, Enlarge AF in Power or Proportion, as L to K, which set from A to B: then by B, draw a parallel Line to FG, to cut AG in C, as BC.

Thirdly, In like manner, from C, draw a parallel Line to GH, to cut AD in D, as CD.

Lastly, A parallel Line from D to HI, as DE being drawn to cut AI, being increased in E; so shall you include the Plot ABCDE similar to AFGHI, and in Proportion thereunto, as the Line K is to the Line L, which was required.

Example I.

Let ABCDE (Fig. 24.) be a Plot given to be so reduced, diminished, or made less, in such Ratio or Proportion as L to K, or 10 to 30: To know the Figure or Shape of the lesser or smaller Plot.

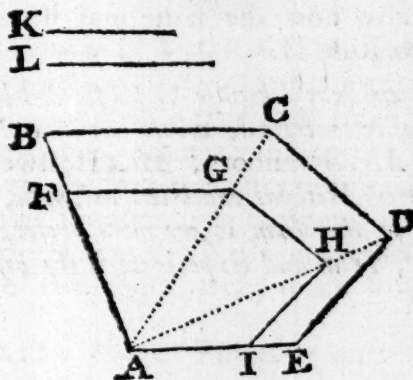
Answ. The Figure AFGHI, which see.

Example II.

Suppose there is given a Plot or Map, represented by the lesser Figure AFGHI; and it is required to have one in the same Form or Shape, as well as Content; which shall represent the larger Figure ABCDE, and bear such Proportion thereunto as K to L, or 30 to 10: To know the Form or Shape of the said Figure.

Answ. ABCDE. See Fig. 24.

Fig. 24.



G g 4

Other-

Other Methods variously performed.

R U L E.

Besides the general Method preceding, there are other Ways very practicable as well as rational : And,

1. The best Way is, if your Plot be not over large, to plot it over again by a smaller Scale : But if it be so large, as the Map of a County, or the like, the only way is to compass in the Plot with one great Square, and afterwards, to divide that into as many little Squares as you shall see convenient, in order that one of those lesser Squares may comprehend something remarkable, as a Church, House, Wind-mill, or any other thing accidental as well as material.

2. Also make the same Number of little Squares upon a fair Piece of Paper by a lesser Scale : supposing that of 20, 30, or 40 Divisions to an Inch, according to the Proportion given : This done, see in what Square, or Part of the same, any remarkable Accident falls, and accordingly put it down orderly and exactly in your lesser Squares ; and that you may not herein mistake, it is a good way to number your Squares both ways, as well as to dot their Lines, according to the Nature of the Figure given, when performed by the Artist ; by which means the same will be particularly illustrated, seeing this small *Enchiridion* will not admit of the Performance.

Example.

Admit there is a Map or Plot of Land, (*viz.* ABCD) made by a Scale of 10 Chains in an Inch ; but it is found needful to reduce the same to a lesser Plot (*viz.* EFGH) of 30 Chains in an Inch : To know how the same may be accurately performed according to Rule II.

N. B. *This Problem may likewise be performed by certain Scales, or Rulers purposely contrived, whose Use and Application is recommended by Mr. Rathborn, Mr. Hollwell, and others ; but as the former Methods exhibited as above, are demonstratively rational, so they seem to me more preferable for Exactness : However, Trial and Experience is the best Satisfaction.*

S E C T. IX.

How a fair Map of any Lordship, Manour, or other Estate, may be accurately drawn, and afterwards adorned with proper Colours and other Embellishments.

R U L E.

THAT this Work may be accomplished with all imaginable Plainness and Conciseness, we shall, *First*, shew,

How the Vellum or Paper may be prepared for drawing upon.

I. Maps may be accurately drawn upon Vellum, especially when the Skin is so large as not to need any Augmentation; and in its Nature so white and clear from Grease, that it may want no other Preparation, than only the rubbing it occasionally with a little Pounce, in order to make the Pen slide freely, and keep the Ink from running or spreading; which will still be more effectual, if a little Piece of Gum-Arabic, or white Sugar, be dissolved in your Ink, in order to prevent its sinking, and giving it a handsome Gloss.

II. When Maps are so large as to require Roles and Ledges, then it is most convenient to draw them on strong, fair, and white Paper pasted upon Canvas, as follows:

1st, Let your Canvas be well stretched upon a Table or Door, pulling it tight from one Side and one End to the other; and nailing it round about the Edges with small Nails, to keep it smooth and prevent it from shrinking; always taking care, that your Canvas be larger than you intend the Map to be, that you may the more conveniently cut it square, after it is finished; the Holes that the Nails made being clearly cut off, and the Edges of the Map thereby made straight.

2^{dly}, Take half a Pint of Flour, or more or less, according to the Quantity of Paste your Occasion requires; then put Water

to it, by little and little, keeping it stirring till all the Flour be wet and stiff, or hard like Dough: For this is the best way to break the Lumps, and prevent any dry Knots being in the Paste, then put more Water gradually to it, and keep it stirring till it be almost as thin as Milk; which done, set it upon a gentle Fire, keeping it continually stirring, and let it boil a Quarter of an Hour, or half an Hour, more or less, as you would have it strong or weak; and then take it off, and when it is cold it is fit for use.

3dly, Then with a Brush for that Purpose, spread the Paste upon the Canvas, beating it in, till the Grain of the Canvas be all filled up (for this, when dry, will prevent the Canvas from shrinking when it is taken off the Stretch) and then, having cut the Edges of your Paper straight, paste one Side of every Sheet, and lay it upon the Canvas Sheet by Sheet, joining to one another, or reaching over one another, till you have covered all the Canvas intended for the Map.

4thly, If you happen to use strong Paper, it is best to let every Sheet lie five or six Minutes after you have laid the Paste upon it (and in the mean time, you may be laying the Paste upon more) for as the Paste soaks in, the Paper will stretch and may be better spread smooth upon the Canvas; whereas, if it is laid on before the Paste has moistened the Paper, it will stretch afterwards and rise in Blisters after it is laid upon the Canvas.

Lastly, The Paper being thus pasted and thoroughly dried, it is then fit to draw upon: but it is best to be dried in the Sun, or in a dry Room, if Time permit; but if in the Winter Season, or for Want of Time, the Help of the Fire is necessary, provided you let it not come too near any scorching Heat.

S E C T. X.

To draw a Map from the protracted Draught you cast up the Land by.

R U L E.

HAVING in the last *Section* shewn the Method of preparing the Vellum or Paper for drawing upon; we shall next shew, how any Draught truly protracted may be fairly and beautifully mapped, with all imaginable Exactness and Facility. And,

I. If the fair protracted Draught be done to the same Scale, or of the same Dimensions the Map is intended to be; then, in this Case, you are to lay carefully the protracted Draught upon the Vellum or Paper you intend to draw the Map upon; and with small Weights here and there upon it, keep it from moving: Then, with the Point of a Pair of Compasses, or such like Tracer, trace all the Lines of the protracted Draught, pressing softly, that the Impression may appear on the Paper you are to draw a Map upon; and having traced all the Lines, take off the protracted Draught, and you will see underneath all the Lines in the Map, which being traced over with a fine hard Pen and good Ink, you have what some call the Out-lines of the Map, and may proceed to colouring and adding other Ornaments, as shall hereafter be directed.

II. But in case you are confined to a lesser Scale for your Map, than what will be sufficiently large to cast the Land exactly up by, you must reduce your protracted Draught to the Size proposed for the Map, according to the various Methods prescribed in *Seet. VII.* of this *Chapter*, which see.

S E C T. XI.

How Maps of any Form or Dimensions may be ingeniously shaded, beautified, and adorned, with proper Colours, in order to render them (when truly drawn or protracted) pleasantly delightful.

R U L E.

I. **W**HEN the Map is truly adjusted to the Dimensions proposed, and every Field laid down in its proper Shape, it adds very much to the Beauty of the Draught to adorn it with proper Colours, as Grass, whether Meadow or Pasture, with green; arable or ploughed Land, with a brown, earthen Colour, &c. and when you have writ the Title in a handsome Print-hand, colour it over (when thoroughly dry) with a transparent yellow, and embellish your Compartments, or other Ornaments, as the Compasses, Scale, &c. with any other Colours, at pleasure, which you may procure cheaper than can be made in small Quantities, as Occasion requires; seeing it would require a Treatise of itself, to shew the Manner of making and compounding them truly and exactly, according to the Method of the best Artists herein, to whom the Curious may have reference, particularly in *Chap. XVIII. Page 263, of Surveying Improved*, which see.

II. It is not only ingenious, but even also convenient, that the Colouring of Maps should be performed by the Surveyor himself; because, after he hath drawn the Draught of a Manour or Lordship, and reduced it to his intended Bigness, it would be very troublesome to repair to a Painter to finish his Work, since the thing itself is very commendable and easy to be obtained: And besides, a Painter is not to be found in every Country-town or Village; nor is every Painter furnished with Colours fitting for such a Purpose; they, for the most part, using more gross and ordinary Colours. Now, for the Benefit of such who desire to exercise themselves in this kind of Practice, I have added these Directions following.

First, The black Lines by which the Bounds of every Field or Enclosure is limited, may be shaded (outwardly and inwardly) with

with a small Pencil and some transparent Colour: so shall you have a transparent Stroke or Margent on either Side of your black Line; which being thus shadowed, will add a great Lustre and Beauty to your Plot.

Secondly, In your Wood-land Grounds, draw divers little Trees in the most material Places, and shadow your mountainous and uneven Grounds of Hills and Vallies; expressing all kinds of Bogs, Groves, Highways, Rivers, &c. distinguishing them by lively Colours, according to their Similitudes.

Thirdly, Then, in some convenient Part of the Plot, without the Enclosures, draw a Circle, and therein describe the 32 Points of the Mariners Compass, according to the Situation of the Grounds with a Flower-de-luce at the North Part thereof.

Fourthly, Let likewise some convenient Place of a Plot be chosen for making a Scale, equal to that by which your Plot was protracted, with a Pair of Compasses upon it.

Lastly, In some other convenient Place, towards the upper Part thereof, draw the Coat of Arms belonging to the Lord of the Manour, with Mantle, Helm, Crest, and Supporters, or in a Compartment: But be sure you blazon the Coat in its true Colours.

These things being well performed, your Plot will be a neat Ornament for the Lord of the Manour to hang in his Study or other private Place; so that, at Pleasure, he may see his Land before him, and the Quantity of all and every Parcel thereof, without any farther Trouble.

Also in your Plot must be expressed the Manour-house, according to its Symmetry or Situation, with all other Houses of note; also all Water-mills, Wind-mills, and whatever else is necessary, that may be put into your Plot without Confusion.

The ARGUMENT.

Having in six Chapters preceding shewn the Performance of the Chain in all the Occurrences of Surveying, whether ordinary or extraordinary, we shall now in this Chapter, shew arithmetically, &c. the Use and Application of certain Problems, whereby the Surveyor will be enabled to reduce compendiously irregular Figures and Plots, into those which are regular, for the more readily casting up the same: Also, they will be no less helpful to him, in the Division and Separation of Land, and in the laying out of any assigned Quantity; whereby large Parcels may be readily divided into several smaller Parts, and those again subdivided if need be.

C H A P. VII.

Shewing practically by Problems, how any large Plot or Figure, tho' never so multangular and irregular, may be accurately reduced into a Triangle, Parallelogram, or Trapezium. Also, the Method of dividing any plain Superficies into two or more Parts, according to any Proportion by Lines drawn from any Angle, or from a Point in any Side, &c. with several other things both curious and useful.

P R O B. I.

A certain Quantity of Acres being given, how to lay out the same in a Square Figure.

R U L E.

TH E R E being no less than 40 Problems for Transmutation of Figures or Plots, from one Form or Shape into another; as well as their Division or Separation into any
Number

Number of proportionable Parts assigned, which ought of necessity to be known by every Man, that intendeth to attain a competent Proficiency in the Practice of Surveying; but as the Geometrical Demonstration, &c. thereof will require a larger Treatise than this Essay will possibly permit, I shall only briefly treat of such as are more practical and useful; and (for the most part) can be comprehended in the Arithmetical Performance; and then refer the Reader to the Works of such as have more copiously treated thereupon, particularly the Author of *Surveying Improved*, Chap. IX. Page 158. which see.

II. Now for the true Performance of this Problem, you are always to annex to the Right-hand of the Number of Acres, &c. given, five Cyphers, with a Point prefixed, which will turn those Acres, &c. into the Product of Square Links, because 100000 Square Links make an Acre: Then from the Number thus increased extract the square Root, which shall be the Side of the proposed Square, to be protracted or laid down according to the Method prescribed in *Chap. V. Sect. 1.* which see.

Example. I.

Suppose it is required to lay out in a square Figure 200 Acres: To know the Side of the Square that shall comprehend the said Quantity.

Ch. Lin.
Answer, 44.72 +

Example. II.

Admit two Acres of Ground is required to be cut off from any Corn-field: To know the Length or Side of those two Acres that shall constitute a true geometrical Square.

Ch. Lin.
Answer, 4.47 +

Explanation.

1. In *Example I.* unto 200 Acres let five Cyphers be annexed (thus 200.00000) and the square Root of the Product extracted, gives 44.72 Chains, &c. for the Side of the Square sought

2. For

2. For the second Example let the square Links in 2 Acres be extracted (*viz.* 2.00000) the Root is 4.47 Chains +, for the Side of the Square sought.

Note, When the Side of the Square is required for Roods and Perches, as well as Acres, you are to take particularly the Number of square Chains or Links, equal to the odd Roods and Perches given; and having added them (as before) unto the square Chains and Links in each Acre, the Square-root of that Sum will be the Side sought.

P R O B. II.

How to lay out any given Quantity of Acres, &c. in a Parallelogram, whereof one Side is given.

R U L E.

Let the Acres, &c. given, be first turned into square Links, by adding (as before) five Cyphers; that Number thus increased; divide by the given Side, the Quotient is the other Side sought.

Example.

A Parallelogram is required that shall contain 96 Acres, 3.25 Roods: Now allowing its Breadth to be 18.30 Chains: To know its Length, or other Side sought.

Ch. Lin.

Answer, 52 . 95 +

Explanation.

First 96 Acres, 3.25 Roods, being reduced into square Links (*viz.* 9690625) by *Chap. III. Table 8. &c.* and that Sum divided always by the given Side in Links (*viz.* 1830) the Quotient (*viz.* 5295 +) is the other Side required.

P R O B. III.

How any Piece of Land in the Form of a Parallelogram may be laid out, that shall be 4, 5, 6, or 7, &c. times longer than it is broad.

R U L E.

Let first the given Quantity of Acres, &c. be turned into square Links, as before, which Sum divide by the Number given

given for the Proportion between the Length and Breadth, as 4, 5, 6, 7, &c. then the Square Root of the Quotient will be the shortest Side of such a Parallelogram in Links, which being always multiplied by that Number, which was made your Divisor, the Product exhibits likewise the longest Side thereof.

Example.

Admit it were required to lay out 300 Acres in a Parallelogram, that shall be six Times as long as broad: To know the Breadth and Length severally.

Ch. Lin.

Ch. Lin.

Answer, Breadth = 22.36. Length = 134.16.

Explanation.

First, to the 300 Acres given, I add five Cyphers, and it makes 300.00000; which Sum I divide by 6, the Quotient is 50.00000, the Square Root of which is nearest 22.36 Chains, for the Breadth or shortest Side of such a Parallelogram; which being multiplied by 6, the given Proportion between the Length and Breadth, the Product, (*viz.* 134.16 Chains) is the longest Side thereof.

P R O B. IV.

How to make a Triangle that shall contain any Number of Acres proposed, being confined to a certain Base.

R U L E.

Multiply the Number of Acres given by 100000, and that Product again by 2; then the last Product being divided by the given Base, the Quotient will be the Height of the Perpendicular sought, whose Triangle shall contain the Quantity given.

Example.

A certain Triangle is required, that shall contain 64 Acres: Now supposing the given Length of the Base to be 50 Chains; to know the just Height of the Perpendicular that is needful to contain the said Quantity.

Answer, 25.60 Chains for the Perpendicular sought.

H h

Expla-

Explanation.

This Example, and all others of the like kind, may be readily performed by doubling the given Quantity (*viz.* 64 Acres) and annexing thereunto five Cyphers, the Product is 128.00000; which being divided by the limited Base (*viz.* 50.00 Chains) the Quotient (*viz.* 25.60 Chains) is the Perpendicular Height sought.

N. B. *The Figure or Form of this Triangle, and all others comprehended in this Problem, may be readily and easily discovered by Geometrical Projection, which, for Brevity sake, I am forced here to omit.*

P R O B. V.

To divide a Right-line given into two Parts, which shall have such Proportion one to the other, as two given Right-lines.

R U L E.

First. Add the Length of the two Right lines given, which are so proportionable, and observe their Sum: Then,

Secondly, By the Rule of Proportion direct, say, As the Sum of the two Terms or Right-lines given, is to the whole Line given to be divided into Parts; so is each of the two Terms or Right-lines given, to its proportionable Part, or Segment of the Line given to be divided; which being subtracted from the whole Line, the Remainder will be the other Part or Segment sought.

Example.

Suppose a Field in the Form of a right-angled Triangle, hath for its Base AB 40 Chains, and its Hypothenuse (*viz.* AE = 50 Chains) or Sum of the two Terms or Right-lines given (*viz.* AF = 20 Chains, and FE = 30 Chains :) To know the two Parts or Segments of the said Base, that shall be exactly proportionable to the two Right-lines given.

*Analogy.**Ch.*

As $\left\{ \begin{matrix} AE \\ 50 \end{matrix} \right\}$ to $\left\{ \begin{matrix} AB \\ 40 \end{matrix} \right\}$ so $\left\{ \begin{matrix} FE=30 \\ AF=20 \end{matrix} \right\}$ to $\left\{ \begin{matrix} GB=24 \\ AG=16 \end{matrix} \right\}$ for Answ.

Explanation.

This useful Problem and its Example, is of itself so obvious, that it needs no farther Illustration.

P R O B.

P R O B. VI.

How to divide a triangular Piece of Land, into any Number of Parts, equal or unequal, by Lines proceeding from any Point assigned, in any Side thereof.

R U L E.

First, Find the Number of Chains, &c. contained in the Base or longest Side of the Triangle given; then let the said Base be divided into so many Parts proportionable, as there are Shares or Quantities of Acres given for each Man's Part; which may be accurately effected by the Rule of Three, by making the first Term to contain always the whole Number of Acres given; the second Term, the whole Length of the Base in Chains, &c. and the third Term, orderly, the Number of Shares that each Man is to have; by which Analogies you will discover the proportionable Parts that each Man is to have of the Base.

Secondly, In some Part of the said Base, let there be a Point or Mark assigned, from whence the several Lines of Division may proceed, drawing from thence an obscure Line to the opposite Angle, and then, afterwards, from the proportionable Parts or Divisions of the Base, known by certain Letters or Marks, let there be orderly drawn obscure or dotted Lines, parallel to the first Line or Diagonal, that was drawn to the opposite Angle, as above.

Lastly, From the said Mark or Point in the Base assigned, let there be drawn so many several black Lines, as there are parallel Lines, in such Manner, that each of them may touch the Extremity of the parallel Line that is nearest, as well as the outward Lines of the said Figure; so shall the whole Triangle be divided into so many Parts, as was required.

Example.

Let ABC be a triangular Piece of Land containing 60 Acres, to be unequally divided amongst three Men in such Manner that the first may have 15 Acres; the second 20 Acres, and the third 25 Acres: Now supposing the whole Base, viz. AC, to contain 50 Chains, and the first Line of Division to proceed from the Point D in the Base, to the opposite Angle at B: To know each Man's Part of the Base, and consequently, the Parts proportionable of the Triangle's Surface, agreeable thereunto.

Explanation.

There are several ways of doing this by Geometry, without the Help of Arithmetic; but my Business is not to shew you what may be done; but to shew you, how it may be done the most easy and practical way: Therefore, having found by the Rule of Three, the Length of each several Base, much after the same manner as Merchants discover their Gains by the Rule of Fellowship, it will then follow, that the first Man's Part of the Base is $AE = 12.50$ Chains, whose parallel Line to DB is EH ; the second Man's Part of the Base is $EF = 16.66$ Chains, whose Parallel-line to the said Diagonal-line DB , is FG ; then consequently, the third Man's Base must be $FC = 20.84$ Chains. Hence it is, that the two black Lines, viz. DH and DG , will divide the whole Triangle into three Parts, as was was required.——In like manner, all other Triangles may be thus divided, let the Number of Parts be more or less.

Note, The Figure of this Example, and all others relating to Geometrical Division and Reduction, is left intirely to the Learner's Exercise and Improvement; seeing the Rule, Example, and Explanation shew the Method of Performance.

P R O B. VIII.

How to reduce a Trapezium into a Triangle, by a Line drawn from any Angle thercof.

R U L E.

Admit the Trapezium given is $ABCD$; and it is required geometrically, to reduce the same into a Triangle.

First, Extend the Line DC , and draw the Diagonal BD , from one opposite Angle to another.

Secondly, From the Point A , draw the Line AE parallel to D , extending it till it cut the Side CD , extended in the Point E .

Lastly, From B draw the Line BE , constituting the Triangle EBC , which shall be equal to the Trapezium $ABCD$; as will evidently appear by the Figure, when projected by the judicious Reader.

P R O B.

P R O B. VIII.

How to reduce an irregular Plot of five Sides into a Triangle, which shall contain the same content by Lines drawn from any Angle thereof.

R U L E.

This Problem being well observed, may serve as a Foundation to all others, though the Figures consist irregularly, of ever so many Sides and Angles: To effect which,

First, Let the irregular Plot given be $ABCDE$, to be reduced into a Triangle, which shall contain the same Content.

Secondly, Extend the Side AE both ways to F and G ; and from the Angle C , draw the Lines CA and CE to the Angles A and E .

Thirdly, From the Point B , draw the Line BF parallel to CA , cutting the extended Side AE in F : Also from the Point D , draw the Line DG parallel to CE , cutting also the extended Side AE in G .

Lastly, From the Angle C draw the Lines CF and CG , constituting the Triangle CFG ; which is equal to the Plot $ABCDE$.

O B S E R V A T I O N S.

Note 1. As the Author of *Surveying Improved* in Chap. IX. Page 158, has treated very accurately upon several useful Problems, relating to the dividing and reducing of irregular Plots of Land; so I have here annexed upon this Subject such Illustrations and Improvements, as may happily tend to the Reader's Benefit and Satisfaction.

2. Although Brevity, and the Smallness of this Compendium, has constrained me to omit the Figures of the several Problems of this Chapter; yet the judicious Reader will find the Rules, Examples, and Explanations, so plain, easy, and complete, that he may occasionally project the same with all imaginable Exactness and Facility.

P R O B. IX.

How to divide an irregular Plot of Land, consisting of any Number of Sides, according to any given Proportion, by a streight Line through it, the easiest way.

R U L E.

There are several Ways of performing this Problem, according to the various Methods of Geometricians, but none that I know of, more plain and easy than the following.

First, Let there be supposed an irregular Field in a Pentagonal Form, viz. ABCDE; and let it be required to divide the same exactly between two Men in equal Halves, by a Line of Separation.

Secondly, For the Performance hereof, let a blank or obscure Line, (viz. AF) be drawn by guess thro' the Figure, from any Vertical Angle, supposing that of A, until it bisect the opposite Side or Base ED, near the Middle thereof, as at G, by the said obscure or dotted Line of Separation AF.

Thirdly, the whole Figure being thus divided, as above, let the Content of either Half, especially that on the Left-hand, be exactly cast up; and observing what it wants, or what it is more than the Half should be, let the said Quantity be so comprehended in a Triangle, that when it is added to that Part of the Figure where the Defect lies, the true Line of Separation may thereby be discovered.

Lastly, To make this Triangle, let, 1st, the blank or dotted Line (viz. AG) be measured in Chains and Links, and reserved always for a Divisor; then 2^{dly}, for a Dividend, let the Content in Acres, &c. which is to be comprehended in a Triangle, be doubled and then reduced into Square Links, by annexing thereto five Cyphers, and dividing the same by the said Divisor, the Quotient, in Chains and Links, is the Perpendicular of the Triangle sought (viz. HI) which being taken from your Scale and its Extent so set from the Base AFG, that the End of the Perpendicular may just touch the Base or Side ED of the given Figure, which will be at I; then from the opposite Angle at A, draw the Line AI, which make the Triangle AGI, to contain the Number of Acres that is deficient, or wanting, and divides the whole Figure as desired,

Example,

Example.

Admit a Field of five unequal Sides, containing 46 Acres, is required to be divided into equal Halves between two Men; and supposing the streight Line drawn by guess through the Figure to be AF ; to know the just Length of that Line, that shall proportionably divide the same Figure into two such equal Parts, as each may contain just 23 Acres.

Answer, $AI =$

Explanation.

1. After the streight Line AF is drawn accidentally, or by guess, thro' the Figure, it cuts off from the whole Figure $ABCDE$, the Triangle AEG remaining on the Left-hand, which being measured is found to be 15 Acres.

2. Now there being 8 Acres wanting of the true half more than AEG , it is needful, according to the Rule, to make another Triangle (*viz.* AGI) whose Base and Perpendicular shall likewise depend upon the same Side of the Figure ED , containing just 8 Acres; which being added to the Triangle first found, *viz.* AEG , the whole Figure is thereby divided into two equal Parts by the Line AI sought.

3. If it had been required to have set off the Perpendicular the other way, you must still have made the End of it but just touch the Side or Line ED , as LK does: For the Triangle AKG is equal to the Triangle AGI , each containing 8 Acres.

And thus you may divide any Piece of Land, of ever so many Sides and Angles, according to any Proportion, by streight Lines through it, with as much Certainty, and more Ease, than the other ways frequently used.

P R O B. X.

To reduce customary Measure to statute, and the contrary.

R U L E.

I. The Area or Superficial Content of any Piece of Land being given, according to one kind of Pole or Perch, to find how much the same Piece of Land would contain, if it were measured with a Pole or Perch of another Length differing from the former.

H h 4

II. In

II. In several Parts of *England*, as well as in the Principality of *Wales*, the customary Measure made use of, differs much from that which is appointed by the Statute; seeing in several Places, they account for Wood-lands, Forests, &c. 18, 21, 24, 28, &c. Feet in Length to the Pole or Perch: Whereas, on the contrary, our true Measure for Land, by Act of Parliament, is but 160 Square Poles or Perches for one Acre, at $16\frac{1}{2}$ Feet in Length to the Perch: Therefore to reduce the one into the other, you are to observe,

III. That like Planes are in Proportion one to another, as are the Quadrats or Squares of their Homologal Sides; and, as such, it follows, by Analogy, thus:

As the Square of the Perch of one Sort of Measure given, is to the Square of the Perch of the other required, so is the given Content of the one Measure, to the Content required of the other.

Hence it is, that the given Content in Acres, &c. must always be multiplied by the Number of Square Feet in that Pole or Perch the Ground was measured by; and the Product divided by the Number of Square Feet in the Pole or Perch, it must be reduced into; the Quotient, in this Case, will always be the Number of Acres, &c. desired.

But when it happeneth, as frequently it doth, that the given Content contains Acres, Roods, and Perches; then must those three Denominations be first reduced into the least Name mentioned, before Multiplication and Division can be performed, as above, the Quotient, in this Case, will be always (for Answer) the Number of Square Poles or Perches sought, which may be accurately reduced into Acres, &c. by *Chap. II. Sect. 7.* of this Treatise, which see.

Example I.

Suppose a Field measured by a customary Perch of 18 Feet in Length, accounting 160 Square Poles or Perches to the Acre, and allowing the Content of the said Field to be 96 Acres, 3 Roods, 27 Perches: How many Acres, &c. shall the same Field contain by the Statute Perch of $16\frac{1}{2}$ Feet in Length.

Sq. Perch. A. R. P.

Answer, 18455 = 115 — 1 — 15 +.

Example

Example II.

There is a Field measured by the Statute Perch, containing $16\frac{1}{2}$ Feet in Length: Now allowing the Content of the said Field to be 115 Acres, 1 Rood, 15 Perches +, to know how many Acres, &c. shall the said Field contain, when measured by the customary Perch of 18 Feet in Length.

Sq. Perch. A. R. P.

Answer, $15507 = 96 - 3 - 27 +$.

Explanation.

I. Example 1. the given Content (*viz.* 96 Acres, 3 Roods, 27 Perches) being reduced into the least Name mentioned, (*viz.* Perches,) make 15507 Square Perches; which being multiplied by 324, the Square of 18 Feet, or 1 Perch in Length customary Measure, and divided by 272.25, the Square of $16\frac{1}{2}$ Feet, or 1 Perch in Length, Statute Measure, the Quotient is 18455 Square Perches; which being reduced, make 115 Acres, 1 Rood, 15 Perches +, for Answer, as above.

II. In Example 2. the given Content (*viz.* 115 Acres, 1 Rood, 15 Perches, or 18455 Square Perches) being multiplied by 272.25, the Square of the Statute Perch of $16\frac{1}{2}$ Feet in Length, and the Product divided by 324, the Square of the customary Perch of 18 Feet in Length, the Quotient is 15507 Square Perches; or 96 Acres, 3 Roods, 27 Perches, for Answer, in customary Measure; which is exactly the Converse of Example 1. and consequently, an undeniable Proof to the same.

— In like Manner, all other customary Measures whatsoever, may be thus reduced into Statute, *vel è contra*; regard being always had unto the proper Multiplier and Divisor, according to the Rule above.

OBSERVATIONS.

1. *Note*, The proper Multiplier and Divisor for either Statute or customary Measure, may likewise be found and expressed in Integral Numbers, no less concise, by reducing them both into one Denomination, and then abbreviating them, until they are brought into their least Terms, and afterwards into their Quadrats or Squares, by multiplying each of them by itself, they then become respectively Integral Numbers proportionable to

to one another, which, with less Trouble in working, will produce the same Effect as before. — As for Instance, the Statute Pole in Length, as well as the customary, being both of them reduced into Halves; because one of them retains that Fraction, they make $\frac{3}{4}$, which being abbreviated and brought into their least Terms, make $\frac{11}{12}$, whose Squares particularly are 121 and 144; that is to say, the Factor for the customary Pole of 18 Feet in Length, is always 144, when the Statute is but 121. — In like Manner, you may likewise discover proper Factors, for all other customary Measures whatsoever.

2. When the given Content of any Field happens to be only Acres, there needs no Reduction, seeing the Answer will likewise be Acres; but when otherwise there is given Acres and Roods, or Acres, Roods, and Perches, they must first be reduced into the least Name mentioned in order to Operation, as above.

P R O B. XI.

Knowing the Content of a Piece of Land, to find what Scale it was plotted by, supposing the Scale to be either lost or concealed that was first used in taking the given Plot.

R U L E.

This Problem is not only very useful, but even curious in finding out the Divisions of the original Scale that the Figure of the given Content was plotted by; seeing any other Scale upon a second Protraction, will find it either more or less than just: Now, for a true Discovery of the same, you are first, by any Scale whatever, to measure the Content of the given Plot, and then proceed by way of Analogy, in arguing thus:

As the Content last found, is to the Square of the Scale I tried by; so is the true Content given, to the Square of the true Scale it was plotted by.

Hence it is that if the original Content of the Plot given be multiplied always by the Square of so many Divisions in an Inch of that Scale you measured last by, and the Product divided by the second Content last found, the Quotient will be always the Square of that Scale inquired for, whose Square Root declares, how many Divisions to the Inch is contained therein.

Example.

Example.

Suppose there is given the Figure or Plot of a Piece of Land, containing exactly 8 Acres; but upon a second measuring of it, by a Scale of 12 Divisions in an Inch, it is found to contain 11 Acres, 2 Roods: To know by what Scale the original or first Content was plotted by.

Answer, By the Scale of 10 Divisions to an Inch.

A. Pa. *Sq. Pa.* *A.* *Sq. Pa.*

Analogy, As 11.5 : 144 :: 8 : *Answer,* 100 Root = 10.

Explanation.

By the Rule above, the original Content, or Quantity given (*viz.* 8 Acres) being multiplied by the Square of the Scale you measured last by (*viz.* $12 \times 12 = 144$) and the Product (*viz.* 1152.0) divided decimally by the Content last found (*viz.* 11.5 Acres) the Quotient is the Square of the Scale inquired for (*viz.* 100) whose Square Root (*viz.* 10) is the Number of Divisions in an Inch thereof: Therefore, I conclude, that the original Plot was made by a Scale of 10 Divisions to an Inch. — In like Manner, all other Scales may be thus discovered.

P R O B. XII.

Concerning several Errors in measuring of Land by the Vulgar, too frequently practised and depended upon, tho' most justly refuted and detected.

R U L E.

As the Chain and measuring Pole, here recommended to use, must needs meet with the Approbation of the Candid and Judicious, particularly the honest Countryman, for whom this Essay is principally intended; so I am persuaded their Accuracy and Veracity will likewise appear, if not confounded with those false Methods and erroneous Notions, that Custom has ren-

rendered too prevalent, for which Reason, they are explained and detected as follow. And,

I. There are some who say, that if two Pieces of Land are of an equal Number of Poles, Perches (or other Measure) about, that those two Pieces of Land do contain equal Quantities of Ground; but this will prove evidently false from *Reason I.* hereafter.

II. In measuring of Ground for ploughing, sowing, mowing, reaping, &c. there are likewise some who erroneously conceive, that, if a Piece of Ground consisting of four Sides (how unequal soever they be) if you add the two opposite Sides one to the other, and take their Halves, those two Halves multiplied together, shall be the true Content of the Piece of Ground, which is also refuted from *Reason II.*

III. There are also others, who say, if in a four-sided Piece of Ground, you measure cross the Field from the Middle of one Side to the Middle of the other Side, both ways; that those two Lengths multiplied into each other, shall give the true Content of that Piece; which is likewise confuted from *Reason III.*

An Explanation and Detection of the three pretended Axioms above, by Reasons, &c.

R E A S O N I.

1. For a Refutation of the first pretended Axiom, let there be a Piece of Ground in the Form of a true Square, whose Angles are all Right, and every Side thereof to contain 80 Perches, it then follows, that 4 times 80 is 320; and so many Perches is that Piece of Land about: And 80 Perches, one of the Sides, being multiplied in itself, that is 80 by 80, the Product will be 6400 Perches, or 40 Acres; which is the true Content of this Square Piece in Perches.

2. Again, let there be another Piece of Ground in the Form of an oblong or long Square, whose Angles are likewise right, but the Length thereof is 110 Perches, and the Breadth 50 Perches. Now this Piece of Land is as much about as the other; for twice 110, and twice 50, added together, make like-
wise

wise 320 Perches for the Perimeter: But if you multiply 110 the Length, by the Breadth 50, the Product will be 5500 Perches; which is less than the former Area by 900 Perches, or 5 Acres, 2 Roods, 20 Perches; which renders this Maxim entirely false.

3. Suppose then, that a Piece of Land retaining the same Length in the Sides, and the same Breadth at the Ends, but two of the Angles obtuse, and two acute, like the Figure of a Rhomboides; it will then follow, that tho' the Perimeter be the same as the last, yet it cannot be so measured, by multiplying the Length by the Breadth; but the Length of the longest Side, by the deepest Distance between those two Sides, which we suppose to be 32 Perches; so that 110, the Length, multiplied by 32, that Distance produceth but 3520 Perches, which is less than the Square by 2880 Perches, or eighteen Acres: A farther Confirmation of the Fallacy, as above.

4. But for Brevity's sake, omitting all irregular Figures, whose Perimeters, respectively, may be made equal to that of the Square; yet, nevertheless, it is observed, that their Areas shall be no Ways agreeable to the same, which is directly contrary to the pretended Axiom, as above: Yet, lest regular Figures should be thought more agreeable to this Maxim, we will consider regular Polygons of any Number of equal Sides, which of all right-lined Figures are most capacious, and therefore must approach nearest the Content of the Square; yet, upon Trial, their particular Areas will be found vastly different, tho' their Perimeters are exactly equal to the same; and if so, it must needs follow, that if a Piece of Land were to lie in a circular Form, whose Periphery being exactly equal to the Perimeter of the given Square, shall nevertheless contain 1742 Square Perches more than the Geometrical Square; because of all plain Figures whatsoever, the Circle is the most capacious.

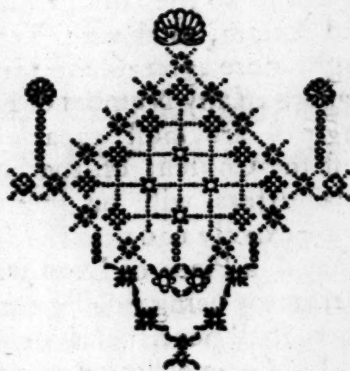
REASON II.

The second pretended Maxim is likewise discovered to be false and irrational, by supposing a Piece of Land to lie in the Form of a Trapezium; yet when its Area is found, according to the pretended Rule above, and compared with the true Way, it will likewise appear to have no better Foundation than the former.

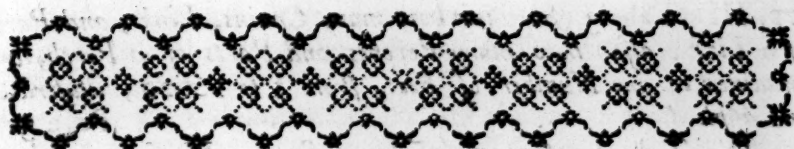
REASON III.

As the third Rule, or Maxim, hath some Affinity with the first and second, and no better Foundation for its Support; so it must likewise be condemned as absurd and irrational, seeing, after all, it is not the Arithmetical, but the Geometrical Mean, that reduces any Trapezium, &c. into a right-angled Parallelogram, whose Contents shall be equal.

From what hath been said, we may observe how that darling favourite Custom prevails, beyond the greatest Reason and Demonstration, for Want of due Examination.



THE



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OF THE

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by the CHAIN only.*

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Summarium Steriometriae :

O R, A

N E W E S S A Y

U P O N

S O L I D S;

COMPENDIOUSLY PERFORMED.

I N T W O P A R T S.

PART I. DIRECTIONS shewing how the Dimensions of Timber Trees, both standing and lying, may be readily taken at one View, without the Trouble and Charge of Mathematical Instruments.

PART II. MEASURING by INSPECTION only; or the Method of finding readily (by a new Table) the *Area* or Solid Content of Timber and Stone, whether round or square, and consequently its Value, with all imaginable Exactness and Accuracy, according to the Dimensions given.

T H E W H O L E

Being newly composed, methodized, and illustrated by proper Propositions, Rules, Examples, Operations, Explanations, &c. performed five Manner of Ways, viz. vulgarly, decimally, duodecimally, instrumentally, and tabularly.

By *WILLIAM HUME, Philomath.*

Doctrina vana est, ratio ni accesserit.

L O N D O N,

Printed in the Year MDCCLXIX.

NEW E S S A Y

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COMPTON'S

И Т Е М

Mathematical Induction. Ask at one View, without the aid of any other, the truth of the proposition of the induction.

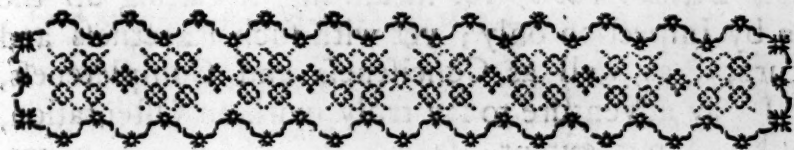
and accounts, according to the location, nature and quantity of the material given the character and character of the material, and the method of finding results (by a new method the method of entering the material into the material).

SECRET

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NO. 10

THE YEAR 1900



T H E
P R O E M.

THAT this Treatise of SURVEYING may be rendered every way complete, I think it will not be unacceptable to the PUBLIC, to treat succinctly of SOLID-MEASURE; especially so far as relates to the *Measuring of Timber*, both standing and lying; and to give an Estimate of the Value of it; seeing it is one of the necessary Qualifications belonging to a Surveyor, who, by Virtue of the same, may give a great Light to Purchasers, whether Buyer or Seller, when hereunto required.

For which Reason, the Author, in the former Impressions of this Book, hath given the Public a Table for the ready casting up the same; but as the Errors of the Press, with other Defects, have prevented it from being so serviceable as was intended; so I think it will not be unacceptable, to grant it such Amendments, Alterations, and Improvements, as will render it acceptable to the Public in general, as well as any learned and judicious Practitioner in particular.

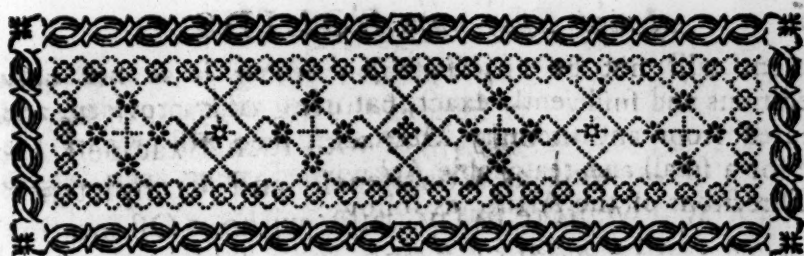
In order to which, I have newly methodized and illustrated his Directions, for taking the Dimensions of Timber-trees, both standing and lying, at first View, by proper Positions, Rules, Examples, &c.

and afterwards have purposely composed or calculated a *New Decimal Table*, for measuring or casting up the same by Inspection only; yet with such Exactness and Accuracy, as well as Conciseness and Completeness, that I may adventure to say truly, without Ostentation, the like is not extant; seeing it is seldom or never, that any thing new is offered to the Press, and from thence to the Public, without a View of making Improvements upon the Subject proposed; and hence, no doubt, but the Emulation of Authors have contributed very much to the Improvement of Arts; and sometimes with that Warmth, that even private Interest is sometimes postponed for a public Advantage.

But, after all, when Work is well done, there wants neither Apology nor Commendation to set it forth; by reason the Reader cannot be supposed to suspend his Judgment, and believe that Truth is false; or that which is erroneous and imperfect, is directly otherwise, because the Author's Preface requests that Favour: Therefore shall leave this Performance intirely to the Judgment of the candid and judicious Reader, whose Satisfaction, as well as the Public Benefit, is sincerely endeavoured by him, who is a true Lover and Admirer of ingenious Arts,

March 28,
1740.

William Hume.



A
NEW ESSAY
UPON
SOLID S.



PART I.
CHAP. I.

How the Dimensions of Timber-trees, both standing and lying, may be readily taken, in order to Mensuration.

SECT. I.

Concerning the Description and Use of the Artificial-pole, &c. in taking the Dimensions of Timber-trees that are standing.



WHEN the Artift or Surveyor is required to take the Dimensions of Timber-trees, standing or growing, in order to Mensuration, it is needful for greater Exactness, that the same should be effected by some proper Instrument. Now, as Quadrants, Sectors, and other Mathematical Instruments for taking Heights and Distances, cannot always be had when wanted, nor indeed easily purchased by Artifts of mean Circumstances, I conceive the following Instrument

strument will not be unacceptable; seeing it is not only expeditious and sufficiently exact, but even easily provided, and procured from any ordinary Mechanic, who will prepare the same for a small and reasonable Acknowledgment, according to the Directions of the Artift, as follow :

The DESCRIPTION.

How to prepare the said Instrument according to Rule and Diagram, in the following Manner.

DIAGRAM.



To effect which, let there be a square Pole provided, as A B, in the Diagram, consisting of five, six, eight, or ten Feet long, at pleasure; and let it be marked at every Foot with brass Nails, or little Notches: And after the Feet are thus equally divided, let them be orderly numbered from the Top downwards, with 1, 2, 3, 4, &c. to the lowest End. Then take a Piece of dry Wainscot-board, and whatever the Breadth be, let the Length be twice as much; as you see in the marginal Diagram, and near the bottom Corner towards the Right-hand, as at c, drive in a small Wire-pin, so as to be fast in the Board, and to stand up about half a Quarter of an Inch above the Surface of the Board; and drive another such, at the other Corner, at d; and draw the Line c d, or imagine it to be so drawn. Then along the Side of the Board, and perpendicular to c d, draw the Line d e g; and upon the said Line set the exact Distance c d, from d to e; and there put another Wire-pin, as at e; and setting the same Distance d e, from e to g, set another Pin at g; and then is the Board ready to put in its Place, that is, to be fixed upon the plain Side of the Pole, the bottom Part c d being about five Feet from the lower End of the Pole; or at such a convenient Distance from the Ground, when the Pole stands upright, that the two bottom Pins, c and d, may be about the Height of your Eye from the Ground; for the Conveniency of looking without stooping much, or discommoding yourself when

when you are upon Observation; which, that it may be done regularly and exactly, the Pole must be fixed perpendicular, by putting a very small Wire-pin in the Middle of the Top of the Board at n ; on which there must be hung the String np ; with the Plummets p at the End of it, and draw the Line np ; and then setting up the Pole, bring the Plummets-string upon the Line np , and then is the Pole perpendicular.

N. B. *It is best to have the Pole a little broader one way than the other, for sake of having a broader Side to fix the Board upon, and yet to be narrower or thinner the other way, in such manner, that it may not too much gripe or incumber your Hand in using or carrying it.*

S E C T. II.

The Use and Application: How the said Instrument may be rendered singularly useful, in taking readily the Height of any Tree, standing or growing, with all imaginable Exactness and Expedition.

HAVING thus prepared and described the said Instrument, as above; you are next to observe its Use, &c. And, *First*, In going to take the Height of a Tree, you are to erect your Pole perpendicular at such a Distance from the Tree, as that you can just see the Top of the Tree, thro' the two Sights, c , and e : which done, measure with your Pole, how many Feet, &c. it is from the Foot of the Pole, to the Root of the Tree; for that is the exact Height of the Tree without Calculation: especially, if there be added thereunto so much as is the perpendicular Height of your Eye above the Ground; because, where the Ground is plain from the Place of your standing, to the Tree growing that is to be measured, you only measure that part of the Tree which is above the Horizontal-line, that passes from your Eye to the Tree: As for instance, suppose you are upon a plain, level Ground, and you can just see, or bring the Top of the Tree in a right-line with the said two Sights c and e when the Pole is perpendicular; your Distance then from the Place of your standing, from the Foot of your Pole to the Root of the Tree, being measured, is found to be 37 Feet; then it follows, that the Height of the Tree is likewise 37; to which

if

if there be added more, 5 Feet 7 Inches, the supposed Height of your Eye from the Ground when Observation is taken; then the complete Height of the Tree sought will be 42 Feet 7 Inches, for Answer.

Secondly, But in case the Trees grow too rank, or near together; or by reason of some other Obstacle, as Brambles, or the like, you cannot conveniently go so far from the Tree, as to bring the Top of it in a Line with the said two Sights *c* and *e*, when the Pole is perpendicular, as above; you may, in this Case, stand so near the Tree, that you may see the Top of it thro' the first Sight *c*, and the top Sight *g*: Which done, measure your Distance from the Tree, and that Distance doubled is just the Height of the Tree, without farther Trouble. As for Instance: Suppose your Distance from the Tree, according to these two Observations, is 19 Feet, for the just Half of its Height, then you may infer, that the Height or Length of the Tree is just its double, *viz.* 38 Feet; which is to be understood without farther Study or Calculation.

Thirdly, It is also to be observed, that when a Tree leans or declines from the Perpendicular, then, in such Case, you must not stand on that Side that it leans towards; for then the lower Part of the Body of it will be farther from you than the Perpendicular let fall from the Top, and make your Distance from it (and consequently the Height of the Tree) too much—Nor should you stand on that Side that it leans from, for then, for the same Reason, you will bring in the Height of the Tree too little, or less than it is; but stand on one Side of it, that it may lean towards the Right-hand or the Left (and not from you, or towards you) and then the Body, or lower Part of it, will be nearly the same Distance from you with the Perpendicular let fall from the Top, and give the true Height of it.

Fourthly, Having found the Height of the Tree, or Length of the Timber, according to the Observations above, your next Business is to find the Girt or Side of the Square equal, according to the customary way, by which is meant the Quarter of the Circumference, which, amongst all Trees, are those that are regularly tall and tapering, and such as have neither been lopped nor topped; then may they be particularly girt within two or three Feet of the Ground, towards the spreading abroad the Root; and to that Girt, or Quarter of the Circumference, add 6, and the Half of that Sum is the mean Girt sought; and, as such, may generally be depended upon in all common Cases, where Trees do so regularly taper, seeing they become (in effect) equal to those whose Girts are taken in the Middle. For suppose you were not to measure a Tree any further towards the
Top,

Top, then it will bear 6 Inches Girt; then let the Thickness of the Tree be what it will below, yet if it tapers regularly, and let it be long or short, the Girt at the Top being supposed 6 Inches, that 6 added to the Girt at the Bottom, the Half of that Sum would be the Girt in the Middle, where you would girt it if it was cut down, according to the common way of measuring.

Example.

Suppose the Top, or small End of a tapering Tree, was always to terminate at 6 Inches Girt; and suppose the Girt near the Bottom or thick End, was 18 Inches (or 6 Feet in Circumference) then 18 and 6 is 24 Inches, the Half of which is 12; which would be the exact Girt of the Tree in the Middle, if it taper all alike.

But in Pollards or Trees that have had their Tops cut off, and their Branches frequently lopped for Fuel, as is the Custom in some Countries, you may commonly, by small discretionary Allowance, making the middle or mean Girt a little less than it is 4 or 5 Feet from the Ground, where you can reach to girt, give a Guess sufficiently exact, some of those Trees being almost as thick at the Top as at the Bottom; or, however, by a little Practice, the Difference may be easily accounted for.

S E C T. III.

Observations very curious and useful concerning the Pole, in taking Heights and Distances.

THE preceding Directions, tho' conspicuously easy and plain of themselves, yet these following Remarks will be found very useful, viz.

First, In taking your Observations by the said Instrument, as above, you are never to take the Height of any Tree, further or higher than it may be called Timber, which Measurers commonly allow at 6 Inches and more, but not less, in the Girt or Quarter of the Circumference; but if you measure for a Gentleman who gives particular Orders for limiting the Girt of what he allows to be Timber, at either more or less than six Inches, his Orders are to be your Directions in that Case.

Secondly,

Secondly, You are likewise to have regard to the Situation or Position of your Pole, as well as the Place upon which the Tree standeth or groweth; for whether the Ground be plain or declining, it is the most exact (having fixed your Pole perpendicular) to look first at the Bottom of the Tree, by your horizontal Sights, cd ; and there cause your Assistant to make a Mark upon the Tree with Chalk, or the like; and having found the Height of the Tree above the said Mark, by the foregoing Directions, measure that below the Mark with your Pole or Rule, and add it to the Height found above the Mark, the Sum is the true Height of the Tree.

Thirdly, It must also be observed, that the Distance from the Tree must always be esteemed the Height or Half the Height of the Tree, without farther Trouble: For if you see the Top of the Tree, or rather the upper Part of the Trunk or Body, so far as may be termed Timber, by the Sights ce , your Distance from the Tree is just the Height of the Tree; but if you are nearer the Tree, and see the Top of it, as before, by the Sights eg ; your Distance then from the Tree is just Half the Height of it, according to the preceding Examples, which render the Directions so plain, that I need not multiply Instances; and so easy, as to be intelligible to any common Capacity that can but read and write, seeing this no way depends upon Mathematical Calculation.

S E C T. IV.

A Geometrical Reason and Demonstration, shewing why the Pole may be made a proper Instrument for taking Heights and Distances.

THE Truth and Reason of this easy Method of taking the Height of a Tree, will manifestly appear by Demonstration, especially to those who understand Geometry. For it is proved (*Euclid, Lib. VI. Prop. 2.*) that any Line drawn parallel to any Side of a right-lined Triangle, cuts the other two Sides proportionally: (See *Prob. XIV. Page 58. and Fig. 41.*) Now, since the Line cd , continued to the Tree, is horizontal (because the Line np is perpendicular, and consequently the Line dg , which is parallel to it and the Tree) is also [supposed] perpendicular; it will, therefore, be as the Base cd upon the Board

Board to the Perpendicular $d e$ upon the Board (which, by the Construction of the Board, are equal: So the Base (the Distance from the Pole to the Tree) to the Perpendicular (the Height of the Tree) are consequently also equal.

And for the same Reason, when you use the first Sight c , and the highest g , where the Perpendicular $d g$ is just double to the Base $c d$; by Construction, the Height of the Tree is just double your Distance from it, which was also to be proved, and is further confirmed by *Euclid, Lib. VI. Prop. 4.* which, though I shall not enlarge upon, because those Directions are intended for common Capacities, yet it is proper to take notice of, for the Satisfaction of the more Learned and Curious. —The same Truth holding good in all Proportions, as well as where the Bases and Perpendiculars are equal, or where they are double to one another (as in *Fig. 93*) it is, as Base $A E$ to the Perpendicular $E F$; so the whole Base $A B$, to the greater Perpendicular $B C$.

S E C T. V.

To take a near and general Estimate of many Timber-Trees, standing or growing in a Wood, when Opportunity permits not the Survey of each Tree in a particular Manner, as above.

WHEN it happens that a Gentleman has a View of selling his Timber standing, or growing in a Wood, without such a particular Survey as the measuring of every Tree, as above; then to avoid Tedioufness, as well as for the sake of Expedition, a general Estimate of the Quantity and Value of the Timber may be deemed sufficient, according to the Directions following.

1. In order to which, let your Pole be fixed as far from the Tree, as you imagine or conceive the Tree to be high; especially when you intend to take the Top of it by the Sight $c e$; but if upon trial you observe that you are rather too near, and instead of seeing the Top of the Timber-part of the Tree, you see two or three Feet below it, yet you need not remove the Pole any further back, but guess at the Height of that Part which is above the Part which you see by the Sight of c and e , it being
easy

easy to guess within Half a Foot in 3 or 4 Feet ; and half a Foot in Length at the Top, where the Tree is supposed but 6 Inches square, being but one Eighth of a Foot, is but inconsiderable in such a Case. — It is therefore best to place your Pole rather too near the Tree than too far off ; but yet it is better to place it as near the right Distance as you can ; which a little Practice will make you more ready at, than many Instructions.

2. Having by your Instrument, obtained the Height and Girt of each Tree, as before directed, let them be orderly numbered and placed under their proper Titles and Columns in your Memorandum-book ; by which means you will find methodically and readily, the Solid-content of the same by your Table, when you return home.

3. When the Timber in a Wood, or Part of a Wood, is mostly of one Growth, you may frequently find 6, 8, 10 or 12 Trees, or more, so near of one Dimension, that you may measure one of the middling ones for all ; and so let your Assistant mark them as fast as he can go to them, while you write them down : And thus, in case of so much Haste, you may measure as fast as he can mark, by which means you may write and cast off a vast quantity of Timber in a little Time.

4. If this Method be not thought expeditious enough, and a more general Estimate will answer the Purpose, it is best to take a Walk round the Ground with your Instrument ; and, with as much Exactness as your Time will permit, find the Quantity of Ground contained in the Wood, by the Directions given, *Part IV. Chap. 4. Page 129*, which done, measure off Half an Acre, or an Acre, in some middling Part of the Wood, where the Timber is neither the best nor the worst, and measure the Trees therein contained : Which done, multiply the Feet in that half Acre, &c. by the Number of half Acres in the Wood ; it will serve for a general Estimate of the Quantity, and consequently, the Value of the Timber in the whole Wood, where Time will not permit for a more exact Admeasurement.

5. If the Gentleman have a Map of the said Wood, or has had it surveyed, and can give an authentic Account of the Content, it may save the Trouble of taking any Survey at all, as to the Quantity of Land the Timber grows upon.

6. Where still more Expedition is required, the experienced Surveyor may, by Practice, give a great Guess, by a general View of the Wood ; but in that Case, I need not give any particular Direction ; Experience best teacheth the Surveyor how to perform, and the Circumstance of the Affair best shewing him

him how far to dispense with strict Exactness for the sake of Expedition.

But when you have Time to be exact, which is most to be wished, both for the Satisfaction of the Gentleman, and the Credit of the Surveyor, keep a little Book, in which set down orderly, in proper Columns, according to their Titles, *1st*, The Number of Trees; *2^{dly}*, The Length of each Tree in Feet, &c. *3^{dly}*, The Girt or Circumference, taken in Feet, &c. *4^{thly}*, The Square or *4th* Part of the Girt, in Inches, &c. And *5^{thly}* or *lastly*, The solid Content in Feet and Decimal Parts: So have you orderly the Content of each Tree; and casting up every Column every Day, or at what convenient Opportunity you shall think fit; carrying one for every ten found in the Decimal Parts of the Feet, as in common Addition: and when you have added all the Feet together, divide the Sum by 40, the Feet in a Tun, the Quotient is Tuns of Timber in the whole; or if you divide by 50, the Feet in a Load, the Quotient is Loads in the whole Quantity, and the Remainder is Feet, or the Fractional Parts of a Load.

N. B. *In measuring or taking the Dimensions of each Tree, it may be proper to number it (by Chalk, or some such thing) with the same Figure that you enter into your Memorandum or Pocket-book; by which Means you will know those that are measured, from those that are otherwise.*

SECT. VI.

The Method for using the Theodolite, or Mathematical Instrument for taking Heights and Distances.

ALTHOUGH the Directions in the preceding Propositions are performed without any Trouble of Calculation, which render it more agreeable to the Understanding of those who aim at Surveying of Timber in general, as well as in particular: But those Gentlemen and others, who chuse to use the Theodolite, may see likewise its Performance by the following Directions.

Let the said Instrument be placed vertically or pendicularly to the Horizon, at any possible or convenient Distance from the

K k

Tree;

Tree; and having fixed it horizontally, take the Angle that the Top of the Tree (so far as it is Timber) makes with the Horizon; and then measuring the Distance from the Instrument to the Tree (with the Allowance before-mentioned, for the Height of the Instrument) the Analogy or Proportion is,

As Sine-Complement of the Angle,
To the Distance of the Tree;
To Sine of the Angle,
To the Height of the Tree.

Note, *The same Proportion holds in the Height of the Tower; for as Sine-Comp. A (or Sine ACB) to Side AB; so Sine CAB to Side BC, &c. And the same Proportion holds in the Height of a Tree, Tower, Steeple, or any inaccessible Altitude.* Fig. 93.

S E C T. VII.

Of Timber-Trees felled or lying; how their Dimensions may be accurately taken, in order to Mensuration.

IF the Tree to be measured happens to be one that regularly and straitly tapers from the Butt-end to the Extremity of the small; or so far that it may, at least, contain 24 Inches Girt, being the least Circumference that is usually taken for Timber; then in such Case, the Dimensions may be accurately taken as follow,

1. Let the Length of the said Tree be measured from the Root or Butt-end, to the Extremity of the small End, or so far as it may, at least, contain 24 Inches girt; which may be exactly measured with a Two-foot Rule, or rather a Pole, orderly divided into Feet and equal Parts, and accordingly noted your Memorandum-book.

2. With a Packthread, or Chalk-line, girt it in 3 several Places, viz. at each End, as well as at some Place near the Middle; then it will follow that the Sum of those three Girts or Circumferences in Inches, and Quarter Inches, being divided by 3, the Quotient is the Mean-girt, or Circumference sought, whose

whose 4th Part is the Side of the Square-equal, according to customary Measure; which being likewise set down in your Book, as above.

Or otherwise thus:

In all tapering Timber that carry not the same Square from End to End, throughout the whole Piece, the Mean-girt, or Circumference, may be taken in the Middle of the Tree, especially if the same appear to be strait from End to End: Or it may likewise be effected by taking one Fourth of the Girt of each of the Ends, and adding them together, let half of their Sum be taken for the Side of the Square, which the Tree will carry throughout for the second Dimension sought.

3. The Length being thus found in Feet, &c. and the Side of the Square equal, or the fourth Part of the Mean-girt, or Circumference in Inches, &c. you have obtained the proper Data for finding readily the solid Content, either by the Table or the Pen, according to Pleasure.

Example.

Admit there is a Tree 36 Feet long, regularly tapering from the Butt-end to the small, or only so far as its least Girt or Circumference may contain 24 Inches, and not under: To know its proper Dimensions.

Explanation.

1. Suppose the Tree to be measured, and its just Length found to be 36 Feet; let it be noted down in your Memorandum-Book, as above.

2. Admit its Girt and Circumference at the But-end to be $56\frac{3}{4}$ Inches; the 2d Girt, or that near the Middle, $38\frac{1}{2}$ Inches, and the least, at the small End of the Tree, only $26\frac{1}{4}$ Inches; to know the Mean-girt or Circumference, and consequently the 4th Part or Side of the Square equal, according to customary Measure.

		<i>Inches.</i>
Girts at	{ But-end	$= 56\frac{3}{4}$
	{ Middle	$= 38\frac{1}{2}$
	{ Small End	$= 26\frac{1}{4}$
	Sum	$= 121\frac{1}{2}$

Inch. Pa.

2dly, $121\frac{1}{2}$, or $121.5 \div 3 (= 40.5$ for the Mean-girt sought.

Inch. Pa.

3dly, $40.5 \div 4 (= 10.125$ for Side of the Square-Equal.

K k 2

Answ.

Inch. Pts.

Answer, $40\frac{1}{2}$, or 40.5 for Mean-girt sought, whose 4th Part or Side of the Square-equal is 10.125, or 10 Inches only; the Fraction being so small that it may be omitted, seeing it is so inconsiderable. Hence the *Data* now found, hath for its entire Length 36 Feet, and the Side of the Square-equal, or one 4th Part of the Girt, 10 Inches; which is all that is required in order to find the solid Content of the said Tree, by either the Table or the Pen, according to *Chap. II. Sect. 3.* which see.

S E C T. VIII.

How to take proper Dimensions when the Tree to be measured happens to be irregular, as to bulge in the Middle; or to contain from End to End many Knots, and consequently diverse Thicknesses.

First, **L**ET the Length be always found in Feet, &c. as before directed.

Secondly, For the Mean-girt or Circumference sought, let it (for greater Exactness) be orderly girt in four several Places, equally distant from one another, *viz.* the first Girt beginning at the Butt-end; the second about a Quarter, or fourth Part of the Length of the whole Tree distant from the first; the third Girt, a fourth Part more distant from the second; and the fourth or last Girt at the Extremity or small End of the Tree, when long: Then having found the Sum of those four Girts in Inches and Quarter Inches, and divided their Total by the Number of Girts (*viz.* four) the Quotient is the Mean-girt, or Circumference sought, whose fourth Part is the Side of the Square-equal, according to customary Measure, and consequently, the second Particular in the *Data*; as before.

Or otherwise thus:

Measure the Diameter or Thickness of the round Piece of Timber, &c. at each End in Inches, &c. and having found the Difference between those Diameters, let Half of it be taken and added to the less Diameter given; the Sum is the mean Diameter, or Thickness, found in the Middle of the Stick, or round Piece of Timber, &c. which being multiplied by 22, and the Pro-

Product divided by 7, the Quotient is the Mean-girt, or Circumference sought, whose fourth Part is the Side of the Square-equal, for one of the Dimensions needful as before.

Example.

Admit a Tree so knotty and irregular as to consist of diverse Thicknesses: Now it happeneth, that the Dimension of its Length is found to be $58\frac{1}{2}$ Feet; but its four several Girts or Circumferences, at the Butt and small End, as well as the intermediate Spaces between, are orderly 68, 46, 32, and 27 Inches: To know the Mean-girt or Circumference, and consequently, the Side of the Square-equal?

Inch. Pts.

Inch. Pts.

Answer, 43. 25, for the Mean-girt sought, and 10. 81, for the Side of the Square-equal thereto

Explanation.

This Example, and all others of the like kind, is so obvious, that there needs no further illustration; only it may be observed, that the more Girts that are taken, the more exactly you find the Mean-girt; and consequently, the Side of the Square-equal, which is always found by dividing the Sum of the Girts in Inches and Quarter Inches, by the Number of Times the Girts were taken; the Quotient is always the Mean-girt sought, as above; whose fourth Part is always the Side of the Square required, as before.

How to measure, or take regular the Dimensions of Timber or Stone, that is square or quadrangular, the Sides being either equal or unequal.

1. **H**AVING taken exactly the Length in Feet, &c. as before directed, let it be accordingly noted in your Memorandum-book, for the first Dimension given.

2. For the Side of the Square-equal, let the Breadth and Thickness, as respectively measured in Inches and Quarter Inches, by clapping two Rules, or other freight Things, to the Side of

the Tree or Stone, that is duly prepared by the Ax or Saw in cutting off the Slabs, or (more properly speaking) the Segments of the circumscribing Cylinder; then it follows, that the Distance between the two Rules, or Things, as above, will be the Side of the Square sought; provided the Dimensions of the Breadth and Thickness are found to be exactly equal; but if otherwise they must be reduced to Equality, as follows:

3. If the Dimensions of the two Breadths, or rather the Breadth and Thickness, are found to be unequal: Then must a Geometrical Mean between them, be exactly found, by multiplying them together, and extracting the Square-root of their Rectangle or Product, according to the Method prescribed in *Chap. II. Sect. 7.* you will thereby find the said Root, or Side of the Square-equal, for the second Dimension sought.

Example I.

There is a Piece of square Timber or Stone, in the Form of a Parallelopipedon or Long-cube, whose Length is found to be 15 Feet; and its Breadth and Thickness so equal, as each of them to contain $7\frac{3}{4}$ Inches. To know the second Dimension sought, or Side the Square-equal.

Answer, $17\frac{3}{4}$ Inches.

Explanation.

This Example needs no Operation, seeing the given Sides, or rather the Breadth and Thickness, are agreeably equal; and consequently, any one of them shews the second Dimension sought

Example II.

Admit a Piece of Timber or Stone is 72 Feet long; but the Sides are so unequal, as one of them to contain $7\frac{3}{4}$ Inches, and the other $19\frac{3}{4}$ Inches: To find the Geometrical Mean, or Side of the Square, equal to the Rectangle, or Product of the two given Sides.

In. Pts. In. Pts. Sec.

Answer, 11.966, &c. = 11 — 11 — 7 + or 12 Inches *ferè.*

Explanation.

In this Example it may be observed that the Arithmetical Mean, used by some Artificers, is more or less erroneous, according as the greater Side exceeds the lesser. For if the two Sides given (*viz.* $7\frac{3}{4} + 19\frac{3}{4}$ Inches) be added together, and Half of that Sum (*viz.* $13\frac{5}{2}$) be taken from the Arithmetical

Mean, appears to be at least $1\frac{1}{2}$ more than just; whereas on the contrary, the Geometrical Mean being found, according to the third Article of this Rule, and *Chap. II. Sect. 7.* it will only be 12 Inches for the true Side of the Square, and consequently, the 2d Dimension sought, in order to find the Solid-Content.

OBSERVATIONS.

I. It is observed, that the Method of taking one fourth of the Mean-girt, or Circumference of round Timber, for the Side of the Square-equal, is so very erroneous, that it finds the solid Content near one fifth Part less than Truth; yet nevertheless, Custom has made it current; and the Reason for continuing the same, is because the fourth Part of the Mean-circumference or Girt, being squared and multiplied by the Length, the Product is but near equal to the Solidity of such Parallelopipedon or Long-cube, as may be inscribed in a Cylinder of the same Dimensions, whose Content is the true Solidity desired.—Hence it is, that the fourth Part of the Girt or Circumference, tho' less than the Side of the Square-equal, yet is somewhat greater than the Side of the Square within, towards which most Timber is hewn, before it can serve to any Square-uses, by making the round Tree, as it were, square, or square-hewed Timber, from quartering of the String or Chord; which (in effect) cuts off almost all the Slabs, or more properly speaking, the Segments of the circumscribing Cylinder; which seem to be a reasonable Allowance, originally and principally attended for the Buyers of green and sappy Oak-timber; tho' others since, have made it so general and universal, as even Artificers, when they measure for Sales, do commonly cast away (for Oak) an Inch out of the Square, or fourth Part of the Mean-circumference, as an Allowance for the Thickness of the Bark; that is, if the Square or the fourth Part of the Mean-girt, or Circumference, be 10 Inches, they measure it, as it were but 9; tho' this Allowance must be esteemed too much for Ash, Elm, Beech, &c. tho' it passes currently for Oak.

II. It is usual with Carpenters and other Artificers, to take the Arithmetical Mean; that is, the half Sum of any two Sides given; or otherwise the Half-sum of the given Breadth and Thickness of any Piece of Timber or Stone; but Reason and Experience manifest in this Case, that is not the Arithmetical but the Geometrical Mean, that finds the Side of the Square-equal; which being multiplied by the given Length, discovers the Solid content desired: Therefore, if the Artist can by either the Pen or Sliding-rule) extract the Square root of the

Rectangle, or Product of the two given Sides, *viz.* the Breadth and Thickness found in the Middle of the Piece, he will there find truly the Geometrical Mean, or Side of the Square-equal.

III. The regular Way of taking Dimensions of Timber and Stone, when the Sides are unequal, is by clapping two Rules, or other streight Things to each of the two Sides, found in the Middle of the Piece; and measuring the Distance between them in Inches and Quarters of an Inch. Also in the same Manner is taken the Breadth and Thickness of the Piece of Timber or Stone; and finding the Geometrical-mean between each of the two Distances, as before, you will thereby discover the Side of the Square-equal; which, with the given Length, is the proper *Data* for finding the Solid-content sought.

IV. When it happens that the Piece of squared or hewn Timber or Stone, hath unequal Bases at each End; that is be greater at one End than the other; then in such Case, it is necessary to multiply the greater Base (*viz.* the Rectangle, or Product of the Breadth, in the Depth or Thickness) by the less; and having extracted the Square Root of that Product, let the Sum, or Product of the two Bases, and the Square-root as above, be multiplied by one third Part of the Length of the Solid propounded; so shall the last Product be the Solid-content required.

V. In all Examples of this kind, as well as others, it is observable, that the Answer or Solution to the Question must always consist of the same Names or Denominations that the Question was given in, or made agreeable to the same by Reduction.

VI. You are likewise to observe, that any Piece of round Timber that tapereth; or any solid Body that resembleth the Frustum of a Cone, as that in the fourth Article does that of a Pyramid's Frustum of a Quadrangular-base; only regard being had, to find, (as before) the Mean-area between the Surfaces of the Cone's two circular Bases, by multiplying the Square of each of their Circumferences by (0.07958) the Area of a Circle, whose Diameter is Unity, gives the Surface of each of the said Bases; which being multiplied together, and the Mean-area, or Square-root of their Product being added to the Sum of the said Bases, and their Total, divided by one third Part of the given Length, gives the true Solidity sought.



S E C T. X.

T A B L E S

F O R T H E

Ready finding by Inspection

T H E

C O N T E N T of any P I E C E

O F

T I M B E R and S T O N E.

T H E

Girt or Side of the Square, as also the
Length, being known.



Feet

Feet long	Side 6 Inc.	Side 6 $\frac{1}{4}$	Side 6 $\frac{1}{2}$	Side 6 $\frac{3}{4}$	Side 7 Inc.
	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	0.2500	0.2712	0.2934	0.3164	0.3402
2	0.50	0.5425	0.5869	0.6329	0.6805
3	0.75	0.8137	0.8803	0.9493	1.0207
4	1.00	1.0850	1.1738	1.2668	1.3610
5	1.25	1.3562	1.4672	1.5832	1.7012
6	1.50	1.6275	1.7607	1.8997	2.0415
7	1.75	1.8987	2.0541	2.2161	2.3817
8	2.00	2.1700	2.3476	2.5326	2.7220
9	2.25	2.4410	2.6410	2.8490	3.0622
10	2.50	2.7125	2.9345	3.1655	3.4025
11	2.75	2.9837	3.2279	3.4819	3.7427
12	3.00	3.2550	3.5214	3.7984	4.0830
13	3.25	3.5262	3.8148	4.1148	4.4232
14	3.50	3.7975	4.1083	4.4313	4.7635
15	3.75	4.0687	4.4017	4.7477	5.1037
16	4.00	4.3400	4.6952	5.0642	5.4440
17	4.25	4.6112	4.9886	5.3816	5.7842
18	4.50	4.8825	5.2821	5.6981	6.1245
19	4.75	5.1537	5.5755	6.0145	6.4647
20	5.00	5.4250	5.8690	6.3310	6.8050
21	5.25	5.6962	6.1624	6.6474	7.1452
22	5.50	5.9675	6.4559	6.9639	7.4855
23	5.75	6.2387	6.7493	7.2803	7.8257
24	6.00	6.5100	7.0428	7.5968	8.1660
25	6.25	6.7812	7.3362	7.9132	8.5062
26	6.50	7.0525	7.6297	8.2279	8.8465
27	6.75	7.3237	7.9231	8.5461	9.1867
28	7.00	7.5950	8.2166	8.8626	9.5270
29	7.25	7.8662	8.5100	9.1790	9.8672
30	7.50	8.1375	8.8035	9.4955	10.2075
31	7.75	8.4087	9.0969	9.8119	10.5477
32	8.00	8.6800	9.3904	10.1284	10.8880
33	8.25	8.9512	9.6838	10.4448	11.2282
34	8.50	9.2225	9.9773	10.7613	11.5685
35	8.75	9.4937	10.2707	11.0777	11.9087
36	9.00	9.7650	10.5642	11.3942	12.2490
37	9.25	10.0362	10.8576	11.7106	12.5892
38	9.50	10.3085	11.1511	12.0271	12.9295
39	9.75	10.5797	11.4445	12.3435	13.2697
40	10.00	10.8510	11.7380	12.6600	13.6100

Feet long	Side 7 $\frac{3}{4}$	Side 7 $\frac{1}{2}$	Side 7 $\frac{1}{4}$	Side 8 Inc.	Side 8 $\frac{1}{4}$
	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	0.3650	0.3906	0.4171	0.4444	0.4726
2	0.731	0.7813	0.8343	0.8889	0.9453
3	1.096	1.1719	1.2514	1.3333	1.4179
4	1.462	1.5626	1.6686	1.7778	1.8906
5	1.827	1.9532	2.0857	2.2222	2.3632
6	2.193	2.3439	2.5029	2.6667	2.8359
7	2.558	2.7345	2.9200	3.1111	3.3085
8	2.924	3.1252	3.3371	3.5556	3.7812
9	3.289	3.5158	3.7543	4.0000	4.2538
10	3.655	3.9065	4.1714	4.4445	4.7265
11	4.020	4.2971	4.5886	4.8889	5.1991
12	4.386	4.6878	5.0057	5.3334	5.6718
13	4.751	5.0784	5.4229	5.7778	6.1444
14	5.117	5.4691	5.8400	6.2223	6.6171
15	5.482	5.8597	6.2572	6.6667	7.0897
16	5.848	6.2504	6.6743	7.1112	7.5642
17	6.213	6.6410	7.0925	7.5556	8.0350
18	6.579	7.0317	7.5096	8.0001	8.5077
19	6.944	7.4223	7.9267	8.4445	8.9803
20	7.310	7.8130	8.3439	8.8890	9.4530
21	7.675	8.2036	8.7611	9.3334	9.9256
22	8.041	8.5943	9.1782	9.7779	10.3983
23	8.406	8.9859	9.5954	10.2223	10.8709
24	8.773	9.3766	10.0125	10.6678	11.3436
25	9.137	9.7672	10.4297	11.1112	11.8162
26	9.503	10.1579	10.8468	11.5557	12.2889
27	9.868	10.5485	11.2640	12.0001	12.7615
28	10.234	10.9392	11.6811	12.4446	13.2342
29	10.599	11.3298	12.0983	12.8890	13.7068
30	10.965	11.7205	12.5154	13.3335	14.1795
31	11.330	12.1111	12.9326	13.7779	14.6521
32	11.696	12.5018	13.3497	14.2224	15.1248
33	12.061	12.8924	13.7669	14.6668	15.5974
34	12.427	13.2831	14.1840	15.1113	16.0701
35	12.792	13.6737	14.6012	15.5557	16.5427
36	13.158	14.0644	15.0183	16.0002	17.0154
37	13.523	14.4550	15.4355	16.4446	17.4880
38	13.889	14.8457	15.8526	16.8891	17.9607
39	14.254	15.2363	16.2698	17.3335	18.4333
40	14.620	15.6270	16.6770	17.7780	18.9060

Feet long	Side 8 $\frac{1}{2}$	Side 8 $\frac{3}{4}$	Side 9 Inc.	Side 9 $\frac{1}{4}$	Side 9 $\frac{1}{2}$
	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	0.5017	0.5316	0.5625	0.5941	0.6267
2	1.0035	1.0633	1.1250	1.1883	1.2534
3	1.5052	1.5945	1.6873	1.7824	1.8801
4	2.0070	2.1266	2.2500	2.3766	2.5068
5	2.5087	2.6582	2.8125	2.9710	3.1336
6	3.0105	3.1899	3.3750	3.5652	3.7603
7	3.5122	3.7215	3.9375	4.1593	4.3870
8	4.0140	4.2532	4.5000	4.7535	5.0137
9	4.5157	4.7848	5.0625	5.3486	5.6405
10	5.0175	5.3165	5.6250	5.9418	6.2672
11	5.5192	5.8481	6.1876	6.5359	6.8939
12	6.0210	6.3798	6.7500	7.1301	7.5206
13	6.5227	6.9114	7.3125	7.7242	8.1474
14	7.0240	7.4431	7.8750	8.3183	8.7741
15	7.5257	7.9747	8.4375	8.9125	9.4008
16	8.0275	8.6064	9.0000	9.5066	10.0276
17	8.5292	9.0380	9.5625	10.1008	10.6543
18	9.0310	9.5697	10.1250	10.6949	11.2813
19	9.5327	10.1013	10.6875	11.2891	11.9077
20	10.0345	10.6330	11.2500	11.8832	12.5345
21	10.5362	11.1646	11.8125	12.4774	13.1612
22	11.0380	11.6963	12.3750	13.0715	13.7879
23	11.5399	12.2279	12.9375	13.6657	14.4147
24	12.0415	12.7596	13.5000	14.2598	15.0414
25	12.5432	13.2912	14.0625	14.8540	15.6681
26	13.0450	13.8229	14.6250	15.4481	16.2948
27	13.5467	14.3545	15.1875	16.0423	16.9216
28	14.0485	14.8862	15.7500	16.6364	17.5483
29	14.5012	15.4178	16.3125	17.2306	18.1750
30	15.0520	15.9495	16.8750	17.8247	18.8017
31	15.5537	16.4811	17.4375	18.4189	19.4285
32	16.0555	17.0128	18.0000	19.0130	20.0552
33	16.5572	17.5444	18.5625	19.6072	20.6819
34	17.0590	18.0761	19.1250	20.2013	21.3086
35	17.5607	18.6077	19.6875	20.7955	21.9354
36	18.0625	19.1394	20.2500	21.3896	22.5621
37	18.5642	19.6710	20.8125	21.9838	23.1888
38	19.0660	20.2027	21.3850	22.5779	23.8155
39	19.5677	20.7343	21.9475	23.1721	24.4423
40	20.0695	21.2660	22.5100	23.7662	25.0690

Feet long	Side 9 $\frac{3}{4}$	Side 10 Inc.	Side 10 $\frac{1}{4}$	Side 10 $\frac{1}{2}$	Side 10 $\frac{3}{4}$
	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	0.6602	0.6944	0.7296	0.7656	0.8025
2	1.3204	1.3889	1.4592	1.5313	1.6051
3	1.9806	2.0833	2.1888	2.2969	2.4076
4	2.6407	2.7777	2.9184	3.0626	3.2102
5	3.3009	3.4722	3.6480	3.8282	4.0127
6	3.9610	4.1666	4.3776	4.5939	4.8153
7	4.6212	4.8610	5.1072	5.3595	5.6178
8	5.2813	5.5555	5.8368	6.1252	6.4204
9	5.9415	6.2469	6.5664	6.8908	7.2229
10	6.6017	6.9444	7.2960	7.6565	8.0255
11	7.2618	7.6388	8.0259	8.4221	8.8280
12	7.9220	8.3332	8.7552	9.1878	9.6306
13	8.5821	9.0277	9.4848	9.6534	10.4331
14	9.2423	9.7222	10.2144	10.7191	11.2357
15	9.9025	10.4166	10.9440	11.4847	12.0382
16	10.5626	11.1111	11.6736	12.2504	12.8408
17	11.2227	11.8055	12.4032	13.0160	13.6433
18	11.8828	12.5000	13.1328	13.7817	14.4459
19	12.5429	13.1944	13.8624	14.5473	15.2484
20	13.2030	13.8889	14.5921	15.3130	16.0510
21	13.8632	14.5833	15.3217	16.0786	16.8535
22	14.5233	15.2778	16.0513	16.8443	17.6561
23	15.1834	15.9722	16.7809	17.6109	18.4586
24	15.8436	16.6667	17.5105	18.3766	19.2612
25	16.5037	17.3611	18.2401	19.1422	20.0637
26	17.1639	18.0556	18.9697	19.9079	20.8663
27	17.8241	18.7500	19.6993	20.6736	21.6688
28	18.4842	19.4445	20.4290	21.4492	22.4714
29	19.1444	20.1389	21.1586	22.2149	23.2739
30	19.8045	20.8334	21.8882	22.9805	24.0795
31	20.4646	21.5278	22.6178	23.7462	24.8790
32	21.1248	22.2223	23.3474	24.5018	25.6816
33	21.7849	22.9167	24.0770	25.2675	26.4841
34	22.4451	23.6112	24.8066	26.0331	27.2167
35	23.1052	24.3056	25.5362	26.7988	28.0892
36	23.7654	25.0001	26.2658	27.5644	28.8618
37	24.4256	25.6945	26.9954	28.3301	29.6943
38	25.0857	26.3890	27.7250	29.0957	30.4969
39	25.7458	27.0834	28.4546	29.8614	31.2994
40	26.4060	27.7779	29.1843	30.6270	32.1020

Feet long	Side 11 Inc.	Side 11 $\frac{1}{4}$	Side 11 $\frac{1}{2}$	Side 11 $\frac{3}{4}$	Side 12 Inc.
	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	0 8402	0.8789	0.9184	0.9587	1.0000
2	1.6805	1.7579	1.8369	1.9175	1.
3	2.5207	2.6368	2.7553	2.8762	2.
4	3.3610	3 5158	3.6738	3.8350	4.
5	4.2012	4.3947	4.5922	4.7937	5.
6	5.0415	5.2737	5.5107	5.7525	6.
7	5.8817	6.1526	6.4291	6.7112	7.
8	6.7220	7.0316	7.3476	7.6700	8.
9	7.5622	7.9105	8.2660	8.6287	9.
10	8.4025	8.7895	9.1845	9.5875	10.
11	9.2427	9.6684	10.1029	10.5462	11.
12	10.0830	10.5474	11.0214	11.5050	12.
13	10.9232	11.4264	11.9398	12.4637	13.
14	11.7635	12.3053	12.8583	13.3225	14.
15	12.6037	13.1842	13.7767	14.3812	15.
16	13.4440	14.0632	14.6952	15.3400	16.
17	14.2842	14.9421	15.6136	16.2987	17.
18	15.1245	15.8211	16.5321	17.2575	18.
19	15.9647	16.7000	17.4505	18.2162	19.
20	16.8050	17.5790	18.3690	19.1750	20.
21	17.6452	18.4579	19.2874	20.1337	21.
22	18.4855	19.3369	20.2059	21.0925	22.
23	19.3257	20.2158	21.1243	22.0512	23.
24	20.1660	21.0948	22.0428	23.0100	24.
25	21.0062	21.9737	22.9612	23.9687	25.
26	21.8465	22.8527	23.8791	24.9275	26.
27	22.6867	23.7316	24.7986	25.8862	27.
28	22.5270	24.6106	25.7166	26.8450	28.
29	24.3672	25.4895	26.6350	27.8037	29.
30	25.2075	26.3685	27.5535	28.7625	30.
31	26.0477	27.2474	28.4719	29.7212	31.
32	26.8880	28.1264	29.3904	30.6800	32.
33	27.7282	29.0053	30.3088	31.6387	33.
34	28.5685	29.8843	31.2273	32.5975	34.
35	29.4087	30.7632	32.1457	33.5562	35.
36	30.2490	31.6422	33.0642	34.5150	36.
37	31.0892	32.5211	33.9826	35.4737	37.
38	31.9295	33.4001	34.9011	36.4325	38.
39	32.7697	34.2790	35.8195	37.3912	39.
40	33.6100	35.1580	36.7380	38.3500	40.

Feet long	Side 12 $\frac{1}{4}$	Side 12 $\frac{1}{2}$	Side 12 $\frac{3}{4}$	Side 13 Inc.	Side 13 $\frac{1}{4}$
	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	1.0421	1.0851	1.1289	1.1736	1.2191
2	2.0842	2.1702	2.2578	2.3472	2.4383
3	3.1263	3.2553	3.3867	3.5208	3.6574
4	4.1685	4.3404	4.5157	4.6945	4.8756
5	5.2106	5.4255	5.6446	5.8681	6.0947
6	6.2527	6.5106	6.7735	7.0417	7.3149
7	7.2948	7.5957	7.9024	8.2153	8.5340
8	8.3369	8.6808	9.0314	9.3890	9.7532
9	9.3791	9.7659	10.1603	10.5626	10.9723
10	10.4212	10.8510	11.2792	11.7362	12.1915
11	11.4633	11.9361	12.4181	12.9098	13.4106
12	12.5044	13.0212	13.5470	14.0834	14.6298
13	13.5476	14.1063	14.6760	15.2571	15.8489
14	14.5897	15.1914	15.8049	16.4307	17.0681
15	15.6318	16.2765	16.9338	17.6043	18.2872
16	16.6739	17.3616	18.0627	18.7779	19.5064
17	17.7160	18.4467	19.1916	19.9516	20.7225
18	18.7582	19.5318	20.3206	21.1252	21.9447
19	19.8003	20.6169	21.4495	22.2988	23.1638
20	20.8424	21.7020	22.5784	23.4724	24.3830
21	21.8845	22.7871	23.7072	24.6460	25.6021
22	22.9266	23.8722	24.8362	25.8207	26.8213
23	23.9687	24.9573	25.9651	26.9943	28.0404
24	25.0109	26.0424	27.0941	28.1679	29.2596
25	26.0530	27.1275	28.2230	29.3415	30.4787
26	27.0951	28.2126	29.3519	30.5151	31.6979
27	28.1372	29.2977	30.4808	31.6887	32.9179
28	29.1793	30.3828	31.6097	32.8623	34.1362
29	30.2215	31.4679	32.7386	34.0359	35.3553
30	31.2636	32.5530	33.8676	35.2095	36.5745
31	32.3057	33.6381	34.9965	36.3831	37.7936
32	33.3478	34.7232	36.1254	37.5568	39.0127
33	34.3899	35.8081	37.2543	38.7304	40.2319
34	35.4321	36.8932	38.3832	39.9041	41.4510
35	36.4742	37.9783	39.5121	41.0777	42.6702
36	37.5163	39.0634	40.6411	42.2514	43.8893
37	38.5584	40.1485	41.7700	43.4250	45.1805
38	39.6005	41.2336	42.8989	44.5987	46.3276
39	40.6426	42.3187	44.0278	45.7723	47.5468
40	41.6848	43.4038	45.1567	46.9460	48.7660

Feet long	Side 13 $\frac{1}{2}$	Side 13 $\frac{1}{4}$	Side 14 Inc.	Side 14 $\frac{1}{4}$	Side 14 $\frac{1}{2}$
	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	1.2656	1.3129	1.3611	1.4101	1.4600
2	2.5313	2.6259	2.7223	2.8203	2.9201
3	3.7969	3.9388	4.0834	4.2304	4.3801
4	5.0626	5.2517	5.4446	5.6406	5.8402
5	6.3282	6.5647	6.8057	7.0507	7.3002
6	7.5939	7.8777	8.1669	8.4609	8.7603
7	8.8595	9.1906	9.5280	9.8710	10.2203
8	10.1252	10.5036	10.8892	11.2812	11.6304
9	11.3908	11.8165	12.2503	12.6913	13.1404
10	12.6564	13.1295	13.6155	14.1015	14.6005
11	13.9221	14.4424	15.9726	15.5116	16.0605
12	15.1878	15.7554	16.3338	16.9218	17.5206
13	16.4534	17.0683	17.6959	18.3319	18.9806
14	17.7191	18.3813	19.0571	19.7421	20.4407
15	18.9847	19.6942	20.4182	21.1522	21.9007
16	20.2504	21.0072	21.7794	22.5624	23.3608
17	21.5160	22.3201	23.1305	23.9725	24.8208
18	22.7817	23.6331	24.4917	25.3827	26.2809
19	24.0473	24.9460	25.8328	26.7928	27.7409
20	25.3130	26.2590	27.2140	28.2030	29.2010
21	26.5786	27.5719	28.5751	29.6131	30.6610
22	27.8443	28.8849	29.9363	31.0233	32.1211
23	29.1099	30.1978	31.2974	32.4334	33.5811
24	30.3756	31.5108	32.6586	33.8436	35.0412
25	31.6412	32.8237	34.0197	35.2537	36.5012
26	32.9069	34.1367	35.3809	36.6639	37.9613
27	34.1725	35.4496	36.7420	38.0741	39.4213
28	35.4382	36.7625	38.1032	39.4842	40.8814
29	36.7038	38.0755	39.4643	40.8944	42.3414
30	37.9695	39.3884	40.8255	42.3045	43.8015
31	39.2351	40.7014	42.1866	43.7147	45.2615
32	40.5008	42.0143	43.5478	45.1248	46.7216
33	41.7644	43.3273	44.9089	46.5350	48.1816
34	43.0321	44.6402	46.2701	47.9451	49.6417
35	44.2977	45.9532	47.6310	49.3553	51.1017
36	45.5634	47.2661	48.9624	50.7654	52.5618
37	46.8290	48.5791	50.3535	52.1756	54.0218
38	48.0957	49.8920	51.7147	53.5857	55.4819
39	49.2603	51.2050	53.0758	54.9959	56.9419
40	50.6260	52.5170	54.4370	56.4060	58.4030

Feet long	Side 14 $\frac{1}{4}$	Side 15 Inc.	Side 15 $\frac{1}{4}$	Side 15 $\frac{1}{2}$	Side 15 $\frac{3}{4}$
	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	1.5108	1.5625	1.6150	1.6684	1.7226
2	3.0217	3.1250	3.2301	3.3369	3.4453
3	4.5325	4.6875	4.8451	5.0053	5.1679
4	6.0434	6.2500	6.4602	6.6738	6.8896
5	7.5542	7.8125	8.1752	8.3422	8.6122
6	9.0651	9.3750	9.6903	10.0107	10.3349
7	10.5759	10.9375	11.3053	11.6791	12.0575
8	12.0868	11.5000	12.9204	13.3476	13.7802
9	13.5976	14.0625	14.5354	15.0160	15.4028
10	15.1085	15.6250	16.1505	16.6845	17.2255
11	16.6193	17.1875	17.7655	18.3529	18.9481
12	18.1302	18.7500	19.3866	20.0214	20.6708
13	19.6410	20.3125	21.0016	21.6898	22.3934
14	21.1519	21.8750	22.6167	23.3583	24.1161
15	22.6627	23.4375	24.2317	25.0267	25.8387
16	24.1736	25.0000	25.8478	26.6952	27.5614
17	25.6844	26.5625	27.4618	28.3636	29.2840
18	27.1953	28.1250	29.0769	30.0321	31.0067
19	28.7061	29.6875	30.6919	31.7005	32.7293
20	30.2170	31.2500	32.3070	33.3690	34.4520
21	31.7278	32.8125	33.9220	35.0374	36.1746
22	33.2387	34.3750	35.5371	36.7079	37.8973
23	34.7495	35.9375	37.1521	38.3763	39.6199
24	36.2604	37.5000	38.7672	40.0448	41.3426
25	37.7712	39.0625	40.3822	41.7132	43.0652
26	39.2821	40.6250	41.9973	43.3817	44.7879
27	40.7929	42.1875	43.6123	45.0501	46.5105
28	42.3038	43.7500	45.2274	46.7186	48.2332
29	43.8146	45.3125	46.8424	48.3870	49.9558
30	45.3255	46.8750	48.4574	50.0555	51.6785
31	46.8363	48.4375	50.0725	51.7239	53.4011
32	48.3472	50.0000	51.6875	53.3924	55.1238
33	49.8580	51.5625	53.3026	55.0608	56.8464
34	51.3689	53.1250	54.9176	56.7293	58.5691
35	52.8797	54.6875	56.5327	58.3977	60.2917
36	54.3906	56.2500	58.1477	60.0662	62.0144
37	55.9014	57.8125	59.7628	61.7346	63.7370
38	57.4123	59.3750	61.3778	63.4031	65.4597
39	58.9231	60.9375	62.9929	65.0715	67.1824
40	60.4340	62.5000	64.6080	66.7400	68.9050

Feet long	Side 16 Inc.	Side 10 $\frac{3}{4}$	Side 16 $\frac{1}{2}$	Side 16 $\frac{3}{4}$	Side 17 Inc.
	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	1.7777	1.8337	1.8906	1.9483	2.0069
2	3.5555	3.6675	3.7813	3.8967	4.0139
3	5.3332	5.5012	5.6719	5.8450	6.0208
4	7.1110	7.3350	7.5626	7.7934	8.0278
5	8.8887	9.1687	9.4532	9.7417	10.0347
6	10.6665	11.0025	11.3439	11.6901	12.0417
7	12.4442	12.8362	13.2345	13.6384	14.0486
8	14.2220	14.6700	15.1252	15.5868	16.0556
9	15.9997	16.5037	17.0158	17.5351	18.0625
10	17.7775	18.3375	18.9065	19.4835	20.0695
11	19.5552	20.1712	20.7971	21.4318	22.0764
12	21.3330	22.0050	22.6878	23.3802	24.0834
13	23.1107	23.8387	24.5784	25.3285	26.0903
14	24.8885	25.6725	26.4691	27.2769	28.0973
15	26.6662	27.5062	28.3597	29.2252	30.1042
16	28.4440	29.2400	30.2504	31.1736	32.1112
17	30.1117	31.1737	32.1410	33.1219	34.1181
18	31.9995	33.0075	34.0317	35.0703	36.1251
19	33.7772	34.8412	35.9223	37.0186	38.1320
20	35.5550	36.6750	37.8130	38.9670	40.1390
21	37.3327	38.5087	39.7036	40.9153	42.1459
22	39.1105	40.3425	41.5943	42.8637	44.1529
23	40.8882	42.1762	43.4849	44.8120	46.1598
24	42.6660	44.0100	45.3756	46.7604	48.1668
25	44.4437	45.8437	47.2622	48.7087	50.1737
26	46.2215	47.6775	49.1569	50.6571	52.1806
27	48.9992	49.5112	51.0475	52.6054	54.1876
28	49.7770	51.3450	52.9382	54.5538	56.1945
29	51.5547	53.1787	54.8288	56.5021	58.2014
30	53.3325	55.0125	56.7195	58.4505	60.2084
31	55.1102	56.8462	58.6101	60.3988	62.2153
32	56.8880	58.6800	60.5008	62.3472	64.2222
33	58.6657	60.5137	62.3914	64.2956	66.2291
34	60.4435	62.3475	64.2821	66.2439	68.2361
35	62.2212	64.1812	66.1727	68.1923	70.2430
36	63.9990	66.0150	68.0634	70.1406	72.2500
37	65.7767	67.8487	69.9540	72.0890	74.2569
38	67.5545	69.6825	71.8447	74.0373	76.2639
39	69.3322	71.5162	73.7354	75.9857	78.2708
40	71.1100	73.3500	75.6260	77.9340	80.2778

Feet long	Side 17 $\frac{1}{4}$	Side 17 $\frac{1}{2}$	Side 17 $\frac{3}{4}$	Side 18 Inc.	Side 18 $\frac{1}{4}$
	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	2.0664	2.1267	2.1879	2.2500	2.3129
2	4.1329	4.2535	4.3759	4.50	4.6259
3	6.1993	6.3802	6.5638	6.75	6.9388
4	8.2658	8.5070	8.7518	9.00	9.2518
5	10.3322	10.6337	10.9397	11.25	11.5647
6	12.3987	12.7605	13.1277	13.50	13.8777
7	14.4651	14.8872	15.3156	15.75	16.1906
8	16.5316	17.0140	17.5036	18.00	18.5036
9	18.5980	19.1407	19.6915	20.25	20.8165
10	20.6645	21.2675	21.8795	22.50	23.1295
11	22.7309	23.3942	24.0674	24.75	25.4424
12	24.7974	25.5210	26.2554	27.00	27.7554
13	26.8638	27.6477	28.4433	29.25	30.0683
14	28.9303	29.7745	30.6313	31.50	32.3813
15	30.9967	31.9012	32.8192	33.75	34.6942
16	33.0632	34.0280	35.0072	36.00	37.0722
17	35.1296	36.1547	37.1951	38.25	39.3201
18	37.1961	38.2815	39.3831	40.50	41.6331
19	39.2625	40.4082	41.5710	42.75	43.9460
20	41.3290	42.5350	43.7590	45.00	46.2590
21	43.3955	44.6617	45.9469	47.25	48.5719
22	45.4619	46.7885	48.1349	49.50	50.8849
23	47.5283	48.9152	50.3228	51.75	53.1978
24	49.5948	51.0420	52.5108	54.00	55.5108
25	51.6613	53.1687	54.6987	56.25	57.8237
26	53.7277	55.2955	56.8867	58.50	60.1367
27	55.7941	57.4222	59.0746	60.75	62.4496
28	57.8606	59.5490	61.2626	63.00	64.7626
29	59.9270	61.6757	63.4505	65.25	67.0755
30	61.9935	63.8025	65.6385	67.50	69.3885
31	64.0599	65.9292	67.8264	69.75	71.7014
32	66.1264	68.0560	70.0144	72.00	74.0144
33	68.1928	70.1827	72.2023	74.25	76.3273
34	70.2593	72.3095	74.3903	76.50	78.6403
35	72.3257	74.4362	76.5782	78.75	80.9532
36	74.3922	76.5630	78.7662	81.00	83.2662
37	76.4586	78.6897	80.9541	83.25	85.5791
38	78.5251	80.8165	83.1421	85.50	87.1921
39	80.5915	82.9432	85.3300	87.75	90.2050
40	82.6580	85.0700	87.5180	90.00	92.5180

Feet long	Side 18 $\frac{1}{2}$	Side 18 $\frac{3}{4}$	Side 19 Inc.	Side 19 $\frac{1}{4}$	Side 19 $\frac{1}{2}$
	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	2.3767	2.4414	2.5069	2.5733	2.6406
2	4.7535	4.8829	5.0139	5.1467	5.2813
3	7.1302	7.3243	7.5208	7.7200	7.9219
4	9.5070	9.7658	10.0278	10.2934	10.5626
5	11.8837	12.2072	12.5348	12.8667	13.2032
6	14.2605	14.6487	15.0417	15.4401	15.8439
7	16.6372	17.0901	17.5487	18.0134	18.4845
8	19.0140	19.5316	20.0556	20.5868	21.1252
9	21.3907	21.9730	22.5626	23.1601	23.7658
10	23.7675	24.4145	25.0696	24.7335	26.4065
11	26.1442	26.8559	27.5765	28.3068	29.0471
12	28.5210	29.2974	30.0834	30.8802	31.6878
13	30.8977	31.7388	32.5904	33.4525	34.3284
14	33.2745	34.1803	35.0973	36.0269	36.9691
15	35.6512	36.6217	37.6043	38.6002	39.6097
16	38.0280	39.0632	40.1112	41.1736	42.2504
17	40.4047	41.5046	42.6182	43.7469	44.8910
18	42.7815	43.9461	45.1251	46.3203	47.5317
19	45.1582	46.3875	47.6321	48.8936	50.1723
20	47.5350	48.8290	50.1390	51.4670	52.8130
21	49.9117	51.2704	52.6460	54.0403	55.4536
22	52.2885	53.7119	55.1529	56.6137	58.0943
23	54.6652	56.1533	57.6599	59.1870	60.7350
24	57.0420	58.5948	60.1668	61.7604	63.3756
25	59.4187	61.0362	62.6738	64.3337	66.0163
26	61.7955	63.4777	65.1807	66.9071	68.6569
27	64.1722	65.9191	67.6877	69.4804	71.2976
28	66.5490	68.3606	70.1946	72.0538	73.9382
29	68.9257	70.8020	72.7016	74.6271	76.5789
30	71.3025	73.2435	75.2085	77.2005	79.2195
31	73.6792	75.6849	77.7155	79.7738	81.8602
32	76.0560	78.1264	80.2224	82.3472	84.5008
33	78.4327	80.5688	82.7094	84.9205	87.1415
34	80.8095	83.0103	85.2363	87.4939	89.7821
35	83.1862	85.4517	87.7433	90.0672	92.4228
36	85.5630	87.8932	90.2502	92.6406	95.0634
37	87.9397	90.3346	92.7562	95.2139	97.7041
38	90.3165	92.7761	95.2631	97.7873	100.3447
39	92.6932	95.2175	97.7701	100.3606	102.9854
40	95.0700	97.6590	100.2770	102.9340	105.6260

Feet long	Side 19 $\frac{1}{4}$	Side 20 Inc.	Side 20 $\frac{1}{4}$	Side 20 $\frac{1}{2}$	Side 20 $\frac{3}{4}$
	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	2.7087	2.7777	2.8476	2.9184	2.9909
2	5.4175	5.5555	5.6953	5.8369	5.9801
3	8.1262	8.3332	8.5429	8.7553	8.9701
4	10.8350	11.1110	11.3906	11.6738	11.9602
5	13.5437	13.8887	14.2382	14.5922	14.9502
6	16.2525	16.6665	17.0859	17.5107	17.9403
7	18.9612	19.4442	19.9335	20.4291	20.9303
8	21.6700	22.2220	22.7812	23.3476	23.9204
9	24.3787	24.9997	25.6288	26.2660	26.9104
10	27.0875	27.7775	28.4765	29.1845	29.9005
11	29.7962	30.5552	31.3241	32.1029	32.8905
12	32.5050	33.3330	34.1718	35.0214	35.8806
13	35.2137	36.1107	37.0194	37.9398	38.8706
14	37.7225	38.8885	39.8671	40.8583	41.8607
15	40.6312	41.6662	42.7147	43.7767	44.8507
16	43.3400	44.4440	45.5624	46.6952	47.8408
17	46.0487	47.2217	48.4100	49.6176	50.8308
18	48.7575	49.9995	51.2577	52.5321	53.8209
19	51.4662	52.7772	54.1053	55.4505	56.8109
20	54.1750	55.5550	56.9530	58.3690	59.8010
21	56.8837	58.3327	59.8006	61.2874	62.7911
22	59.5925	61.1105	62.6483	64.2059	65.7811
23	62.3012	63.8882	65.4969	67.1243	68.7712
24	65.0100	66.6660	68.3446	70.0428	71.7612
25	67.7187	69.4437	71.1922	72.9612	74.7513
26	70.4275	72.2215	74.0399	75.8797	77.7413
27	73.1362	74.9992	76.8875	78.7981	80.7314
28	75.8450	77.7770	79.7352	81.7166	83.7214
29	78.5537	80.5547	82.5820	84.6350	86.7115
30	81.2625	83.3325	85.4305	87.5535	89.7015
31	83.9712	86.1102	88.2781	90.4719	92.6916
32	86.6800	88.8880	91.3258	93.3904	95.6816
33	89.3887	91.6657	93.9734	96.3088	98.6717
34	92.0975	94.4435	96.8211	99.2273	101.6617
35	94.8062	97.2212	99.6687	102.1457	104.6518
36	97.5150	99.9990	102.5164	105.0642	107.6418
37	100.2237	102.7767	105.3640	107.9826	110.6319
38	102.9325	105.5545	108.2117	110.9011	113.6219
39	105.6412	108.3322	111.0593	113.8195	116.6120
40	108.3500	111.1100	113.9070	116.7380	119.6020

Feet long	Side 21 Inc.	Side 21 $\frac{1}{4}$	Side 21 $\frac{1}{2}$	Side 21 $\frac{3}{4}$	Side 22 Inc.
	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	3.0625	3.1358	3.2100	2.2851	3.7611
2	6.1250	6.2717	6.4201	6.5703	6.3223
3	9.1875	9.4075	9.6301	9.8554	10.0834
4	12.5500	12.5434	12.8402	13.1406	13.4446
5	15.3125	15.6792	16.0502	16.4257	16.8057
6	18.3750	18.8151	19.2603	19.7109	20.1669
7	21.4375	21.9509	22.4703	22.9960	23.5280
8	24.5000	25.0868	25.6804	26.2812	26.8892
9	27.5625	28.2226	28.8904	29.5663	30.2503
10	30.6250	31.3585	32.1005	32.8515	33.6115
11	33.6875	34.4943	35.3105	36.1366	36.9725
12	36.7500	37.6302	38.5206	39.4218	40.3338
13	39.8125	40.7660	41.7306	42.7069	43.6949
14	42.8150	43.9019	44.9407	45.9921	47.0561
15	45.9375	47.0377	48.1507	49.2772	50.4172
16	49.0000	50.1736	51.3608	52.5624	53.7784
17	52.0625	53.3094	54.5708	55.8475	57.1395
18	55.1250	56.4453	57.7809	59.1327	60.5007
19	58.1875	59.5811	60.9909	62.4178	63.8618
20	61.1500	62.7170	64.2010	65.7030	67.2230
21	64.3125	65.8528	67.4110	68.9881	70.5841
22	67.3750	68.9887	70.6211	72.2733	73.9453
23	70.4375	72.1245	73.8311	75.5584	77.3064
24	73.5000	75.2604	77.0412	78.8436	80.6676
25	76.5625	78.3962	80.2512	82.1287	84.0287
26	79.6250	81.5321	83.4613	85.4139	87.3899
27	82.6875	84.6679	86.6713	88.6990	90.7510
28	85.7500	87.8038	89.8814	91.9842	94.1122
29	88.8125	90.9396	93.0914	95.2693	97.4733
30	91.8750	94.0755	96.3015	98.5545	100.8345
31	94.9375	97.2113	99.5115	101.8396	104.1956
32	98.0000	100.3472	102.7216	105.1248	107.5568
33	101.0625	103.4830	105.9316	108.4099	110.9179
34	104.1250	106.6189	109.1417	111.6951	114.2791
35	107.1875	109.7547	112.3517	114.9802	117.6402
36	110.2500	112.8906	115.5618	118.2654	121.0014
37	113.3125	116.0264	118.7718	121.5505	124.3625
38	116.3750	119.1623	121.9819	124.8357	127.7237
39	119.4375	122.2981	125.1919	128.1208	131.0848
40	122.5000	125.4340	128.4020	131.4060	134.4460

A new Table of Solid Measure.

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Feet long	Side 22 $\frac{1}{4}$	Side 22 $\frac{1}{2}$	Side 22 $\frac{3}{4}$	Side 23 inc. 5	Side 23 $\frac{1}{4}$
	Ft. Pts.	Ft. Pts.	Ft. P.	Ft. Pts.	Ft. Pts.
1	3.4379	3.5156	3.5941	3.6736	3.7539
2	6.8759	7.0313	7.1883	7.3473	7.5079
3	10.3138	10.5469	10.7824	11.0209	11.2618
4	13.7518	14.0626	14.3766	14.6946	15.0158
5	17.1897	17.5782	17.9707	18.3682	18.7697
6	20.6277	20.0939	21.5649	22.0419	22.5237
7	34.0656	24.6095	25.1590	25.7155	26.2776
8	27.5036	28.1252	28.7532	29.3892	30.0316
9	30.9415	31.6408	32.3473	33.0628	33.7855
10	34.3785	35.1565	35.9415	36.7365	37.5395
11	37.8164	38.6721	39.5356	40.4101	41.2934
12	41.2544	42.1878	43.1298	44.0838	45.0474
13	44.6923	45.7034	46.7239	47.7574	48.8013
14	48.1303	49.2191	50.3181	51.4311	52.5553
15	51.5682	52.7347	53.9122	55.1047	56.3092
16	55.0062	56.2504	57.5064	58.7784	60.0632
17	58.4441	59.7660	61.1005	62.4520	63.8171
18	61.8821	63.2817	64.6947	66.1257	67.5711
19	65.3200	66.7973	68.2888	69.7993	71.3250
20	68.7580	70.3130	71.8830	73.4730	75.0790
21	72.1959	73.8286	75.4711	77.1466	78.8329
22	75.6339	77.3443	79.0713	80.8203	82.5869
23	79.0708	80.8599	82.6654	84.4939	86.3408
24	82.5088	84.3756	86.2596	88.1676	90.0948
25	85.9467	87.8912	89.8537	91.8412	93.8487
26	89.3847	91.4069	94.4479	95.5149	97.6027
27	92.8226	94.9225	97.0420	99.1885	101.3566
28	96.2606	98.4382	100.6362	102.8622	105.1106
29	99.6985	101.9538	104.2903	106.5358	108.8645
30	103.1365	105.4695	107.8245	110.2095	112.6185
31	106.5744	108.9851	111.4186	113.8831	116.3724
32	110.0124	112.5008	115.0128	117.5568	120.1264
33	113.4503	116.0164	118.6069	121.2304	123.8803
34	116.8883	119.5321	122.2011	124.9041	127.6343
35	120.3262	123.0477	125.7952	128.5777	131.3822
36	123.7642	126.5634	129.3894	132.2514	135.1422
37	127.2021	130.0790	132.9835	135.9250	138.8961
38	130.6401	133.5947	136.5777	139.5987	142.6501
39	134.0780	137.1103	140.1718	143.2723	146.4040
40	137.5160	140.6260	143.7660	146.9460	150.1580

A new Table of Solid Measure.

Feet long	Side 23 $\frac{1}{2}$	Side 23 $\frac{1}{2}$	Side 24 Inc.
	Ft. Pts.	Ft. Pts.	Ft. Pts.
1	3.8350	3.9171	4.0000
2	7.6701	7.8343	8.
3	11.5051	11.7514	12.
4	12.3402	15.6686	16.
5	19.1752	19.5857	20.
6	23.0103	23.5029	24.
7	26.8453	27.4200	28.
8	30.6804	31.3372	32.
9	34.5154	35.2543	36.
10	38.3505	39.1715	40.
11	42.1855	43.0886	44.
12	46.0206	47.0058	48.
13	49.8556	50.9229	52.
14	53.6907	54.8401	56.
15	57.5257	58.7572	60.
16	61.3608	62.6744	64.
17	65.1958	66.5915	68.
18	69.0309	70.5087	72.
19	72.8659	74.4258	76.
20	76.7010	78.3430	80.
21	80.5360	82.2601	84.
22	84.3711	86.1773	88.
23	88.2061	90.0944	92.
24	92.0412	94.0116	96.
25	95.8762	97.9287	100.
26	99.7113	101.8459	104.
27	103.5463	105.7630	108.
28	107.3814	109.6802	112.
29	111.2164	113.5973	116.
30	115.0515	117.5145	120.
31	118.8865	121.4316	124.
32	122.7216	125.3488	128.
33	126.5566	129.2659	132.
34	130.3917	133.1831	136.
35	134.2267	137.1002	140.
36	138.0618	141.0174	144.
37	141.8968	144.9345	148.
38	145.7319	148.8517	152.
39	149.5669	152.7688	156.
40	153.4020	156.6860	160.



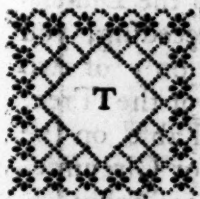
P A R T II.

C H A P. II.

The Mensuration of Solids by the Tables, &c.

S E C T. I.

The Construction and Use of the Table.



HIS new Table is so conspicuously easy and plain, that the Solid-content, according to the *Data*, may be readily and accurately found by Addition only: And, tho' the Inches and Quarters of an Inch, or (according to some Authors) so many twelfth Parts of an Inch, are decimally expressed; yet there wants not easy and concise Rules, to reduce them immediately into those Denominations; either by the Pen, or Inspection only.

Nor are they less valuable for their Conciseness. seeing they comprehend in fifteen Pages, what others (pretending to Brevity) require the greater Part of a Volume for Performance. And, as concerning their Accuracy and Exactness, they will be found (upon Trial) so agreeable to the Pen, that the Answer will prove either equal to the same, or within less than ten thousand Parts of an Inch; which is so inconsiderable (in the way of Business) that the same is not worthy Observation.

As concerning their Extent, they are only calculated for Timber and Stone, not exceeding forty Feet in Length; and for any Girt or Circumference, not exceeding 24 Inches, nor less than $\frac{1}{2}$ Foot, or 6 Inches; each of them being a fourth Part of the Girt given, or Side of the Square, equal thereunto, according to customary measure; seeing the former (in the way of Business) does seldom exceed 8 Feet in Circumference; nor the latter, less than two Feet; yet, when otherwise, this Table (in its Use and Application) will likewise resolve the same by proper Propositions, Rules, Examples, &c.

S E C T. II.

The Table explained and applied.

FOR the ready understanding of this Table, you may observe, that it is comprehended in less than 15 Pages, each Page containing 6 Columns; the first whereof, towards the Left-hand, contains always the Number of Feet given in Length, from 1 to 40 inclusively; and the other five Columns orderly succeeding the first, hath on the Top of each Column, a fourth Part of the given Girt or Circumference in Inches, from 6 to 24 inclusively; orderly increasing $\frac{1}{4}$ of an Inch each time; which, by Artificers, are generally termed a fourth Part of the Girt, or the Side of the Square equal thereunto; according to customary Measure, as abovesaid.

The Table being thus explained, as above, and the Dimensions being found, according to the first Part preceding, you must always observe, that your *Data* must consist only of two Terms or Denominations, *viz.* the given Length of the Tree, &c. in Feet, found in the first Column of the Table on the Left-hand, and the fourth Part of the given Girt or Circumference (known also by the Name of the Side, &c. in Inches being found on the Top of the Table; then in the common Angle of meeting against the Length, and under the said Girt or Side; you have (for Answer) the solid Feet sought, and decimal Parts of a Foot; which, when reduced by Inspection, or otherwise, will be found equal to so many Inches, and twelfth Parts of an Inch; as will more plainly and particularly appear in the following Sections, &c.

S E C T. III.

To know both vulgarly and decimally the Solid-content of any round Piece of Timber or Stone, of equal or unequal Bases, streight or tapering, according to the true Way.

AS the Foundation of all Numbers in Tables and Instruments, depend principally upon an Arithmetical and Mathematical Calculation; so I shall here begin with the Arithmetical

metical Performance, both the true and common way; in order to shew the Veracity of the Table, which performs by Inspection only.

To know (decimally) the Solid-Content of any Piece of round Timber or Stone, that is, either cylindrical or tapering regularly, like the Frustum of a Cone; according to the true Way.

1. When the Piece of Timber or Stone is of equal Circumference or Girt, from one End to the other, like a rolling stone for Walks, &c. then the Solid-content is found (decimally) by multiplying the Area of a Circle, whose Circumference is Unity (*viz.* .05958) by the Square of any Girt or Circumference in Feet, &c. and that Product again, by the given Length in Feet, &c. the last Product is the Solid-content sought. Or otherwise, the whole Girt or Circumference in Feet, &c. being multiplied by the Side of the Square, equal to the Circle whose Periphery is Unity (*viz.* .2821) and the Square of the Product thereof, by the given Length in Feet, &c. the last Product is likewise the Solid-content desired.

2. If the Piece of round Timber or Stone resemble the Frustum of a Cone, or the Segment of a Sugar-loaf, after the small End is cut off, in any Part parallel to the Base; then may the Dimensions of all Timber-trees (so gradually tapering in this Form) be taken and measured, according to the Method prescribed in *Chap. I. Sect. 7.* whose Solid-content may be readily found the customary Way, by the Table above; which again, may be accurately converted into the true Way, by multiplying the said Solidity in Feet and Decimal Parts by 14, and dividing the Product by 11; the Quotient is the Solid-content desired. —Or otherwise, all Timber or Stone bearing this Form or Shape, may have its Solidity found the true Way, by the 6th Observation of *Chap. I. Sect. 9.* which see.

Example. I.

Admit a Piece of Timber or Stone, in the Form of a Cylinder or rolling Stone, hath for its Length 19 Feet, and its Girt or Circumference 7 Feet: To know truly how many solid Feet, &c. is contained in the same.

Ft. Pts. Ft. In. Pts. 3ds.
Ans. 74.08898 = 74 : 1 : 0 : 9 $\frac{3}{4}$ + the true Way: Or,
 58.21277 the customary Way.

Explanation.

In this Example you may observe, that first the Area of a Circle, whose Periphery is Unity (*viz.* .07950) being multiplied by

by the Square of the given Girt or Circumference (*viz.* 7 Feet $\times$ 7 = 49 Square Feet) the Product is the Area, or Superficial Content of the Cylinder's circular Base (*viz.* 3 Square Feet .89942 Parts) which being again multiplied by the given Length (*viz.* 19 Feet) the 2d or last Product *viz.* 74 Feet .08898 Parts) is the Solid-content sought; according to the true Way. See the Rule above.

Area of the Circle, whose Periphery is Unity	.07958
Square of the Girt, or Circumference	- $\times$ 49
	71622
	31832
Area, or Surface of the Base	- - - 3.89942
Length given	- - - $\times$ 19
	3509478
	389942
Solid-content, or Answer	- - - = 74.08898

Example II.

There is a Piece of Timber or Stone, in the Form of the Frustrum of a Cone, whose Length is found to be 36 Feet; and 3 Feet 4 Inches, 6 Parts of an Inch, or .405 Parts for the Mean-girt or Circumference: To know the Solid-content both of the customary and true Way.

Feet. Pts. Feet. In. Pts.
Answer, 25.0002, or 25 : 0 : 0 the customary way : And
 31.8184, or 31 : 9 : 9 10 3ds + for the true way.

Explanation.

1. Having the given Length (*viz.* 36 Feet) and the Mean-girt or Circumference (*viz.* 40 Inches 5 Parts) whose fourth Part (*viz.* 10 Inches) is the Side of the Square-equal, according to customary Measure, and consequently, the proper *Data*, for finding by the Table only, the Solid-content (*viz.* 25.0002 Feet) for the desired Answer.

2. The Solid-content (*viz.* 25.0002 Feet) found the customary Way, as above, being multiplied by 14; and the Product (*viz.* 250.0028) being divided by 11, the Quotient (*viz.* 31.8184 Feet) is likewise the Solidity desired the true Way, as above. See *Chap. I. Sect. 7.* as well as the latter Part of the last Rule.

S E C T. IV.

To find arithmetically, as well as instrumentally, the Solid-content of any round Piece of Timber or Stone, according to any Dimensions given, the customary Way.

I. Arithmetically.

THAT the Veracity of the Table may evidently appear, not only arithmetically by the Pen, but even likewise by Gunter's Line, performed by the Compasses and Sliding-rule, we shall, *First*, shew, that if, (by the Pen) the fourth Part of the Mean-girt or Circumference of any round Piece of Timber or Stone, in Inches, &c. (commonly termed the Side of the Square-equal) be multiplied by itself, and that Product again, by the given Length in Inches; the last Product being divided (decimally) by the Inches in a Cubic Foot (*viz.* 1728) the Quotient is the Solid-content sought the customary way; and consequently, renders the Table every way agreeable to the Pen: As appears by the following Example.

II. Instrumentally.

Secondly, Having shewn the arithmetical Performance by the Pen, as above, you may now see the same agreeably effected by Gunter's Line, thus: Extend the Compasses from the central Number (*viz.* 12) to one fourth Part of the Mean-girt or Circumference, given in Inches, &c. then it will follow, that the same Extent, twice set backwards from the given Length in Feet, &c. will reach or extend to the desired Content in Feet, &c.—Or otherwise by the Sliding-rule, thus, *viz.* As 12, the central Number on the Girt-line, to the given Length on the sliding Part; so is the fourth Part of the given Circumference, or Side of the Square-equal on the Girt-line, to the Solidity sought on the sliding Part, as follows:

Example.

Admit a Piece of round Timber or Stone hath for its Length 16 Feet, and for its Mean-girt or Circumference

rence 44 Inches : To know its solid Content the customary way.

A Fourth of the Girt, or Side of }
the Square-equal, is - - - } = 11

× 11

Square Inches = 121

Given Length 16 Feet = × 192 Inches

242

1089

121

F. P.

1728) 23232.00 (13.44, &c.

1728

.5952

5184

.7680

6912

.7680

6912

Rem. 768

Ft. Pa. Ft. In. Pa.

Answer, 13.4444 = 13 — 5 — 4.

EXPLANATION.

I. *Arithmetically.*

I. In this Example you may observe, that the Side of the Square-equal, or the fourth Part of the Mean-girt, or Circumference, is 11 Inches ; which being multiplied by itself, make 121 Square Inches ; which being again multiplied by the Inches in the given Length (*viz.* 16 Feet, or 192 Inches) the last Product is 23232 solid Inches ; which being divided by the solid Inches ; in a cubic Foot (*viz.* 1728) the Quotient is 13 Feet, and 768 remaining ; to which, if there be orderly annexed and brought down four Cyphers, and divided by the same Divisor (*viz.* 1728) the remaining Part of the Quotient is a repeated Decimal (*viz.* .4444) : Hence it is, that the complete

Quo-

Quotient for Answer is 13 Feet, 4444 Parts, &c. or 13 Feet, 5 Inches, 4 Parts, as above.

II. *Instrumentally.*

2. If the same is effected by *Gunter's Line*, you will find that the Extent of the Compass, from 12, the central Number, to 11, the fourth Part of the Girt or Circumference, or Side of the Square-equal; I say, this Extent from 12 to 11, being twice turned from 16 backward, the given Length in Feet will reach to 13 Feet 45 Parts, *fere*, as before. — Or otherwise by the Sliding-rule, thus: Let 12, the central Number on the Girt-line, be placed against the given Length in Feet (*viz.* 16) on the sliding Part; so then the fourth Part of the given Circumference, or Side of the Square-equal (*viz.* 11 Inches) on the Girt-line, will extend to, or correspond with 13 Feet 44 Parts + on the sliding Part, for Answer.

S E C T. V.

To convert or change the Solid-content of any Piece of Timber or Stone, found the common Way, into the true Way, & sic vice versa: Or to change readily, the Solidity of the one into that of the other, by Analogy, or otherwise.

1. **A**S the Solidity of round Timber, &c. found the true way, hath just as much Reason and Proportion to the common or customary way, as 14 hath to 11; so by this Analogy, the common way may be accurately found, by multiplying always the Solid-content given the true way, by 11, and dividing the Product by 14, the Quotient is the Solid-content desired the common way. Also, on the contrary, if the Solid-content found the common way, be multiplied by 14, and the Product divided by 11, the Quotient is likewise the Solidity sought, the true way; according to the Reasons, &c. shewn in the first Observation of *Chap. I. Sect. 9.* which see.

2. In like manner the same (by Analogy) may be found instrumentally, by the Sliding-rule, thus, *viz.*

1st. As 11 upon A, is to the Solid-content upon B, the common way given; so is 14 upon A, to the Solid-content sought, upon B, for the true way. 2^{dly},

2dly, As 14 upon A, is to the Solid-content upon B, the true way given; so is 11 upon A, to the Solid-content sought, upon B, the common way.

Example.

Admit a Piece of Timber or Stone hath for its Length 24 Feet, and the fourth Part of its Mean-girt or Circumference, usually termed the Side of the Square-equal, 14 Inches: To know the Solid-content, both the common, and true way.

First Way.

By the Table, Solidity the } *Feet.*
common way is - - } 32.6586
 x 14

1306344
326586

÷ 11) 457.2204 = *Prod.*

True way = 41.56516 = *Quot.*

Second Way.

Given Solidity the true way = 41.5656
 x 11

415656
415656

÷ 14) 457.2216 = *Prod.*

Common way = 32.6586 = *Quot.*

	<i>Ft. Pa.</i>	<i>Ft. I. P.</i>
<i>Answer,</i> } 1. Common way = 32.6586 = 32 : 8 : 0 <i>ferē.</i>		
} 2. True way = 41.5656 = 41 : 6 : 9 $\frac{1}{2}$ +		

Explanation.

1. By the *Data*, or particular Contents of the Question, you may find the Solid-content sought the common way, in the Table at once; which when found, if the same (*viz.* 32 Feet, .6586 Parts) is multiplied by 14; the Product (*viz.* 457.2204) divided

divided by 11, the Quotient (*viz.* 41 Feet, 5656 Parts) is the Solid-content sought the true Way; otherwise the same may be accurately performed by *Seet.* 3. which see.

2. When the true Way hath its Solid-content given, in order to find the Solidity the common or customary Way; then, in such Case, you are to multiply the given Solidity by 11; and divide the Product (*viz.* 457.2216) by 14; the Quotient (*viz.* 32 Feet, 6586 Parts) is the Solid-content for the common Way desired. See *Seet.* 4.

The Artift will likewise find, that the Instrument or Sliding-rule will readily effect the same by Analogy; according to the second Article of the Rule above, which is so obvious, that there needs no further Illustration by Examples, or otherwise.

S E C T. VI.

To find accurately the Solid-content of any Square, quaarangular or reētangular Piece of Timber or Stone, whether straight or tapering, consisting of equal or unequal Sides, according to any Dimensions given, the common or customary Way.

HAVING in the preceding Sections shewn how any Piece of round Timber or Stone may be accurately measured, and its Solidity found various Ways, with all imaginable Plainness and Conciseness, we shall likewise shew the Method of finding exactly, as well as expeditiously, the Solid-content of any Square Piece of Timber or Stone consisting of equal or unequal Sides, according to the Particulars following.

1. If the given Piece to be measured, is straight and quadrangular, consisting of four equal Sides; then may its Dimensions be exactly taken according to the second article in *Chap.* I. *Seet.* 9. which see; and its Solid-content found in the Table by Inspection only; or by the Pen thus: Let the Inches and quarter Inches given for the Side of the Square-equal, be reduced into Feet and decimal Parts of a Foot; which being again squared or multiplied by itself, and that Product again by the given Feet, &c. in Length; the last Product is the Solid-content desired.

2. If the solid Piece of Timber or Stone to be measured, happen likewise to be square or quadrangular, but gradually ta-

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pering from the great End to the small ; then having taken its Length in Feet, &c. the Side of the Square-equal may be taken exactly in the Middle ; according to the Method prescribed in the second Article, *Chap. I. Sect. 9.* abovesaid : So have you the proper *Data* for finding the Solid-content of the same, either by the Pen or the Table, as above.

3. If the Sides of the Piece are unequal ; or otherwise, the Breadth and Thickness differing in Equality ; then, in such Case, it is needful, for greater Exactness, to reduce them into a Geometrical-mean for the Side of the Square sought, according to the Directions in article third, *Chap. I. Sect. 9.* abovesaid : So likewise (with the proposed Length) you have given the two proper Dimensions for finding the Solid-content, as above.

4. When it happens that the Bases at each End of the Piece are both quadrangular and unequal ; then having found the Geometrical-mean, or Side of the Square-equal, between them, as before directed, proceed to find the Solid-content, according to the Method prescribed in the fourth Observation of *Chap. I. Sect. 9.* above : Or otherwise, the same may be expeditiously performed by the Table only.

5. It sometimes happens, that the Length, Breadth, Depth, or Thickness of the Piece given to be measured, are all of them unequal one to another ; then in such Case, it may be proper, 1st, To reduce the given Inches, &c. found in the Breadth and Thickness respectively, into the Decimal Parts of a Foot equivalent : Then, 2^{dly}, The Rectangle, or Product of those two Decimal Fractions, thus found, being multiplied by the given Length in Feet, &c. the last Product is decimally the Solid-content desired.

Example I.

There is a Piece of Timber or Stone whose Length is 15 Feet, but the Breadth and Depth are equal, and consequently, exactly square, each of them containing $17\frac{3}{4}$ Inches : To know the Solid-content.

	<i>Ft. Pts.</i>	<i>F.</i>	<i>I.</i>	<i>P.</i>
<i>Answer,</i> {	1. By the Table = 32.8193	= 32	: 9	: 10 <i>fers.</i>
	2. By the Pen = 31.8155	= 32	: 9	: 9 +

Example II.

Admit a Piece of Timber or Stone consists of four equal Sides, gradually tapering from the great End to the small ; and allowing the Side of the Base at the great End, to be $19\frac{3}{4}$ Inches, but that of the small End only, $7\frac{1}{4}$ Inches ; it now happeneth that the
Length

Length is 27 Feet : To know the Geometrical-mean, and consequently the Solid-content sought.

N. B. That the Geometrical-mean, or Side of the Square-equal is $11.966 = 11 : 11 : 7 +$, or 12 Inches fere.

Ft. Pts.

Answer, { 1. By the Table = 27.0000
2. By the Pen = 27. fere.

Example III.

Suppose a Piece of Timber or Stone is 35 Feet long, and the Breadth and Thickness so unequal, as the former to contain 18 Inches, and the latter but only 3 Inches : To know the Side of the Square-equal, and also the Solid-content.

		<i>In. Pts.</i>	<i>In. Pts.</i>
<i>Ans.</i> {	1. Side of the Square	7.34, = 8. or 7 : 5. fere.	
		<i>Ft. Pts.</i>	<i>Ft. In. Pts.</i>
	2. By the Table -	13.0935 = 13 : 1 : 1.	
	3. By the Pen - -	13.1250 = 13 : 1 : 6.	

Example IV.

There is a Piece of Timber, &c. representing the Frustum of a quadrangular Prism, whose Bases at each End are unequal in their Sides : Now, supposing the Breadth and Depth of their greater Base to be 14 Inches by 6, and that of the lesser to be 7 Inches by 5 ; also allowing the Length of the whole Piece to be 18 Feet : To know the Mean-area, and consequently, the Solid-content of the whole.

Sq. Ft. In. Pts.

Answer, { 1. The Mean-area = 4 : 6.22, &c.
2. Solid-content = 86 : 7.32, &c.

Example V.

There is a Tomb-stone of Marble, whose Length is $7\frac{1}{4}$ Feet, Breadth $5\frac{1}{4}$ Feet ; and Depth or Thickness $6\frac{1}{2}$ Inches : To know the Number of Solid Feet that are contained therein.

Ft. Pa. Ft. In. Pts. 3ds. 4ths.

Answer, $22.93635 = 22 : 0 : 5 : 2 : 9\frac{3}{4} +$.

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Expla-

Explanation.

1. In *Example 1.* The Breadth and Depth given being equal (*viz.* $17\frac{3}{4}$ Inch.) render the Question to be solved at once by the Table, or otherwise by the Pen, according to the first Article of the Rule, which see.

2. In *Example 2.* the Sides given at each End (*viz.* $19\frac{3}{4}$ and $7\frac{1}{2}$ Inch.) being unequal and the Piece gradually tapering; the Geometrical-mean, or the Side of the Square-equal, (*viz.* 11.11 Inches 7-3ds, &c. or 12 Inches *ferè*) being found in the Middle; and the given Length (*viz.* 27 Feet) will readily find the Solid-content sought, by either the Table or the Pen; according to the second Article of the Rule above.

3. In *Example 3.* as well as all others of the like kind, it may be observed that the given Breadth and Thickness (*viz.* 18 and 3 Inches) are unequal; therefore the Geometrical-mean between them, or Side of the Square-equal must first be found (*viz.* 7.34 or 7.5 Inches) in order to find, with the given Length (*viz.* 35 Feet) the Solidity desired by either the Pen or Table, as above.

4. In *Example 4.* the Bases at each End are both quadrangular and unequal; and consequently the Sides must be tapering; therefore the Mean-area, (*viz.* 4 Feet 6.22 Inches) must be found, in order to find with the given Length (*viz.* 18 Feet) the Solid-content desired, according to the fourth Article of the Rule above, which see.

5. In *Example 5.* all the three Dimensions given (*viz.* $7\frac{1}{2}$, $5\frac{1}{2}$, and $6\frac{1}{2}$ Inches) are unequal one to another, therefore the Solidity desired may be likewise accurately found, either by the Pen or Table, as above, according to the fifth Article of the Rule.

S E C T. VII.

AS the Author of the Treatise, entitled, *Surveying Improved, &c.* hath thought it indispensibly needful, to begin with the essential Parts of Decimal Arithmetic; so, by way of Digression, I think it necessary to complete the same, by annexing thereunto the Extraction of the Square-root, with all imaginable Plainness and Conciseness; in order to the exact and expeditious finding the Geometrical-mean, or Side of the Square-equal, between any two Dimensions given; by which Means, tedious and erroneous Tables, for finding the Superficies of

of Ends or Squares of unequal-sided Timber, &c. will thereby be avoided, by solving (occasionally) all manner of Questions of this Kind, with the utmost Accuracy and Expedition.

To extract the Square-root ; or the Method of finding decimally, the Geometrical-mean between any two Numbers or Dimensions given.

1. All square Numbers given to be extracted, are one of these three Species or Kinds, *viz.* single, compound, or irrational.

2. A single or simple square Number is composed or made up of nine Digits, whose Root is always one of them, and the Square thereof, never exceeds two Places at the most ; of which sort are those in the marginal Table, which (for greater Accuracy) ought to be retained in Memory.

T A B L E.

Roots.			Square.	
1	×	1	=	1
2	×	2	=	4
3	×	3	=	9
4	×	4	=	16
5	×	5	=	25
6	×	6	=	36
7	×	7	=	49
8	×	8	=	64
9	×	9	=	81

3. A compound square Number consists of three Places, at least, and is composed of more Figures than one ; that is, such whose Root is never less than two Places ; as 100, whose Root is 10 ; 121, whose Root is 11 ; or 144, whose Root is 12, &c.

4. An irrational or surd Number, is such, whose Square-root may be made to approach near the Truth, yet cannot be exactly discovered by Art, neither in whole Numbers or Fractions ; but something will still remain, there being no Proportion yet found between an irrational Number and its Root : Such Numbers are 3, 7, 19, 74, 156, 751, &c.

5. When any square Number, is required to be extracted, the Operation is not much unlike Division ; only there our Divisor is fixed ; here we are to look for a new one, for each Dividual or Resolvend.—Also, it is to be further observed, that the Root of a single or simple square Number, is found by Inspection only ; as may be seen in the foregoing Table ; but if it be a compound square Number, then may it be performed according to the general Rule following.

R U L E.

I. Let the compound Number given, whether rational or irrational, be placed with a crooked Line at the End thereof (as in common Division) to separate the Quotient or Root from the Square or Number given ; as well as a curved straight Line

on the Left-hand of each Resolvend to separate the Divisor therefrom.

II. In order to Operation, let the given Number be prepared by pointing thus : Make a Point above your Units Place, and omitting one, point every other Figure from the Right-hand to the Left ; and as many Points as your Number contains, so many Figures will your Root consist of.

III. Seek the greatest Root of your first Period on the Left-hand, which by your Table foregoing, or rather your Memory, may be readily found and placed in the Quotient ; and then the Square thereof being placed under the said Period, and deducted therefrom, whose Remainder is your first Work, and is no more to be repeated.

IV. To the last Remainder, bring down the Figures of your next Period, for a new Dividual or Resolvend.

V. Double your entire Quotient or Root, and place it for your first Divisor on the Left-hand of the Dividual or Resolvend, as in common Division.

VI. Ask how oft your Divisor can be had in all the Figures of your Dividual or Resolvend, excepting the Units Place thereof (which must always be reserved for the Square of the Figure sought :) And place your Answer in the Quotient, as well as the same Figure on the Right-hand of your Divisor as Part thereof : Then having multiplied the said Divisor by the Figure last placed in the Root or Quotient ; and subtracting the Product from your Dividual or Resolvend, discovers the Remainder ; to which bring down the Figures of your next Period, for a new Dividual or Resolvend, as before ; and so proceeding and repeating orderly the Directions of the 4th, 5th, and 6th Steps, until the Work is finished.

VII. When Extraction is finished, and there happens no Remainder, then it is a true Square or rational Number ; but if, to the contrary, a Remainder happen, then you may know it is irrational, and the Root cannot be got exact ; although you may come as near the Truth as you please, by continuing the Work, and annexing unto each Remainder a pair of Cyphers, for each decimal Place you intend in the Root.

VIII. For Proof, if the Figures in the Work stand regularly under one another, then you may set a Cross or Mark on the Left-hand of each Product, as well as at the last Remainder, if any happen, to distinguish them from others ; by reason whereof it will follow, that the Addition of all those Numbers so marked and distinguished, from the Right-hand to the Left (according to their Places and Columns) will readily revert in the Total, the given Number to be extracted, if the Work is right ; other-

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wife false.—Or Proof will likewise appear, by squaring or multiplying the Root, when found by itself, adding unto the Product the last Remainder, if any happen, the Sum will be equal to the Number given to be extracted, if right; otherwise not.

Example.

Suppose a Piece of Timber or Stone in a quadrangular Form, hath two of its Sides, or rather its Breath and Depth, so unequal, as the former contain $19\frac{3}{4}$ Inches, and the latter $7\frac{1}{4}$ Inches, whose Rectangle or Product is found decimally to be 143.1875 Square Inches: To know the Geometrical-mean, or Side of the Square equal thereto.

See the Work.

$$\begin{array}{r} \text{Divisors }) \quad 143.187500 \quad (11.966, \&c. = \text{Roots.} \\ \times 1 \end{array}$$

$$(1.) \quad 21 \quad) \quad 043 \quad = \text{Resolvend (1.)} \\ \times 21$$

$$(2.) \quad 229 \quad) \quad 2218 \quad = \text{Resolvend (2.)} \\ \times 2061$$

$$(3.) \quad 2386 \quad) \quad 15775 \quad = \text{Resolvend (3.)} \\ \times 14316$$

$$(4.) \quad 23926 \quad) \quad .145900 \quad = \text{Resolvend (4.)} \\ \times 143556 \quad +$$

Remains 2344

Proof, 143.187500 Total.

In. Pts. In. Pts. 3ds. 4ths. 5ths.

Answer, 11.966 r = 11 : 11 : 7 : 1 : 3 fere.

Explanation.

Let the given Number (*viz.* 143.187500) to be extracted, whether rational or irrational, be placed with a straight, or curved Line on its Left-hand, and a crooked Line on its Right, for the Reception of the Quotient or Root as in the Work.

on the Left-hand of each Resolvend to separate the Divisor therefrom.

II. In order to Operation, let the given Number be prepared by pointing thus : Make a Point above your Units Place, and omitting one, point every other Figure from the Right-hand to the Left ; and as many Points as your Number contains, so many Figures will your Root consist of.

III. Seek the greatest Root of your first Period on the Left-hand, which by your Table aforegoing, or rather your Memory, may be readily found and placed in the Quotient ; and then the Square thereof being placed under the said Period, and deducted therefrom, whose Remainder is your first Work, and is no more to be repeated.

IV. To the last Remainder, bring down the Figures of your next Period, for a new Dividual or Resolvend.

V. Double your entire Quotient or Root, and place it for your first Divisor on the Left-hand of the Dividual or Resolvend, as in common Division.

VI. Ask how oft your Divisor can be had in all the Figures of your Dividual or Resolvend, excepting the Units Place thereof (which must always be reserved for the Square of the Figure sought :) And place your Answer in the Quotient, as well as the same Figure on the Right-hand of your Divisor as Part thereof : Then having multiplied the said Divisor by the Figure last placed in the Root or Quotient ; and subtracting the Product from your Dividual or Resolvend, discovers the Remainder ; to which bring down the Figures of your next Period, for a new Dividual or Resolvend, as before ; and so proceeding and repeating orderly the Directions of the 4th, 5th, and 6th Steps, until the Work is finished.

VII. When Extraction is finished, and there happens no Remainder, then it is a true Square or rational Number ; but if, to the contrary, a Remainder happen, then you may know it is irrational, and the Root cannot be got exact ; although you may come as near the Truth as you please, by continuing the Work, and annexing unto each Remainder a pair of Cyphers, for each decimal Place you intend in the Root.

VIII. For Proof, if the Figures in the Work stand regularly under one another, then you may set a Cross or Mark on the Left-hand of each Product, as well as at the last Remainder, if any happen, to distinguish them from others ; by reason whereof it will follow, that the Addition of all those Numbers so marked and distinguished, from the Right-hand to the Left (according to their Places and Columns) will readily revert in the Total, the given Number to be extracted, if the Work is right ; other-

wise

wife false.—Or Proof will likewise appear, by squaring or multiplying the Root, when found by itself, adding unto the Product the last Remainder, if any happen, the Sum will be equal to the Number given to be extracted, if right; otherwise not.

Example.

Suppose a Piece of Timber or Stone in a quadrangular Form, hath two of its Sides, or rather its Breath and Depth, so unequal, as the former contain $19\frac{1}{4}$ Inches, and the latter $7\frac{1}{4}$ Inches, whose Rectangle or Product is found decimally to be 143.1875 Square Inches: To know the Geometrical-mean, or Side of the Square equal thereto.

See the Work.

$$\begin{array}{r} \text{Divisors }) \quad 143.187500 \quad (11.966, \&c. = \text{Roots.} \\ \times 1 \end{array}$$

$$(1.) \quad 21 \quad) \quad 043 \quad = \text{Resolvend (1).}$$

$$\times 21$$

$$(2.) \quad 229 \quad) \quad 2218 \quad = \text{Resolvend (2).}$$

$$\times 2061$$

$$(3.) \quad 2386 \quad) \quad 15775 \quad = \text{Resolvend (3).}$$

$$\times 14316$$

$$(4.) \quad 23926 \quad) \quad .145900 \quad = \text{Resolvend (4).}$$

$$\times 143556 \quad +$$

Remains 2344

Proof, 143.187500 Total.

In. Pts. In. Pts. 3ds. 4ths. 5ths.

Answer, 11.966 r = 11 : 11 : 7 : 1 : 3 fore.

Explanation.

Let the given Number (*viz.* 143.187500) to be extracted, whether rational or irrational, be placed with a straight, or curved Line on its Left-hand, and a crooked Line on its Right, for the Reception of the Quotient or Root as in the Work.

M m 4

2. Then

2. Then having pointed out or prepared the same by Periods, according to the second Step of the Rule, the Work shews, that the Root or Quotient will have as many Places as there are Points, viz. 5.

3. Seek the greatest Root of your first Period on the Left-hand (viz. 1.) which, either mentally, or by the marginal Table aforegoing, you will likewise find to be 1; which place in your Quotient; and the Square thereof (viz. 1.) under your said Period (viz. 1.) and subtracted therefrom, there remains 0. — This is your first Work, and is no more to be repeated.

4. To the Remainder (viz. 0.) bring down your next Period (viz. 43.) make a new Dividual or Resolvend, as in the Work.

5. Double your Quotient (viz. 1.) makes 2 for a Divisor; then seek how often 2 in 43 (reserving always the Units Place for the Square of the Figure sought.) *Answ.* Once: Then 1 being put in the Quotient, as well as the same on the Right-hand of the Divisor, as Part thereof, making 21; then multiplying 21 by 1, and placing the Product thereof, (viz. 21. under the said Dividual or Resolvend, which work is to be as often repeated, as there are Periods to bring down; according to the 6th Step of the Rule.

6. Subtract 21 from 43, rest 22, to which bring down the third Period, make 2218, for a new Dividual or Resolvend; as you may see in the Work itself.

7. Double the Quotient (viz. 1.) makes 22 for a new Divisor, then I ask, How oft 22 in 2218? (still reserving the Units Place in the Resolvend.) *Answ.* 9 times; which being placed in the Quotient, and likewise on the Right-hand of the Divisor making it 229: Then multiplying 229 by 9, the Product (viz. 2061) being placed under the Dividual or Resolvend, and subtracted therefrom, there remains 157; to which bring down the Figures in the 4th Period (viz. 75.) make 15775, for the 3d Dividual or Resolvend, as you may see in the Work.

8. Double the Quotient (viz. 119.) make 238, for a new Divisor, then ask, How oft 238 in 15775, still the Units Place in the Resolvend being reserved) or, more briefly, How oft 2 in 15? *Answ.* 6 times; which being placed in the Quotient, as well as on the Right-hand of the Divisor, making it 2386: Then multiplying 2386 by 6, the Product (viz. 14316) being placed under the last Dividual or Resolvend (viz. 15775) and deducted therefrom, there remains 1459; to which bring down the 5th or last Period (viz. 00) make 145900, for the 4th or last Resolvend.

9. Then the Quotient (viz. 1196) being doubled, make 2392 for a new Divisor: Then ask, How oft 2392 may be had in

145900

145900 (still reserving the Units Place of the Resolvend, as before) or which is the same, How oft 2 in 14? *Answ.* 6 times; which being placed in the Quotient, as well as on the Right-hand of the Divisor, making it 23926: Then multiplying 23926 by 6, the Product (*viz.* 143556) being placed under the last Dividual or Resolvend (*viz.* 145900) and deducted therefrom, there remains 2344; which indicates that the Number given to be extracted, is a surd or irrational Number; and the Root thereof somewhat imperfect, seeing it is near equal to the Side of the Square sought; as by the Work appears.

10. For Proof; let only the Products and last Remainder of the Operation, be particularly distinguished from the rest of the Figures, by some certain Sign or Character (namely $\times$, as above) and then added together in the same Order as they stand; their Sum, for Proof, will be equal to the Number given to be extracted; as in the Work; according to the 8th Step of the Rule, which see.

N. B. The Work and Explanation of the last Example being duly observed, all others may likewise be as truly performed, regard being only had to the particular Steps of the Rule and its Application.

OBSERVATIONS.

I. The Reader may observe, that if Decimal Places in the Number given to be extracted happen to be odd: then, in order to Extraction, must one Cypher be annexed, in order to make them even; by which means, the Prick or Point will always happen upon the Units Place of the integral Part, and the rest proceeding orderly from the Right-hand to the Left, as above.

II. In all surd or irrational Numbers given to be extracted, for each Pair of Cyphers or decimal Places found therein, you will likewise find so many single Figures contained in the fractional Part of the Root: and tho' the more they are in Number the greater Exactness, yet only two or three of them are esteemed sufficiently exact in the way of Business.

III. When the Square-root or Side of the Square-equal, is found (as above) for either round or square Timber, &c. the arithmetical Performances may be accurately effected by *Chap. II. Sect. 6.* which see.

S E C T. VIII.

To find readily by Inspection, or otherwise, the Inches, Parts, &c. contained in the decimal Parts of a Solid Foot. Or, which is the same, to convert or change the one into the other alternately, with all imaginable Exactness and Accuracy.

R U L E.

1. **T**HE decimal Parts of a Solid Foot may be readily valued, converted, or changed into duodecimal Terms of Inches, Parts, &c. by multiplying orderly the decimal Fraction given, by 12, after the concisest way, and that Product again by 12, if need be; cutting off from the Right-hand of each Product (by a Point or Comma) so many decimal Places as are given in the Multiplicand; then will it follow, that the Increase or overplus Figure or Figures, happening on the Left-hand of the Point, after the decimal Places are so cut off, will be orderly the Inches, Parts, &c. sought; as in the following Example, and its Explanation.

2. When the Value of the decimal Parts of a Foot-solid is required by Inspection only, you are first, for every half Foot, or five Tenths (*viz.* 5.) to reckon 6 Inches; and for every one Tenth (*viz.* 1.) superior or inferior to the same, you are to reckon so many Inches and Quarter of Inches, each Quarter of an Inch containing 3 Parts. *2dly*, For every 2, in the second Place, add 1 Quarter or 3 Parts of an Inch: And this will not err a Quarter of an Inch in the Generality of Decimals, if in any.

3. Having solid Inches given, they may immediately be converted or changed into decimal Parts equivalent, by annexing to their Right-hand 3 or 4 Cyphers, and dividing that Number, mentally, by 12; the Quotient (after a Point is prefixed on its Left-hand) is the decimal Parts sought.—*2dly*, When solid Inches and Parts are given duodecimally, they may likewise be accurately reduced into decimal Parts equivalent, by multiplying the Inches given by 12; taking in the Parts; and (after Cyphers are annexed to their Right-hand, as before) dividing that Number by 144, the Quotient with a Point on its Left-hand, is the decimal Parts desired.—*3dly*, When there are three Names given duodecimally, *viz.* Inches, Parts, and Thirds, then having reduced them into the least Name mentioned (*viz.* Thirds)

Thirde) as before; and annexing thereto Cyphers, as above; and dividing that Number by 1728, the Quotient (with the Point prefixed) will be the decimal Parts required.

Example I.

A square Piece of Timber or Stone, whose Solid-content is (decimally) 35.9375 Feet: To know duodecimally the Solidity in Feet, Inches, and Parts, by the Pen, as well as by Inspection.

First, By the Pen.

	<i>Ft. Pts.</i>
	35.9375
	× 12
Inch.	11.2500
	× 12
Parts	3.0000

Secondly, By Inspection.

<i>Ft. Pts.</i>		<i>Ft. In. Pts.</i>
1 st , 35.0	- - - is	35 : 0 : 0
2 ^{dly} , .5	according to the Rule, is	0 : 6 : 0
3 ^{dly} , .4	is 4 <i>Inch.</i> 4 <i>grs.</i> or	0 : 5 : 0
4 ^{thly} , .03	<i>℥c.</i> is 1 <i>gr.</i> or	0 : 0 : 3
Sum 35.93 <i>℥c.</i> =		Sum 35 : 11 : 3
	<i>Ft. In. Pts.</i>	
<i>Ans.</i>	35 : 11 : 3	

Explanation.

1. In the Example above you may observe, that the decimal Parts of a Solid-foot (*viz.* 9375) being given, they may (by the Pen) be accurately reduced into Inches, by multiplying them concisely in one Line, by 12; and cutting off from the Right-hand of the Product by a Point or Comma, so many decimal Places as are found in in the Multiplicand (*viz.* 4) then it follows, that the Figures on the Left-hand of the Point are Inches (*viz.* 11.) 2^{dly}, The four decimal Places remaining on the Right-hand of the Point (*viz.* 2500) being again multiplied by 12, as before; and the same Number of Places (*viz.* 4.) being cut off, there appears on the Left-hand of the Point 3 Parts

Parts more. Hence you may infer, that the exact Value of the decimal Parts given, are 11 Inches 3 Parts—The like is to be observed in all other Examples of this kind, according to the 1st Article of the Rule, which see.

2. By Inspection only, from the same Example, it appears, that the Solid-feet given, are the same without Alteration. 2dly, The Primes or Tens in the given Decimal (*viz.* 9) being valued according to the Rule, imply, that there are always 6 Inches to be reckoned for five Tenths (*viz.* .5) and 4 Inches 4 qrs. more, or 5 Inches, for the four Tenths (*viz.* .4) 3dly, The Figure in the 2d Place of the decimal Parts, towards the Left-hand (*viz.* .03) imply, that for its Value there must be reckoned 1 qr. of an Inch more, or 3 Parts; to be added to the rest; which make in the whole 11.3 Inches, for the complete Value of the decimal Parts given (*viz.* .9375) as before: See the second Article of the Rule.

Example II.

There is a Piece of Timber or Stone, whose Solid-content is found duodecimally to be 25 Feet, 5 Inches, 8 Parts, and 6 Thirds: To know the Solidity of the same decimally.

Et. Pts.

Answ. 25.47569 +.

Explanation.

According to the latter Part of the 3d Article of the Rule, let all the inferior Denominations given (*viz.* 5 Inches, 8 Parts, 6 Thirds) be reduced into the least Name mentioned (*viz.* Thirds) their Sum will be 822 Thirds; to which annex 5 Cyphers, the Number will be 822.00000; which being divided by 1728, the Quotient, after a Point prefixed (*viz.* .47569) is the decimal Parts sought; and consequently, equal to 5 Inches 8 Parts, and 6 Thirds; as above.

OBSERVATIONS

1. In valuing the decimal Parts of a Solid-foot, or finding duodecimally, the Number of Solid-inches, Parts, &c. that may be equal unto the decimal Fraction given, it oftentimes happens, that the first Figure on the Left-hand of the last decimal Fraction, after the Inches, Parts, &c. are orderly found, amount to either the Figure of 8 or 9; then, in such Case, the Parts or Thirds, or any other Name that happens to be last found, must be augmented or increased with Unity (*viz.* 1.) that is, termed one more than it appears to be.

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2. In the 2d Article of the Rule you are likewise to observe, that when the Figure of the decimal Parts in the 2d Place, happen to be even, you are always to take it just half, for so many Quarters, or 3 Parts of an Inch; but when otherwise odd, you may omit one (*viz.* 1) and take half of the even Number remaining, according to the Rule.

3. Hence it may be observed, that the immediate writing down any Inches and Quarters, or Parts of an Inch, in the Decimal of a Foot-solid, is done so easily by the Rule above, that there need no other Example for Illustration.

S E C T. IX.

To find readily from the Table, by Inspection only the solid Content of any square or round Piece of Timber, &c. for any Girt or Circumference from 2 to 8 Feet inclusively, whose 4th Part in Inches, &c. (according to Custom) is the Side of the Square-equal; and consequently, a proper Dimension, agreeable to the given Length in Feet and fractional Parts of a Foot.

R U L E.

1. **T**HE Mean-girt or Circumference being found, as before directed, let always one fourth Part of the same be taken in Inches, &c. which being found on the Top of the Table in its proper Column, guide your Eye down the first Column on the Left-hand, until you likewise find the given Length in Feet; then directly forward from thence to the Right-hand under the Title of the given Side in Inches, you will find in the Angle of meeting, the Solidity desired in Feet and decimal Parts, which may immediately be converted into solid Inches, &c. by *Sett.* VIII. aforegoing.

2. When it happens that the given Length in Feet, found in the first Column on the Left-hand, is any Number under 40, and the given Side in Inches, &c. found on the Top of the Table, in any Number from 6 to 24 inclusively, then the Solid-content is found at once by Inspection only; but when on the contrary, the given Length in Feet happen to be any Number superior to 40; then adding unto the Solid-content first found, the Solidity
for

for the Number of Feet that is wanting in Length, their Sum for the same Side of the Square given in Inches, &c. is the complete Solidity desired.

3. If the Solid-content is required for any Number of Feet in Length, with the fractional Part of a Foot; then having found the Solidity for the integral Part, according to the 1st and 2d Articles of the Rule above, let the Solid-content for 1 Foot, according to the given Side, be multiplied by the Numerator of the given Fraction in its least Term, and the Product divided by the Denominator, the Quotient is decimally the Solidity for the fractional Part sought; which being added unto the former, their Sum is the complete Solidity desired. See Example 4. and its Explanation.

Example I.

There is a Piece of Timber or Stone 40 Feet long, and the Side of the Square is $19\frac{3}{4}$ Inches: What is the Solid-content of it?

$$\text{Answ. } 108.3500 = 108 : 4 : 2 : 5, \text{ \&c.}$$

Ft. Pts. Ft. In. Pts. 3ds.

Example II.

Admit a square or round Piece of Timber, &c. hath for its Length 74 Feet; and the Side of the Square, equal to one-fourth Part of its Girt, is $21\frac{1}{4}$ Inches: To know the Solid-content.

$$\text{Answ. } 232.053 = 232 : 0 : 7 : 8$$

Ft. Pts. Ft. In. Pts. 3ds.

Example III.

How many Solid-feet is contained in that round Piece of Timber or Stone, whose Length is 100 Feet, and the Side of the Square-equal only $7\frac{1}{2}$ Inches?

$$\text{Answ. } 39.067 = 39 : 0 : 9 : 8$$

Ft. Pts. Ft. In. Pts. 3ds.

Example IV.

Suppose a Piece of Timber or Stone hath for its Length $17\frac{3}{4}$ Feet, and the Side of the Square-equal $23\frac{3}{4}$ Inches: To know the Solid-content.

$$\text{Answ. } 69.0397 = 69 : 0 : 5 : 8$$

Ft. Pts. Ft. In. Pt. 3ds.

Explanation.

In *Example 1.* you are to look on the Top of the Table for the Side of the Square, equal to $\frac{1}{4}$ of the Girt or Circumference; and

and keep your Eye down the first Column on the Left-hand, till you find the given Length of the Piece in Feet (*viz.* 40) and over-against it in the Angle of meeting, stands the Content of the Piece sought in Feet and decimal Parts, *viz.* 108.3500, which being valued by Inspection, according to *Set*, VIII. it will be duodecimally 108 Feet, 4.2 Inches, 5 Thirds, &c.

2. In *Example 2.* there is given for the Mean-girt or Side of the Square-equal $21\frac{1}{2}$ Inches, and 74 Feet for the complete Length: Now as the Table extends no farther than only 40 Feet, you are first to find the Content for the said Number (*viz.* 40 Feet) whose Solidity is 125.4340 Feet; and then afterwards for the remaining Part of 74 Feet, the given Length (*viz.* 34 Feet) whose Solidity is 106.6189 Feet: Hence the complete Solidity sought is found by Addition to be 232.0529 Feet; whose Value duodecimally, by Inspection only, is 232 Feet, 7 Inches, 8 Thirds. See the Work below, as well as the 1st and 2d Articles of the Rule above

	<i>Ft.</i>	<i>Ft. Pts.</i>	
Given Length {	40 =	125.4340	} Solidity.
	34 =	106.6189	
Sum or Answer	74 =	232.0529	

3. In *Example 3.* you have given for the Side of the Square-equal to $7\frac{1}{2}$ Inches; which being found on the Top of the Table, as before; and the 40 Feet being Part of the given Length on the 1st Column on the Left-hand, the Solidity for the same is found to be 15.627 Feet; which being doubled, or rather twice set down, with the Solidity of 20 Feet more, found in the same Column; because twice 40, with the Addition of 20 more, make just 100 Feet, the given Length; the Sum of those Particulars in the whole, is 39.0675 Feet for the Solid-content desired.

Given Length {	40 =	15.6270	} Solidity.
	40 =	15.6270	
	20 =	7.8130	
Sum or Answer	= 100 =	39.0670 =	<i>Ft. In. Pts. 3ds.</i> 39 : 0 : 9 : 8

4. In *Example 4.* there is given for the Side of the Square $23\frac{1}{2}$ Inches, whose Solidity is required for $17\frac{5}{8}$ Feet long; which by the Table may be accurately found, by finding, *1st*, The Solidity for the integral Part (*viz.* 17 Feet) the Content for the same will be 66.5915 Feet; to which, *2dly*, There must be added $\frac{5}{8}$ of one Solid-foot, for the same Side of the Square (*viz.* 3.9171 Feet) which being multiplied by the given Numerator (*viz.* 5) the Product is 19.5855; which being divided

vided by the given Denominator (*viz.* 8) the Quotient is 2.4482, the Solidity sought for $\frac{1}{8}$ of one Solid-Foot, according to the same Dimensions given; which being added to the integral Part, *viz.* 66.5915 Feet) found as above their Sum, (*viz.* 69.0397) is the complete Solidity desired. See the 3d Article of the Rule above and the Work which follows.

	<i>Ft.</i>	<i>Ft. Pts.</i>	
Given Length	17	= 66.5915	} Solidity.
	$\frac{1}{8}$ of 1	= 2.4482	
Sum or Answer	= 17 $\frac{1}{8}$	= 96.0397	<i>Ft. In. Pts. 3ds.</i> = 69 : 0 : 5 : 8

S E C T. X.

To extend the Table, and render it useful in finding readily Solidities for all other Dimensions whatsoever, beyond those in the Table already expressed and discovered.

R U L E.

1. **W**HEN it happens that a 4th Part of the Mean-girt or Circumference given, commonly termed the Side of the Square-equal, happen to exceed 24 Inches, the last tabular Dimension; then, in this Case, it is needful to bring it within the Limits or Compass of the Table, by dividing the given Number in Inches and Quarter-inches, by 2; or otherwise, by 4; until the Quotient appears to be less than 24 Inches, and consequently, within the Limits of the Table; which being found on the Top thereof, and the given Length in Feet, in the first Column of the Left-hand; then, in the Angle of meeting, you will find the Solidity for the said Quotient or Dimension given; which being multiplied by the Square of that Number you divided by, (*viz.* 4 or 16) the Product is (decimally) the true Solidity sought; which, by Inspection only, may be duodecimally expressed in Feet, Inches, Parts, &c. according to *Sett.* 8. which see.

2. It sometimes happens that a Division of the 4th Part of the Girt given, or Side of the Square-equal by either 2 or 4, will not bring the Number of Inches and Quarter-inches, within the Limits or Compass of the Table; then, in this Case, it is needful to divide the same by 8; in order to find in the Quotient, the 8th Part of the given Side in Inches, &c. whose Solidity for the given Length in Feet, &c. being found in the Table

as above directed; let it be multiplied by the Square of 8, the given Divisor (*viz.* 64) the Product is likewise (decimally) the true Solidity sought the customary way, as above.

3. When it happens that the Side of the Square given, is any Number less than 6 Inches, or half a Foot, and consequently, not comprehended in the Table; then, in this Case, the Solidity may be accurately found for any Number of Feet in Length, &c. by multiplying always the decimal Parts of a Foot corresponding to 1 Solid-inch (*viz.* .0069444, &c. by the Square of the given Side in Inch and Quarter Inches, and that Product again by the given Length in Feet, &c.) the last Product is (decimally) the Solid-content sought, which may immediately be expressed duodecimally, in Feet, Inches, Parts, &c. according to *Set.* 8. which see.

Example I.

To know the Solidity of that Piece of Timber or Stone, whose 4th Part of its Mean-girt or Circumference, commonly termed the Side of the Square-equal, is 54 Inches, and its Length 37 Feet

$$\text{Answ. } 1012.5200 = 1012 : 6 : 3 \text{ fere}$$

Example II.

Admit a square or round Piece of Timber or Stone hath for its Length 37 Feet, and the Side of the Square-equal 76 Inches: What Quantity of solid Timber or Stone does that Piece contain?

$$\text{Answ. } 1484.0995 = 1484 : 1 : 3 +$$

Example III.

Let the Mean-girth of a Piece of Timber be 33 Feet 4 Inches; $\frac{1}{2}$ thereof is 100 Inches for the Side of the Square; and let the Length of the Piece be 84 Feet: How much solid Timber doth this Piece contain?

$$\text{Answ. } 5833.4728 = 5833 : 5 : 8 : 0$$

Example IV.

The Length of a Piece of Timber is 100 Feet long, and the Side of the Square $3\frac{1}{4}$ Inches: What is the Solid-content?

$$\text{Answ. } 7.335 = 7 : 4 : 0 : 3$$

N n

Ex-

Explanation.

1. In *Example 1.* the 4th Part of the given Girth or Side of the Square-equal, is 54 Inches; which is a Dimension so great, that one half thereof (*viz.* 27 Inches) cannot be had in the Table; therefore, to bring it within the Compass of the tabulary Dimensions, I take only the 4th Part (*viz.* $13\frac{1}{2}$ Inches) whose Solidity for the given Length (*viz.* 50 Feet) is found by the Table to be 63.2825 Feet; which being multiplied by 16, the Square of the Divisor (*viz.* 4) the Product (*viz.* 1012.5200 Feet) is the Solidity desired. See the 1st Article of the Rule above.

2. In *Example 2.* the Mean-girth, or Circumference given, appears to be 25 Feet 4 Inches; whose 4th Part, or Side of the Square-equal is 6 Feet 4 Inches, or 76 Inches: Now seeing the 4th Part of 76 Inches is 19; which being found on the Top of the Table and the given Length in Feet (*viz.* 37) on the Left-hand, you will find in the Angle of Meeting the Solidity agreeable to the same (*viz.* 92.7562) which being multiplied by 16, the Square of the given Divisor (*viz.* 4) the Product is 1484.099, for the complete Solidity sought, as above.

3. In *Example 3.* there is given for the Side of the Square-equal 8 Feet 4 Inches, or 100 Inches. Now seeing this Dimension is so very great, that neither its Half, nor 4th Part can be had in the Table; therefore, according to the 2d Article of the Rule its 8th Part must be taken (*viz.* $12\frac{1}{2}$ Inches) which being found on the Top of the Table, as before, and underneath in the same Column, and opposite to the given Length (*viz.* 84 Feet) on the Left-hand, is the Sum of the particular Solidities (*viz.* 91.148) according to the Method prescribed in the 2d Article of that Rule in *Sec.* 9. which being multiplied by 64, the Square of the given Divisor (*viz.* 8) the Product is decimally the complete Solidity desired (*viz.* 5833.4720 Feet) or duodecimally, 5833 Feet, 5.8 Inches, *ferè*, &c. as above.

4. In *Example 4.* there is given for the 4th Part of the Mean-girth, or Circumference, known also by the Name of the Side of the Square-equal, only $3\frac{1}{4}$ Inches, (which is less than 6 Inches, the 1st tabulary Dimension: Therefore, I multiply decimally the Square of 325 Inches) *viz.* 10.5625 Inches) by the given Length in Feet (*viz.* 100) the Product is 1056.2500 Feet; which being multiplied again by the fixed Number, being the decimal Parts of a Foot, corresponding to one Solid-inch (*viz.* .0069444, &c.) the last Product (*viz.* 7.335 Feet, &c.) is the true Solidity sought, whose Value, being expressed duodecimally

is 7 Feet, 4 Inches, 3 Thirds. In like manner, may all other Examples be performed, whose given Side is less than 6 Inches. See the third Article of the Rule above.

OBSERVATIONS.

I. The Reader may observe, that this *New Treatise of Solids* is not only concise, but even conspicuously remarkable for its Variety; seeing it shews the Performance of Questions five manner of Ways, viz. 1st. Vulgarly; 2^{dly}, Decimally; 3^{dly}, Duodecimally; 4^{thly}, Instrumentally; and 5^{thly}, Tabularly, or by Inspection: All of them agreeable to one another; in solving and proving Examples, &c. with all imaginable Plainness and Exactness.

III. It may be also observed, that both the true and customary way of measuring Timber and Stone, is likewise discussed and illustrated by proper Rules, Examples, Explanations, &c. with a concise Method of changing alternately, the one into the other, by Analogy, or otherwise: As also, Reasons why the customary way of measuring, though less exact, is more practically used than the other.

III. The Table and Rules preceding, are so very excellent in their Use and Application, that they discover accurately, Solidities for all manner of Dimensions whatsoever, whether they be greater or less than those exhibited in the Table foregoing; which had been farther confirmed by Geometrical Reason and Demonstration, if Space and Time had so permitted.

IV. If the Smallness of each Column in the Table had but admitted of two or three decimal Places more in the Calculation; then the Solution of each Example had attained to the greatest Perfection imaginable, especially in all commensurable Quantities or Dimensions given; but, seeing otherwise, the Calculation being confined to four decimal Places only, the Table, notwithstanding that Defect, will readily and accurately discover Solidities in the way of Business, with all imaginable Plainness and Facility, without the Trouble of those large and voluminous Tables now extant; whose tedious Calculations duodecimally, in their many Degrees, Terms, and Denominations, serve rather to amuse than instruct.

S E C T. IX.

How the young Surveyor may methodically reduce his Solidities, &c. into a Scheme, in order to prove and compare them with others.

LET the young Surveyor set down their Number and Dimensions, as in the following Columns, and then find their Contents, as already taught; and set that down against each Tree, as under, according to the following Scheme.

The S C H E M E.						
No. <i>Trees</i>	Side Sq. <i>Inches.</i>	Length <i>Feet.</i>	By this New Method. <i>Ft. Pts.</i>	By Mr. Hoppus. <i>Ft. In. Pts.</i>	By Mr. Worgan. <i>Ft. I. P. 3ds.</i>	
1	9	10	5 6250	5 7 6	5	7 6 0
2	10 $\frac{1}{2}$	14	10.7191	10 8 7	10	8 7 6
3	15	11	17.1875	17 2 3	17	2 3 0
4	20 $\frac{1}{2}$	16	46.6952	46 8 4	46	8 4 0
5	16	24	42.6660	42 8 0	42	8 0 0
6	16	23	40.8882	40 10 8	40	10 8 0
7	36	18	162.0000	162 0 0	162	0 0 0
8	26	28	131.4492	131 5 4	131	5 4 0
9	31	17	113.4544	113 5 5	113	5 5 0
10	35	30	255.2100	255 2 6	255	2 6 0
Total			825.8946	825 10 7	825	10 7 6
			Or,			
			F. I. P. 3ds.			
			825:10:8:9			

N. B. The Reader, upon due Examination, will find the particular Contents of this Scheme, so agreeable to the Tables themselves from which they are abstracted, as well as the Performance by the Pen only, that the Difference between one another, is within less than 10000 Parts of an Inch; which is so inconsiderable in the way of Business, that it is not worthy Observation.

S E C T. XII.

Animadversions upon the whole ; shewing, briefly, the Nature and Quality of the Subject : Recommended to the Public for the due Encouragement of Authors, according to their several Abilities and Qualifications.

IN perusing several Treatises upon Solids, tabularly performed, particularly those of Mr. Hoppus and Mr. Worgan, I find that neither of them are unworthy of Esteem, with regard to their several Methods, &c. but their endeavouring to raise themselves a Name or Character, by way of Reflection or Disparagement, seem to me, no ways honourable ; especially when we consider, that no Person professing any of the liberal Arts and Sciences, and appearing in Print, ought to be slighted for his Labours and good Intention to the Public, provided his Works appear to be truly genuine ; though, perhaps, not with that Fineness of Stile and Aptness of Expression as some others ; yet, nevertheless, I say, this Author may justly claim from the Public, not only the Fruit of his Labours, but even also due Esteem and Respect, seeing no man can act beyond the Talent he possesses.

From these Considerations, we may rationally infer, that when Epistles to Books of this Nature consist of cringing Apologies, and frivolous Excuses, they are so far from augmenting the Credit and Worth of the Author, that they are rather the means of diminishing his Character ; seeing, in the Judgment of the Learned, nothing can appear more absurd, impertinent, and useless, than Self-encomiums of this kind, to any thing that is valuable : For if the Author has inserted nothing but what is true and genuine, and has advanced something new, it cannot be denied, but that the Obligation is due to him from the Public, and not the contrary, as such Epistles seem to imply. I am sure, in all Cases of personal Information, the Learner always looks on himself obliged to the Teacher, not the Teacher or Tutor to the Learner ; and why this Method of conveying useful Knowledge by Books, should not be the same, is altogether unaccountable ; especially if it be considered, that the communicating any Secret by the Press, is a way abundantly more generous ; because more universally useful, not only to the Author's most intimate Friends, but even likewise to many others, that are entirely remote from his Knowledge or Acquaintance.

I know

I know in most *Prefaces* or *Introductions*, it is not only usual, but even sometimes needful, for the Author to give particular Descriptions of the Nature and Quality of the Subject; yet, nevertheless, this may be done without Self-commendation, &c. because the Work itself, when evidently supported with Truth, Reason, and Demonstration, cannot miss of its due and just Esteem from all impartial and judicious Readers; but when otherwise, it is in vain to expect it by flattering Insinuations, by reason the Reader cannot be supposed to suspend his Judgment, and believe that Truth and Falshood are agreeable in their Nature; or that which is erroneous and imperfect is directly otherwise, merely because the Author's *Preface* intreats that Favour: Much less can the greatest of Performances be honourably supported by flandering or defaming those of others; seeing he must needs be foolish that brags much, either what he will do, or what he has done: For if what he speaks of fall not out accordingly, then will the world mock him with Derision and Scorn; but if otherwise, he shall at best be reputed a vain-glorious Boaster and unwise; besides, it is intirely needless, because every Work proves itself effectually, to be either worthy or unworthy.

As concerning the Errors of the Press, there are few Books wholly exempt from them, in a greater or less Degree; and an Author has hard Measure, if his Work is condemned on their Account; but where there are no other Faults, there are some (judicious Readers, as they fancy themselves) who for the sake of the Deficiency or Superfluity of a Letter, Figure, or Comma, will (like hungry Curs over their Viands) snarl; though at the same time, they are replenishing their empty Pericranies, with the plentiful Crops of the Author's Industry, which they are scarce apprehensive enough to understand, much less to correct or amend.

I am also apprehensive, I shall be blamed by some for prescribing to the Practitioner, the Knowledge and Use, not only of the Decimal Table, newly calculated, but even also, certain Parts of Vulgar and Decimal Arithmetic; especially, so far as only the four initiating Rules, with the Extraction of the Square-root, as the best Introduction to Subjects of this kind; seeing, when once learned, the young Surveyor may proceed with Boldness, Certainty, and Expedition, when he is thereby enabled to examine and correct Tables of this kind, as above, by reason they may be subject to the Errors of the Press, as before hinted; which seem to be the only Cause, why the ingenious Author of the Book, entitled, *Surveying Improved*, &c. has begun that Essay with a short System of Arithmetic; to which is here annexed the Extraction of the Square-root, and its Use,

in finding accurately (without the Trouble of tedious Tables, &c.) the Superficies of Ends or Squares of Timber, &c. whose Sides are unequal; by which means, the Geometrical-mean between any two Sides, Ends, or Bases given, may be found with less Trouble and more certainty, than by any duodecimal Table now extant.

As concerning Objections being made against this new Table, decimally performed, they can be of no Validity; seeing its Veracity is undeniably evident, and its Construction, as well as Application, supported with the greatest Reason and Demonstration; which shews that Solidities of this kind, may be readily found with all imaginable Exactness and Accuracy, as well as Plainness and Facility; seeing the decimal Parts of any Solid-content may be immediately expressed duodecimally, in Inches, Parts, &c. by Inspection only; and, as such, must be abundantly preferable to Duodecimal Calculations, which contain so many troublesome Names or Denominations; especially, when we consider that there is not, nor can be any Fraction invented so easy to work as the Decimal, because so like an intire Number, for there is the same Increase of the Places and Values in these, as in Integers, appearing even from the Nature of the Table itself.

In brief, though I believe this Essay to be the only one of this kind extant, with regard to its Extensiveness and Completeness, as well as Variety and Conciseness; yet I shall readily yield the Laurel to any Author, that shall happen to improve the same, by something more excellent of his own; seeing Emulation in Arts and Sciences is no ways to be contemned, when they generously and honourably act, without Slight and Disparagement to any of their Inferiors, who may happily give them an Idea or Apprehension of the same: Therefore, hope that this (at least) may be the means to incite, without Slight or Reflection, a more ingenious Pen, in order to the farther cultivating and improving so useful a Part of Learning.

Having thus far, for the Satisfaction of my Reader, shewn the Nature and Quality of this Subject, with regard to the essential Parts, I shall now intirely leave it to the Censure of the Public; not doubting, but if worthy, it will meet with the Approbation of the Impartial and Judicious; but if otherwise, neither of them nor I can reasonably be its Advocate: However, I shall never be inclinable to build myself a Reputation on the Ruin of others; seeing a Jewel of an intrinsic Value, after Probation, will naturally and advantageously shew itself.

To

To conclude, if this new and concise Table, with its Use and Application, illustrated by Propositions, Rules, Examples, &c. should happily meet with due Encouragement from the Public, it will be no small Satisfaction to the Author hereof, who (next unto Heaven) aims principally at the Promotion of Trade and Commerce, as well as that of Arts and Sciences, according to the utmost Ability of,

Yours, &c.

W. HUME.



THE
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SUPPLEMENT,
RELATING TO
TIMBER MEASURE, &c.

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